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A TENSORIAL MODELLING OF THE 3D INFLUENCE OF THE FIXTURE ERRORS ON THE ORIENTATIONAL GEOMETRIC SPECIFICATIONS

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Abstract:
In this work, we present a model of a three-dimensional manufacturing tolerancement by the use of a tensorial approach. We are interested in the study of the influence of the fixture errors of a workpiece on a machined surface. The orientation variation of a workpiece is caused by the fixture errors. Consequently three-dimensional geometric defaults are generated when the workpiece is machined. Using a tensorial approach, these defaults are modelled by the positional variations of a set of surface points.

By a numerical simulation, we validate the developed model and show the influence of the fixture errors on the geometric orientation specifications. We also show that the choice of the fixture location can be verified by the 3D influence of the geometric fixture errors on the generated defaults of machined surfaces.

Key Words: Modelling, Three-dimensional, Defaults, Tolerancing, fixture

1. INTRODUCTION

When a workpiece is fixtured for a machining or inspection operation, the accuracy of an operation is mainly determined by the efficiency of the fixturing method. Variability in manufactured workpiece is hardly inevitable. When such variability is found at contact areas between the workpiece and the fixture errors in location are expected. The errors will affect quality of features to be produced. Shawki and Abdel-Aal [1] experimentally studied the impact of fixture wear on the positional accuracy of a workpiece. Asada and By [2] performed the kinematical modelling, analysis, and characterization of adaptable fixturing. Screw theory was developed as an attempt to estimate locating accuracy under a rigid body assumption (Ohwovoriole and Roth [3]). Weill et al.[4] have developed optimization approaches to minimize positional errors. Rong and Bai [5] verified fixture locating schemes by considering machining accuracy. Cai et al. [6] developed a variational method to conduct robust fixture design to minimize the workpiece positional errors. There were also algorithms to predict a deviation of a prismatic workpiece located by 3-2-1 fixturing method (Salisbury and Peters [7]). Choudhuri and De Meter [8] considered the contact geometry between the locators and workpiece in investigating the impact of fixture locator tolerance on the geometric error of a feature. Marin and Ferreira [9] analyzed the influence of dimensional locator errors on the tolerance allocation problem. Djurdjanovic and Ni [10] developed procedures for determining the influence of fixture errors on dimensional errors in machining. These studies were conducted when a static case is assumed. Although variability existing in a production line is playing an obstructive role in gaining an efficiently precise control over manufacturing operations, only a few researchers employed a variational model to evaluate fixture performance. A model in the absence of workpiece variability consideration would not be applicable and functional in most cases. A probabilistic fixturing model was also developed by Sangnui [11] and Weipinz Zhong [12] and geometric variation of a feature was determined.

This paper developed an algorithm to determine variant final locations of a displaced workpiece given normally distributed errors at contact points. Resultant geometric variation of workpiece location reveals interesting information which is beneficial in tolerance planning. Then a tensorial model of manufacturing tolerancement is developed. It studies the influence of three-dimensional fixture errors on the geometric orientation of a machined surface. This tensorial model is developed by formulating these four stages:

- Uses the Monte Carlo Simulation in order to calculate the geometric fixture errors.
- Develops the global homogeneous matrixes that are necessary to describe the geometric transformations of positions of n surface points.
- Computes the new positions of the n surface points.
- Deducts the dispersions caused by the geometric defaults, according to the geometric orientation specification to verify.

2. QUANTIFICATION OF THE GEOMETRIC FIXTURE ERRORS

2.1 Computing the geometric fixture errors

The fixture of the workpiece is presented by the 3-2-1 fixture scheme (Fig.1). We have a system of coordinates $(O',\vec{n}'_x,\vec{n}'_y,\vec{n}'_z)$ attached to the workpiece (WCS) and another system of reference coordinates $(O,\vec{n}_x,\vec{n}_y,\vec{n}_z)$ attached to the fixture (FCS). These coordinate systems are useful to define the point positions of the surface to machine.

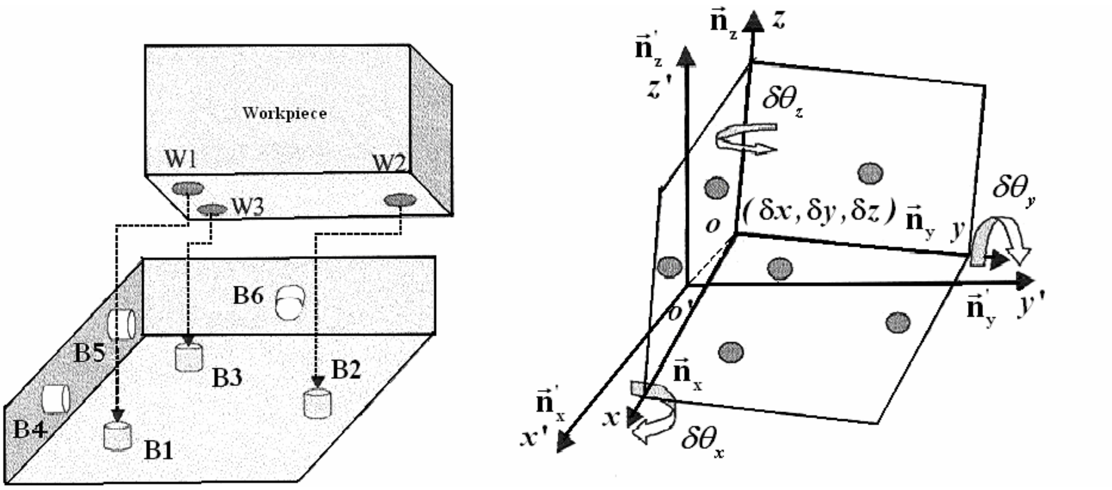

Figure 1: A 3-2-1 fixturing scheme and the related geometric errors ([12]).

If surface variation exists at contact points between a workpiece and a fixture, deviation of the workpiece from its nominal location is expected. A typical 3-2-1 fixturing method is composed of six locators forming three mutually perpendicular datum planes. The primary datum plane as shown in Fig.1 is constructed from the first three locators (B1, B2, B3). Perpendicular to the primary datum plane, the secondary datum plane is established from the contacts of the locator B4 and B5. Finally, the tertiary datum plane is the plane perpendicular to the preceding ones in which the last contact point B6 lies. Note that the workpiece surfaces in contact with the datum planes are called workpiece datum features. Given a distribution of surface errors at the contact locations, statistics allows us to determine variability of final position of the fixtured workpiece.

The errors on each locator can be generated using the Monte Carlo simulation given the tolerance of fixture. The detailed procedures are summarised as follows:

- Randomly generate the coordinates (e.g., $\vec{B}_i = (x_i, y_i, z_i)$) of the locators based on the fixture tolerance (i.e.Tf) and the ideal positions (e.g., $\vec{B}_{i,0} = (x_{i,0}, y_{i,0}, z_{i,0})$), under the assumption of normal distribution.
  Mathematically:

$$x_i \approx N(x_{i,0}, \left(\frac{T_f}{3}\right)^2)$$
$$y_i \approx N(y_{i,0}, \left(\frac{T_f}{3}\right)^2) \quad i = 1:6 \tag{1}$$
$$z_i \approx N(z_{i,0}, \left(\frac{T_f}{3}\right)^2)$$

  The form of the above equation can be represented as $N(\mu, \sigma^2)$, which means x is a random variable and follows a normal distribution with a mean of μ and a standard deviation of σ. For normal distribution, $\mu \pm 3\sigma$ covers 99.73% of data points so that $\pm 3\sigma$ is often used as the tolerance specification (i.e, $\pm T_f$). In this case, $x_{i,0}$ is the designed value and represents the mean for the i[th] locator. Therefore, $\sigma = T_f/3$ and the representation of the x_i variable becomes $x_i \approx N(x_{i,0}, (T_f/3)^2)$.
- Calculate the axis directions of the fixture coordinate system $(O', \vec{n}'_x, \vec{n}'_y, \vec{n}'_z)$ based on generated coordinates, that is:

$$\begin{aligned}
\overrightarrow{n'_z} &= (\overrightarrow{B_1} - \overrightarrow{B_2}) \wedge (\overrightarrow{B_1} - \overrightarrow{B_3}) / \left\| (\overrightarrow{B_1} - \overrightarrow{B_2}) \wedge (\overrightarrow{B_1} - \overrightarrow{B_3}) \right\| \\
\overrightarrow{n'_x} &= (\overrightarrow{B_5} - \overrightarrow{B_4}) \wedge \overrightarrow{n'_z} / \left\| (\overrightarrow{B_5} - \overrightarrow{B_4}) \wedge \overrightarrow{n'_z} \right\| \\
\overrightarrow{n'_y} &= \overrightarrow{n'_z} \wedge \overrightarrow{n'_x}
\end{aligned} \tag{2}$$

- Calculate the deviation of (FCS) from its nominal (ideal) position caused by the geometric variation on the six locating points. The deviation can be decomposed into a linear translation $(\delta x, \delta y, \delta z)$ and an angular rotation $(\delta\theta_x, \delta\theta_y, \delta\theta_z)$ in the (FCS). While we refer to "Weipinz Zhong [12]", these deviations can be proven as:

$$\delta\theta_z = \begin{cases} arc\tan g\left(\dfrac{n'_x(y)}{n'_x(x)}\right) & si \;\; n'_x(x) \neq 0 \\ 0 & si \;\; n'_x(x) = 0 \end{cases} \tag{3}$$

$$\delta\theta_y = arc\sin(n'_x(z)) \tag{4}$$

$$\delta\theta_x = -arc\sin(n'_y(z)) \tag{5}$$

$$\begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} = -\begin{bmatrix} n'_z(x) & n'_z(y) & n'_z(z) \\ n'_x(x) & n'_x(y) & n'_x(z) \\ n'_y(x) & n'_y(y) & n'_y(z) \end{bmatrix}^{-1} \begin{bmatrix} n'_z(x).x_2 + n'_z(y).y_2 + n'_z(z).z_2 \\ n'_x(x).x_4 + n'_x(y).y_4 + n'_x(z).z_4 \\ n'_y(x).x_6 + n'_y(y).y_6 + n'_y(z).z_6 \end{bmatrix} \tag{6}$$

  Where $\vec{n}'_x, \vec{n}'_y, \vec{n}'_z$ are the unit vectors of the coordinate system (WCS) with deviations.

2.2 Assumptions

In the following sections, three basic assumptions are made in modelling the geometric variation under the 3-2-1 fixturing:

- The variation sources only include kinematical (i.e., rigid body error) variations.
- A rigid body assumption is made for the fixture locators.
- The fixture errors are assumed to be normally distributed with mean and variance σ^2

3. TENSORIAL MODELLING OF GEOMETRIC DEFAULTS

3.1 Tensorial modelling of the workpiece

The workpiece or surfaces to machine are represented as a set of coordinates of a given number of points on its surface (using a point-based model, which is a set of coordinates of the discrete surface points). These points can be conveniently generated from the CAD models or from the meshed models using the FEA software.
For instance, these points can be represented as a matrix $\mathbf{X_0}$

$$X_0 = \begin{bmatrix} p_1 & p_2 & \ldots\ldots & p_m \end{bmatrix}_0 = \begin{bmatrix} x_1 & \ldots & x_m \\ y_1 & \ldots & y_m \\ z_1 & \ldots & z_m \\ 1 & 1 & 1 \end{bmatrix}_0 \tag{7}$$

$$\text{With } p_i = \begin{bmatrix} x_i & y_i & z_i & 1 \end{bmatrix}' \quad i = 1....m$$

Where the $p_i = \begin{bmatrix} x_i & y_i & z_i & 1 \end{bmatrix}'$ represent the homogeneous coordinates of a point p_i on the workpiece and m is the number of points.

3.2 Tensorial modelling of the geometric defaults generated by the fixture errors

The fixture errors cause some geometric defaults on the manufactured surfaces. An example of drill with the presence of the fixture errors is presented by the Fig.2.

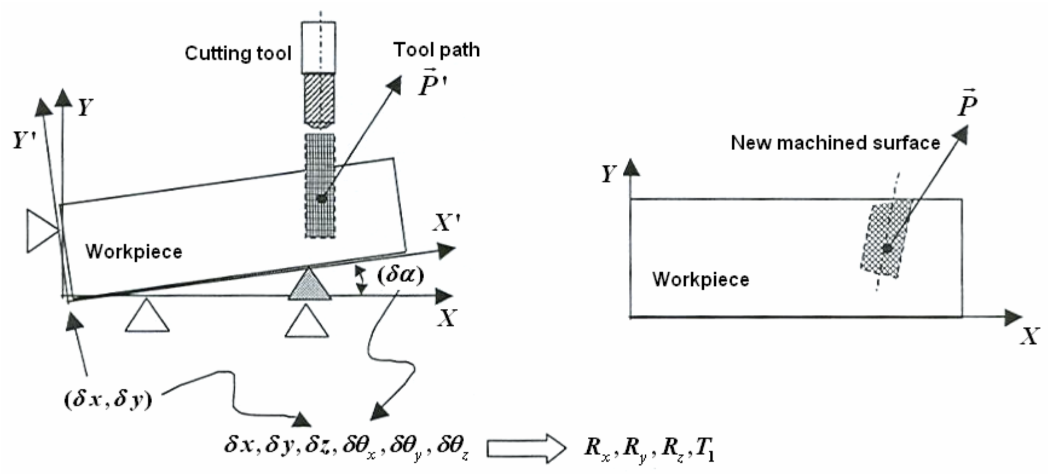

Figure 2: Presentation of the influence of the fixture errors.

The geometric transformations between the ideal position of the machined surface and her real position modified by the effect of the fixture errors, are modelled by the linear displacements and small angular $(\delta x, \delta y, \delta z, \delta\theta_x, \delta\theta_y, \delta\theta_z)$ instituted in the homogeneous matrixes of geometric transformations [13]. These homogeneous matrixes of the geometric transformations generated by the fixture errors are proven as:

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\delta\theta_x & 0 \\ 0 & \delta\theta_x & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_y = \begin{bmatrix} 1 & 0 & -\delta\theta_y & 0 \\ 0 & 1 & 0 & 0 \\ \delta\theta_y & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$R_z = \begin{bmatrix} 1 & -\delta\theta_z & 0 & 0 \\ \delta\theta_z & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_1 = \begin{bmatrix} 1 & 0 & 0 & \delta x \\ 0 & 1 & 0 & \delta y \\ 0 & 0 & 1 & \delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

The matrix T represents the combination of the small linear and angular kinematical displacements generated by the locating workpiece.

$$T = R_x.R_y.R_z.T_1 \tag{9}$$

It is noticed that to simplify the calculation of the matrix T of the combination (9), it is necessary to take account of these approximations:
$\delta v_1.\delta v_2 \approx 0$ where δv_1 and δv_2 represent the small linear $(\delta x, \delta y, \delta z)$ or angular $(\delta\theta_x, \delta\theta_y, \delta\theta_z)$ displacements
That T is expressed by:

$$T = \begin{bmatrix} 1 & \delta\theta_z & -\delta\theta_y & \delta x \\ -\delta\theta_z & 1 & \delta\theta_x & \delta y \\ \delta\theta_y & -\delta\theta_x & 1 & \delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

With the influence of the geometric fixture errors, the workpiece (modelled by X0 without the influence of the geometric fixture errors) will be modelling by a new homogeneous matrix X_1.

$$X_1 = R_x.R_y.R_z.T_1.X_0 = T.X_0 \tag{11}$$

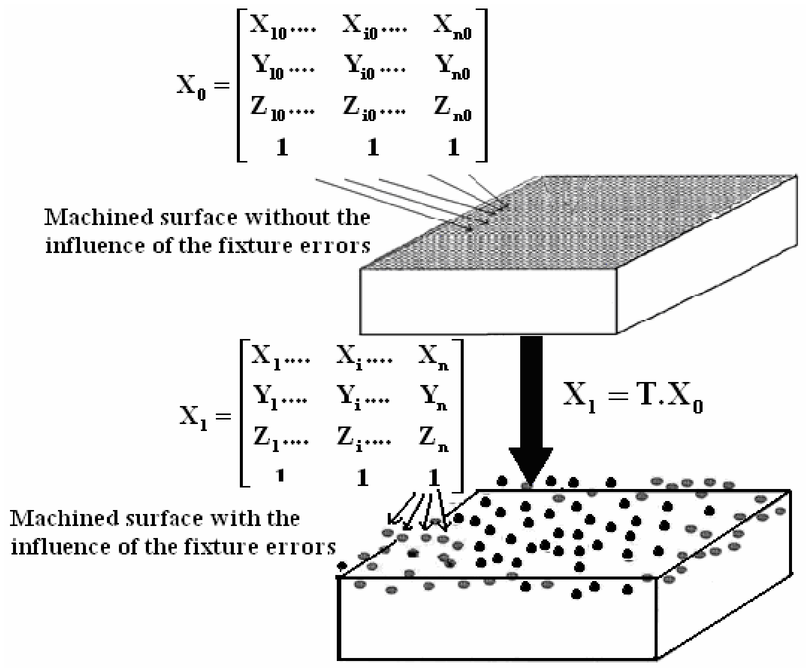

Figure 3: Tensorial modelling of the influence of the fixture errors.

The positions of the representative points of the surface to machine are varied under the geometric fixture errors. While some geometric defaults are generated by the three-dimensional variations of position of these points.

3.3 Quantification of the geometric defaults

- **Geometric condition of parallelism:**
  The parallelism refers to the minimum distance between two parallel planes (or lines) that enclose a set of measurement points such that the two planes (or lines) are parallel to a reference plane (or line). The parallelism property for planes can be illustrated by the following Fig.4.

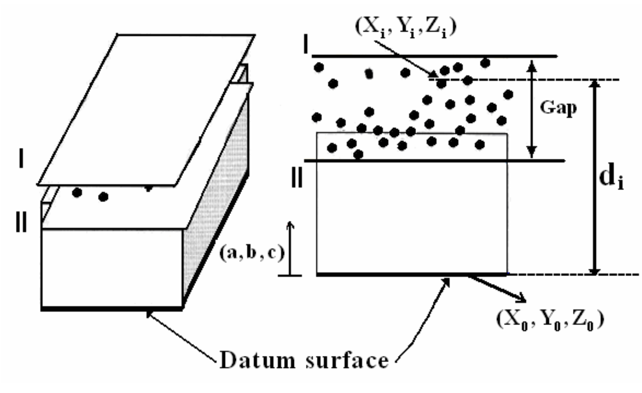

Figure 4: Interpretation of geometric defaults that influences the respect of the condition of parallelism.

Assuming that a point is represented as (xi , yi , zi) and the plane is represented as (x0 , y 0 , z0) and (a, b , c), the deviation from this point to the plane is:

$$d_i = a(x_i - x_0) + b(y_i - y_0) + c(z_i - z_0); i = 1:n \quad (12)$$

For a set of n points, the gap (or distance) between two parallel planes is:

$$Gap = \max(d_i) - \min(d_i); i = 1:n \tag{13}$$

This gap presents the geometric defaults which influence the condition of the parallelism.

- **Geometric condition of perpendicularity:**
The perpendicularity tolerance describes how close to perpendicular is one feature relative to another feature. It can be applied to a plane or an axis relative to a reference feature. For two theoretically perpendicular planes, the perpendicularity means the allowable distance between two parallel planes that are perpendicular to the datum surface.

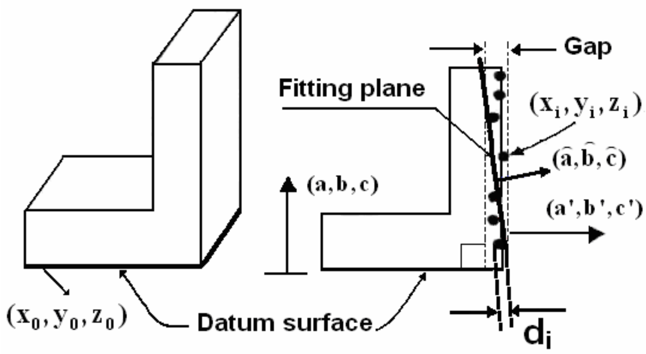

Figure 5: Interpretation of geometric defaults that influences the respect of the condition of perpendicularity

The perpendicularity can be calculated given the reference surface(a, b, c, x0, y0, z0) and the surface points (xi , yi , zi), i =1,…, n. The procedure is the following: (1) rotating the direction vector (a, b, c) with a right angle which forms a new direction vector (a', b', c'), (2) fitting a new surface $(\hat{a}, \hat{b}, \hat{c}, \hat{x}_0, \hat{y}_0, \hat{z}_0)$ using the points (xi , yi , zi), i =1,…, n, (3) calculating the minimum distance between two parallel planes (with a direction vector of (a', b', c')) that enclose the fitted surface.
Assuming that a point is represented as (xi , yi , zi) and the fitting plane is represented as $(\hat{x}_0, \hat{y}_0, \hat{z}_0)$ and $(\hat{a}, b, \hat{c})$, the deviation from this point to this plane is:

$$d_i = \hat{a}(x_i - \hat{x}_0) + \hat{b}(y_i - \hat{y}_0) + \hat{c}(z_i - \hat{z}_0); \quad i = 1:n \tag{14}$$

For a set of n points, the gap (or distance) between two parallel planes (which are perpendicular to the datum surface) is:

$$Gap = \max(d_i) - \min(d_i); i = 1:n \tag{15}$$

This gap presents the geometric defaults which influence the condition of the perpendicularity.

4. NUMERICAL SIMULATION

The example of the numerical simulation presented by Fig.6, consist in machining the surface 1 while respecting the geometric condition of parallelism by report the datum surface 6. In the same phase of machining, it consists in machining the surface 8 while respecting the geometric condition of perpendicularity by report to the datum surface 9.

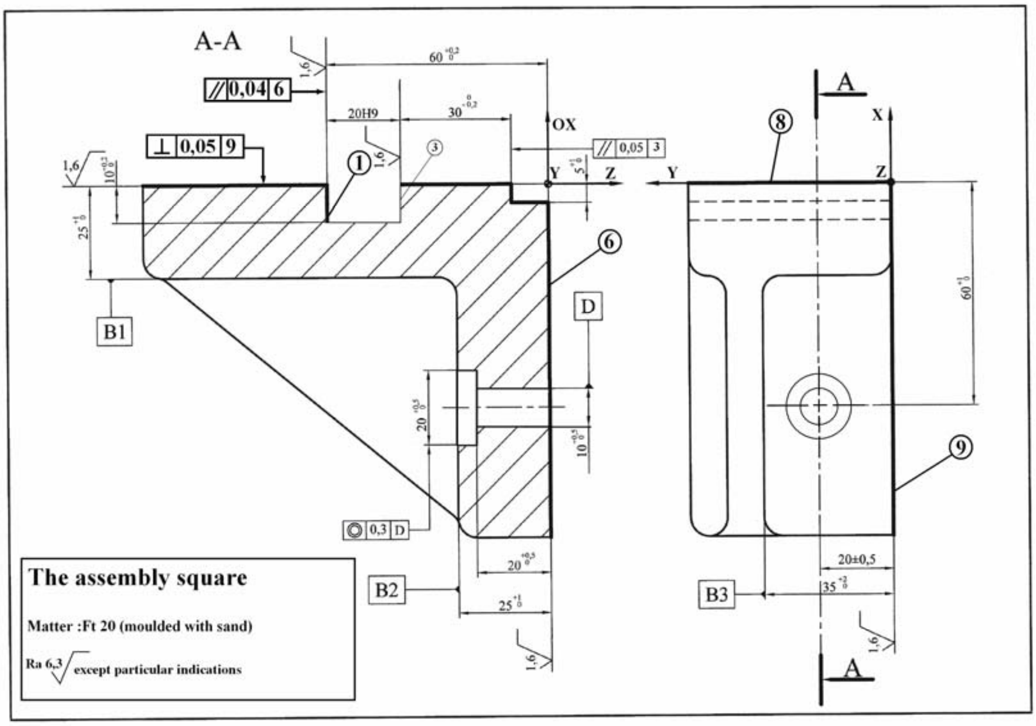

Figure 6: Presentation of the machined workpiece.

In order to respect these geometrical orientation specifications, we show by this numerical simulation that the machined surface 1 and the machined surface 8 cannot be machined by the same fixture location. Indeed, when we use 1D model of geometric tolerancement, we find that these spefications can be respected if the surface 1 and 8 are machined by the same fixture location.

4.1 Fixture location of the workpiece

In order to ensure an optimal localisation of the workpiece, we can use in the numerical simulation two different fixture locations. The first fixture location consists in applying three locators on the datum surface 6, two locators on the datum surface 9 and one locator on the surface B1. While the second fixture location consists in applying three locators on the datum surface 9, two locators on the datum surface 6 and one locator on the surface B1. We assume that:

- T6 is the fixture tolerance of the surface 6 = 0.02 mm.
- T9 is the fixture tolerance of the surface 9 = 0.02 mm.
- TA is the fixture tolerance of the surface B1 = 0.5 mm.

4.2 Study of the geometric condition of parallelism

The objective of the study is to verify the respect of the condition of the parallelism of the machined surface1 by report the datum surface 6.

- **The 1st fixture location:**
  The fixture errors are randomly generated at the points of contacts between the locators and the workpiece. The position of the point of contact between one locator and the workpiece is computed by a normal distribution law and is represented by the vector of position $\vec{B}_i(x_i, y_i, z_i)$.

$$
\begin{aligned}
x_i &\approx N(x_{i,0}, (\frac{T_A}{3})^2) \\
y_i &\approx N(y_{i,0}, (\frac{T_9}{3})^2) \quad i = 1:6 \\
z_i &\approx N(z_{i,0}, (\frac{T_6}{3})^2)
\end{aligned}
\qquad 16)
$$

The fixture errors are randomly generated at the level of points of contacts between the locators and the datum surfaces (using a numerical computing of the equations 3, 4, 5 and 6 by MATLAB software). These are presented below in the table I.

Table I: Geometric errors generated by the 1st fixture location.

Errors	δx (mm)	δy (mm)	δz (mm)	$\delta\theta_x$ (rd)	$\delta\theta_y$ (rd)	$\delta\theta_z$ (rd)
Values	-0.002	0.003	-0.011	$-4{,}29 \times 10^{-4}$	1.4×10^{-4}	-1.66×10^{-4}

The geometric defaults caused by the three-dimensional variation of the orientation of the nominal surface to machine, are presented by the Fig.7 (drawing by the use MATLAB software).

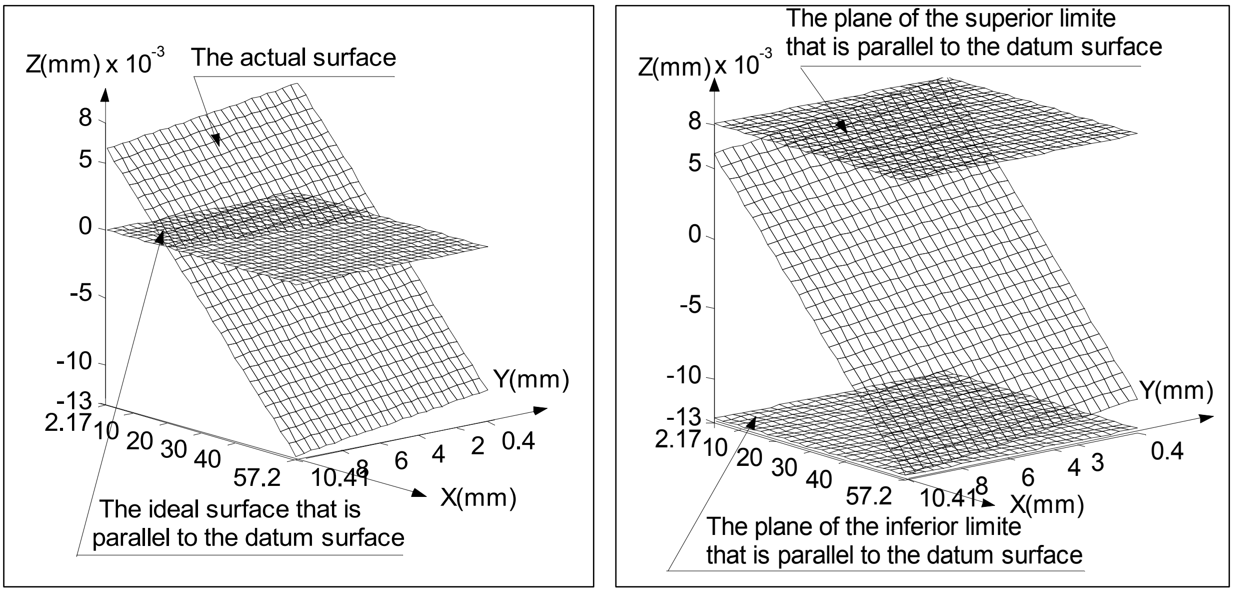

Figure 7: Geometric defaults that influence the parallelism of the machined surface 1 by report the datum surface 6 while using the 1st fixture location.

$$\Delta\text{para.1} = 0.021 \text{ mm} < 0.04 \text{ mm} = \text{specific tolerance of parallelism} \qquad (17)$$

If we use the 1st fixture location, the geometric condition of parallelism of the machined surface 1 by report the datum surface 6 is respected.

- **The 2st fixture location**
  The fixture errors are randomly generated at the level of points of contacts between the locators and the datum surfaces (using a numerical computing of the equations 3, 4, 5 and 6 by MATLAB software). These are presented below in the table II.

Table II: Geometric errors generated by the 2st fixture location.

Errors	δx (mm)	δy (mm)	δz (mm)	$\delta\theta_x$ (rd)	$\delta\theta_y$ (rd)	$\delta\theta_z$ (rd)
Values	-0.126	0.078	-0.039	1.3×10^{-3}	-2.2×10^{-4}	-1.78×10^{-4}

While the three-dimensional geometric defaults of surface 1 are caused by the fixture errors (presented by the table II), are presented by the Fig.8 (drawing by the use MATLAB software).

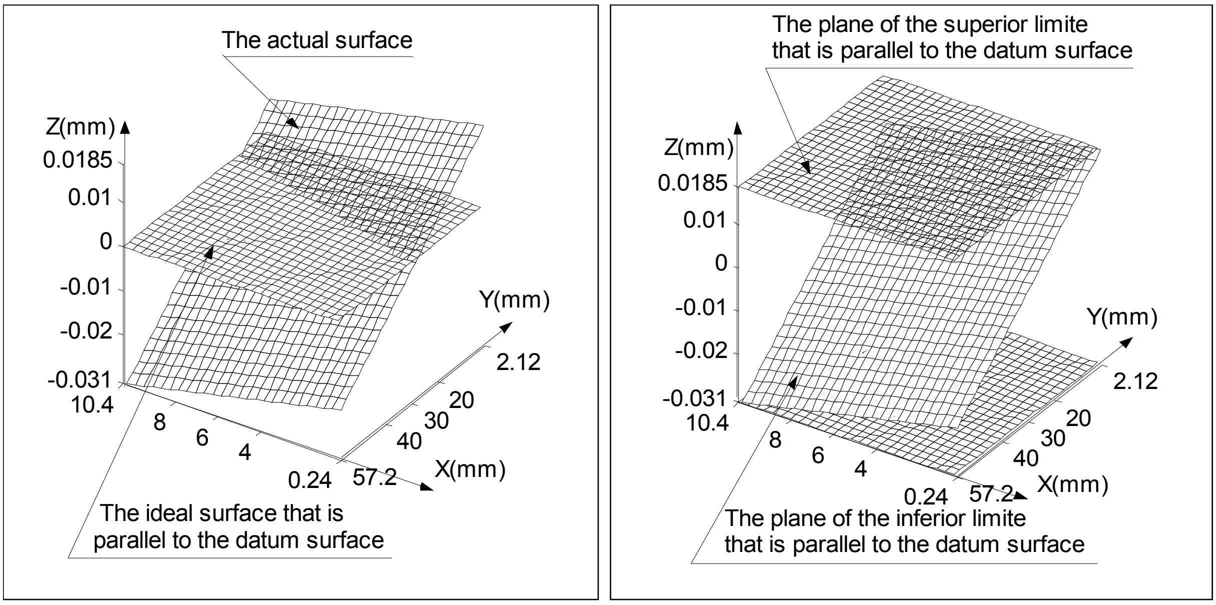

Figure 8: Geometric defaults that influence the parallelism of the machined surface 1 by report the datum surface 6 while using the 2th fixture location

$$\Delta para.2 = 0.049 \text{ mm} > 0.04 \text{ mm} = \text{specific tolerance of parallelism} \qquad (18)$$

It is noticed that the geometric condition of parallelism of the machined surface 1 by report the datum surface6 is not respected while using the 2th fixture location.

4.3 Study of the geometric condition of perpendicularity

The objective of the study is to verify the respect of the condition of the perpendicularity of the machined surface 8 by report the datum surface 9.

- **The 1st fixture location:**
  The three-dimensional geometrical defaults of the machined surface 8 are caused by a three-dimensional variation of this orientation and are presented by the Fig.9 (drawing by the use MATLAB software).

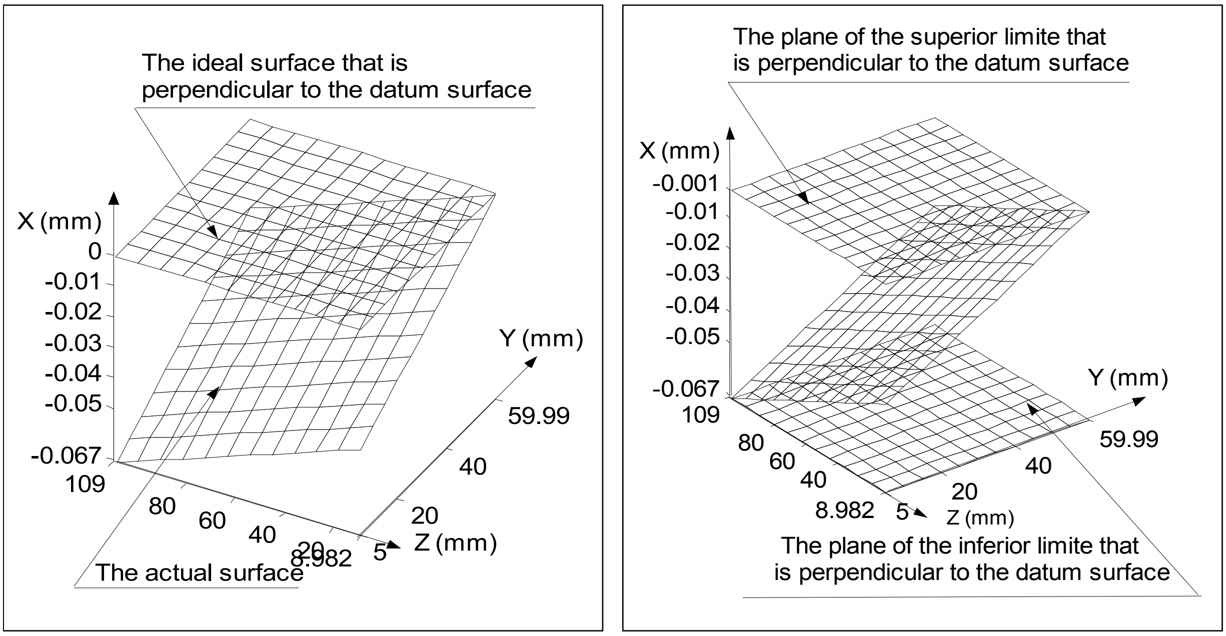

Figure 9: Geometric defaults that influence the perpendicularity of the machined surface 8 by report to the datum surface 9 while using the 1st fixture location

$$\Delta\text{perp.1} = 0.066 \text{ mm} > 0.05 \text{ mm} = \text{specific tolerance of perpendicularity} \quad (19)$$

- **The 2st fixture location:**
  The three-dimensional geometrical defaults of the machined surface 8 are caused by a three-dimensional variation of this orientation presented by the Fig.10 (drawing by the use MATLAB software).

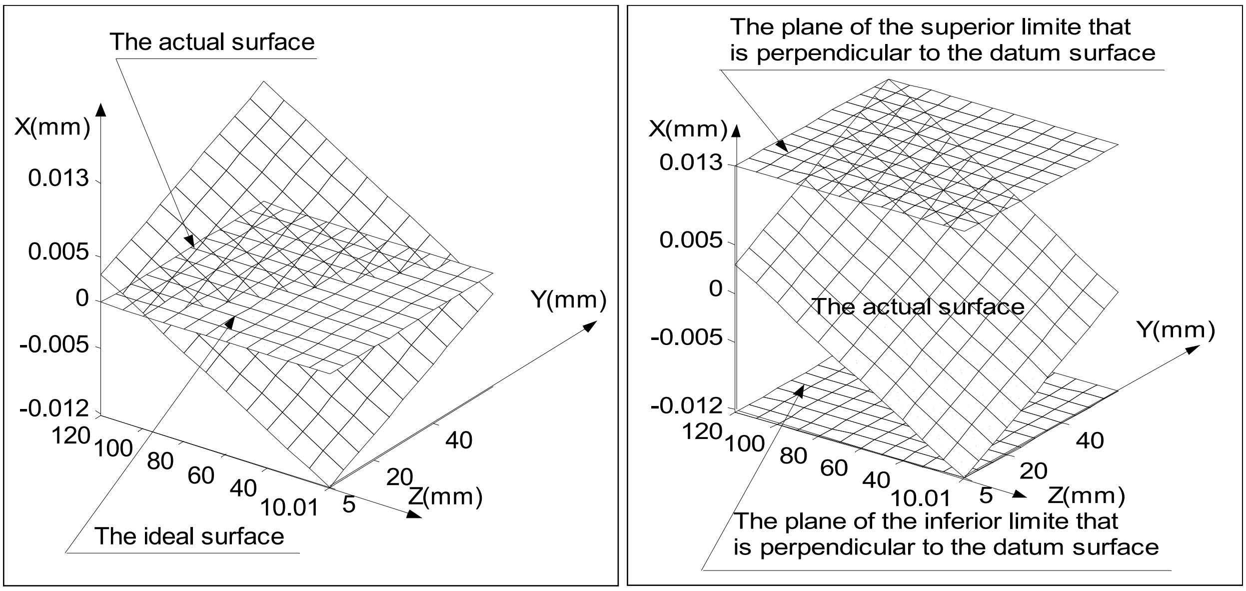

Figure 10: Geometric defaults that influence the perpendicularity of the machined surface 8 by report the datum surface 9 while using the 2th fixture location

$$\Delta\text{perp.2} = 0.025 \text{ mm} < 0.05 \text{ mm} = \text{specific tolerance of perpendicularity} \quad (20)$$

This geometric condition of perpendicularity of the machined surface 8 by report the datum surface 9 is respected when the 2th fixture location is used.

This numerical simulation shows that it is necessary to take account of the three-dimensional effect of the fixture errors at the time of the choice of the fixture location.

5. CONCLUSIONS

This study proposes an analysis of the variability in location of the workpiece as affected by variant errors at contact areas. Unlike previous studies, a step-wise analysis is considered un-necessary and the mathematical formulation is simplified. A Monte Carlo simulation was conducted to assess the variability of workpiece location by assuming a normally distributed fixture errors. A tensorial model of 3D tolerancement is used to analyze resultant variability of a workpiece. The information gained is beneficial to manufacturers as it reveals how 3D fixture errors become influential on geometrical orientation of a machined surface. It is recommended that such the orientational variability of a machined surface should be accounted in tolerancing establishment. Consequently a 3D model of geometrical tolerancement is developed in order to study the influence of the 3D geometric defaults generated by the fixture errors on the respect of the geometric condition of perpendicularity and the one of parallelism. Indeed the results of this study will be beneficial later in future work in order to continue the simulation of the influence of the fixture errors on the other geometrical specifications.

Tighter tolerance ensures a functional assembly; however, usually associated with higher cost. It is the responsibility of a designer to find ways that would benefit the production the most. Implementing the concept proposed in this work would help the designer impose the tolerances more efficiently, and consequently reduce manufacturing cost and improve product quality. In order to ensure the strategy of the improvement of product quality, we will developed in future work a model which also takes account of the influence of the errors caused by the cutting tool and elastic deformations of the fixture and the workpiece.

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