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Deteriorating inventory model using preservation technology with salvage value and shortages

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A B S T R A C T

In this paper, we attempt to develop an inventory model for deteriorating items with the consideration of the fact that the use of preservation technology (PT) can reduce the deterioration rate significantly. In this model the shortages are allowed and salvage value is incorporated to the deteriorated items. Demand rate is constant, deterioration rate is time dependent with Weibull's distribution. The model is solved analytically by minimizing the total cost of the inventory system and the numerical and graphical analysis is provided to illustrate the solution and application of the model. This analysis of the model shows that the solution of the model is quite stable. The model can be applied for optimizing the total inventory cost of deteriorating items inventory for the business enterprises where they use the preservation technology to reduce the deterioration rate of the inventory items.

A R T I C L E I N F O

Keywords:
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1. Introduction

In recent years, many researchers have studied deteriorating items inventory model. Generally, deterioration is defined as decay, change or spoilage to prevent the item from being used for its original purpose. Owing to this fact controlling and maintaining inventory of deteriorating items becomes a challenging problem for decision makers. The inventory system for deteriorating items has been an object of study for a long time, but little is known about the effect of investing in reducing the rate of product deterioration. So in this paper, an inventory model is developed to consider the fact that the uses of preservation technology reduce the deterioration rate by which the retailer can reduce the economic losses, improve the customer service level and increase business competitiveness.

Inventory of deteriorating items first studied by Within [1], he considered the deterioration of fashion goods at the end of prescribed storage period. Ghare and Schrader [2] extended the classical EOQ formula with exponential decay of inventory due to deterioration and gave a mathematical model of inventory of deteriorating items. Philip [3] developed an inventory model with a three parameter Weibull distribution rate without considering shortages. Dave and Patel [4] developed the first deteriorating inventory model with linear trend in demand. He considered demand as a linear function of time. Hollier and Mak [5] considered the constant partial backlogging rates during the shortage period in their inventory model. Deb and Chaudhri [6] derived inventory model with time-dependent deterioration rate. Goyal and Giri [7] gave recent trends of modeling in deteriorating items inventory. They classified inventory models on the basis of demand variations and various other conditions or constraints. Teng and Yang [8] generalized the partial backlogging EOQ model to allow for time-varying purchase cost. Yang [9] made a comparison among various partial backlogging inventory lot-size models for deteriorating items on the basis of maximum profit. Lately, Ouyang et al. [10] developed an inventory model for deteriorating items with exponential declining demand and partial backlogging. Wu et al. [11] developed an inventory model for non-instantaneous deteriorating items with partial backlogging where demand is assumed to be stock-dependent. San José et al. [12] proposed an inventory system with exponential partial backordering.

Dye et al. [13] found an optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging. They assume that a fraction of customers who backlog their orders increases exponentially as the waiting time for the next replenishment decreases. Alfares [14] proposed the inventory model with stock-level dependent demand rate and variable holding cost. In 2008 Roy Ajanta [15] developed a deterministic inventory model when the deterioration rate is time proportional, demand rate is function of selling price and holding cost is time dependent. Skouri et al. [16] developed an Inventory models with ramp type demand rate, partial backlogging and Weibell`s deterioration rate. Mishra and Singh [17] developed a deteriorating inventory model with partial backlogging when demand and deterioration rate is constant. They made Abad [18, 19] more realistic and applicable in practice. He et al. [20] gave an optimal production inventory model for deteriorating item with multiple market demand. Mandal [21] gave an EOQ inventory model for Weibull distributed deteriorating items under ramp type demand and shortages. Chang et al. [22] gave optimal replenishment policy for non instantaneous deteriorating items with stock dependent demand. Hung [23] gave an inventory model with generalized type demand, deterioration and backorder rates. Mishra and Singh [24] developed deteriorating inventory model for time dependent demand and holding cost with partial backlogging.

However, investing on preservation technology (PT) for reducing deterioration rate has received little attention in the past years. The consideration of PT is important due to rapid social changes and the fact that PT can reduce the deterioration rate significantly. By the efforts of investing in preservation technology we can reduce the deterioration rate. In this paper, we made the model of Mishra and Singh [24] more realistic by considering the fact that the use preservation technology can reduce the deterioration rate significantly which help the retailers to reduce their economic losses. Here we attempt to develop an inventory model with controllable deterioration rate by assuming constant demand and finite replenishment rate by allowing preservation technology cost as one of the decision variable in conjunction with inventory system. In the development of this economic order quantity (EOQ) model we also consider that the deterioration rate is time dependent & follow Weibull's distribution and shortages are allowed and completely backlogged. We then find an analytical solution of the model & graphically show that the total inventory cost with the inventory system is a convex function.

The complete assumptions and notations of the model are introduced in the next section. The mathematical model and solution procedure is derived in section 3 and numerical and graphical analysis is presented in section 4. The article ends with some concluding remarks and scope of future research

2. Assumption and Notations

2.1 Assumptions

The mathematical model is based on the following assumptions:

- Demand rate is constant and known.
- The lead time is zero or negligible.
- The replenishment rate is infinite.
- Shortages are allowed and completely backlogged.
- Preservation technologies are used for reducing the deterioration rate.

- Deterioration rate is time dependent and follow Weibull's distribution, i.e., $\theta(t) = t^{\beta-1}\alpha\beta$ where α $(0 \leq \alpha < 1)$ denote scale parameter and $\beta > 1$ denote shape parameter.
- The salvage value $\gamma(0 \leq \gamma < 1)$ is associated to deteriorated units during the cycle time.
- The deteriorated units cannot be repaired or replaced during the period under review.

2.2 Notations

The mathematical model is based on the following notations:

- C: purchase cost per unit.
- A: ordering cost per order.
- h: holding cost per unit.
- $\theta(t)$: the original deterioration rate at time t, where $\theta(t) = t^{\beta-1}\alpha\beta$.
- ξ: preservation technology (PT) cost for reducing deterioration rate in order to preserve the product, $\xi > 0$.
- m: reduced deterioration rate.
- τ_p: resultant deterioration rate, $\tau_p = (\theta(t) - m)$
- γ: salvage value associated with deteriorated items.
- t_1: the time at which the inventory level reaches zero, $t_1 \geq 0$.
- T: the length of cycle time.
- Q: the order quantity (Inventory level) during a cycle of length T.
- R: demand rate.
- IHC: holding cost per order.
- SC: shortage cost per unit per unit time.
- CD: deterioration cost per order.
- IM: the maximum inventory level during [0, T].
- IB: the maximum inventory level during shortage period.
- SV: salvage value per unit time.
- PC: preservation technology cost.
- π: shortage cost per unit short.
- $TC(t_1, T)$: total cost per time unit.

3. Mathematical formulation

The rate of change of inventory during positive stock period [0, t_1] and shortage period [t_1, T], Fig. 1, is governed by the following differential equations:

$$\frac{dQ(t)}{dt} + \tau_P Q(t) = -R \qquad (0 < t < t_1) \tag{1}$$

$$\frac{dQ(t)}{dt} = -R \qquad (t_1 < t < T) \tag{2}$$

with boundary condition

$$Q(t) = 0 \ \text{at} \ t = t_1, \ Q(t) = IM \ \text{at} \ t = 0 \ \text{and} \ Q(t) = IB \ \text{at} \ t = T, \tag{3}$$

where $Q(t) = t^{\beta-1}\alpha\beta$, and $\tau_P = Q(t) - m$.

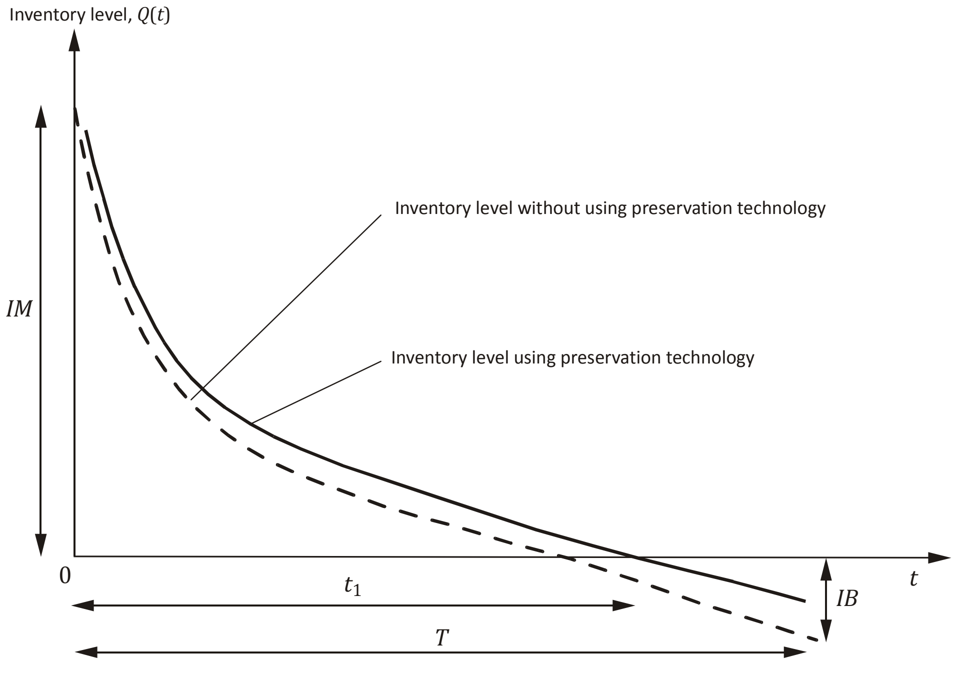

Fig.1 Graphical representation of inventory system

4. Analytical solution of the model

4.1 Case I: Inventory level without shortage

During the period [0, t_1], the inventory depletes due to the reduced deterioration (after using the preservation technology) and demand R. Hence, the inventory level at any time during [0, t_1] is described by differential equation

$$\frac{dQ(t)}{dt} + \tau_P Q(t) = -R \quad (0 < t < t_1). \tag{4}$$

The boundary conditions are $Q(0) = IM$ and $Q(t_1) = 0$, where $\theta(t) = t^{\beta-1}\alpha\beta$. The solution of the linear differential equation (4) is given as follows:

$$Q(t) = R\left(\begin{array}{c} t_1 - t_1 t^{\beta}\alpha + t_1 mt - t + tt^{\beta}\alpha - \frac{1}{2}mt^2 + \frac{t_1^{\beta+1}\alpha}{\beta+1} + \frac{t_1^{\beta+1}\alpha mt}{\beta+1} - \frac{\alpha t^{\beta+1}}{\beta+1} - \frac{t^{\beta+2}\alpha m}{\beta+1} - \\ \frac{1}{2}mt_1^2 + \frac{1}{2}m^2 t_1^2 t^{\beta}\alpha - \frac{1}{2}m^2 t_1^2 t - \frac{1}{2}mt^{\beta+2}\alpha + \frac{1}{2}m^2 t^3 \end{array}\right). \tag{5}$$

(Neglecting the α^2 and higher powers terms).

Since $Q(t) = Q = IM$ at $t = 0$ then

$$Q = R\left(t_1 + \frac{\alpha t_1^{\beta+1}}{\beta+1} - \frac{1}{2}mt_1^2\right). \tag{6}$$

4.2 Case II: Inventory level with shortage

In this case the equation of state of inventory level at time t is given by

$$\frac{dQ(t)}{dt} = -R \quad \text{where} \quad (t_1 < t < T). \tag{7}$$

The solution of differential equation (7) is

$$Q(t) = R(t_1 - t). \tag{8}$$

Since $Q(T) = -IB$ at $t = T$, so

$$IB = R(T - t_1). \tag{9}$$

Therefore the total cost per replenishment cycle consists of the following components:

1. *Inventory holding cost*

$$IHC = h\int_0^{t_1} Q(t)dt$$

$$IHC = -\frac{hR}{24(\beta+1)(\beta+2)}\begin{pmatrix} 3m^2t_1^4\beta^2 - 12t_1^2\beta^2 + 4t_1^3m\beta^2 - 12t_1^{\beta+3}\alpha m\beta \\ -36t_1^2\beta - 24t_1^{\beta+2}\alpha\beta + 9m^2t_1^4\beta + 12t_1^3m\beta \\ +6m^2t_1^4 - 24t_1^{\beta+3}\alpha m - 24t_1^2 + 8t_1^3m \end{pmatrix} \tag{10}$$

2. *Shortage cost during* $[t_1, T]$

$$SC = \pi\int_{t_1}^{T} IBdt$$

$$SC = \pi\int_{t_1}^{T} R(t - t_1)dt$$

$$SC = \frac{1}{2}\pi R(T - t_1)^2 \tag{11}$$

3. *Stock loss due to deterioration*

$$D = IM - Rt_1$$

$$D = R\left(t_1 + \frac{t_1^{\beta+1}\alpha}{\beta+1} - \frac{1}{2}mt_1^2 - t_1\right) \tag{12}$$

4. *Deterioration cost*

$$CD = C\,R\left(\frac{t_1^{\beta+1}\alpha}{\beta+1} - \frac{1}{2}mt_1^2\right) \tag{13}$$

5. *Ordering cost per order*

$$OC = A \tag{14}$$

6. *Salvage value of deteriorated units per time unit*

$$SV = \gamma C\,R\left(t_1 + \frac{t_1^{\beta+1}\alpha}{\beta+1} - \frac{1}{2}mt_1^2 - t_1\right) \tag{15}$$

7. *Preservation technology cost*

$$PC = \xi \tag{16}$$

Thus objective function of this inventory system, total cost function per time unit

$$TC(t_1,T)=\frac{1}{T}(IHC+SC+OC+CD-SV+PC). \tag{17}$$

Putting the values of above cost components in this total cost equation then

$$TC(t_1,T)=\frac{1}{T}\left[\begin{array}{c}\dfrac{-hR}{24(\beta+1)(\beta+2)}\begin{pmatrix}3m^2t_1^4\beta^2-12t_1^2\beta^2+4t_1^3m\beta^2-12t_1^{\beta+3}\alpha m\beta\\ -36t_1^2\beta-24t_1^{\beta+2}\alpha\beta+9m^2t_1^4\beta+12t_1^3m\beta\\ +6m^2t_1^4-24t_1^{\beta+3}\alpha m-24t_1^2+8t_1^3m\end{pmatrix}+\\ \dfrac{\pi R(T-t_1)^2}{2}+A+(1-\gamma)CR\left(\dfrac{t_1^{\beta+1}\alpha}{\beta+1}-\dfrac{1}{2}mt_1^2\right)+\xi\end{array}\right]. \tag{18}$$

The necessary condition for the total cost per time unit to be minimize is

$$\frac{\partial TC}{\partial t_1}=0 \quad\text{and}\quad \frac{\partial TC}{\partial t_2}=0$$

Provided

$$\left(\frac{\partial^2 TC}{\partial t_1^2}\right)\left(\frac{\partial^2 TC}{\partial t_2^2}\right)-\left(\frac{\partial^2 TC}{\partial t_1\partial t_2}\right)>0 \quad\text{and}\quad \left(\frac{\partial^2 TC}{\partial t_1^2}\right)^2>0. \tag{19}$$

Since the nature of the cost function is highly non linear thus the convexity of the function shown graphically in the next section.

5. Numerical illustration and graphical analysis

Consider an inventory system with the following parameter in proper unit R = 200, h = 0.5, C = 10, π = 50, A = 50, α = 0.85, β = 1.5, γ = 0.10, m = 0.0001, ξ = 2. The computer output of the program by using maple mathematical software is t_1 = 0.2948, T = 0.3224 and TC = 276.8930, i.e., the value of t_1 at which the inventory level become zero is 0.2948 unit time and shortage period is 0.0276 unit time.

If we plot the total cost function (18) with some values of t_1 and T such that t_1 is 0.20 to 0.40 and T is 0.20 to 0.40 with equal interval, fixed t_1 at 0.29 and T varies from 0.20 to 0.40, fixed T at 0.32 and t_1 varies from 0.20 to 0.38 then we get strictly convex graph of total cost function (TC) given by the Fig. 3, 4 and 5, respectively.

From Fig. 3, 4 and 5 we observe that the optimal replenishment schedule uniquely exists and the total inventory cost with the inventory system is a convex function and Fig. 2 indicate that as we increase the reduced deterioration rate by the use of preservation technology the total cost of the inventory decreases.

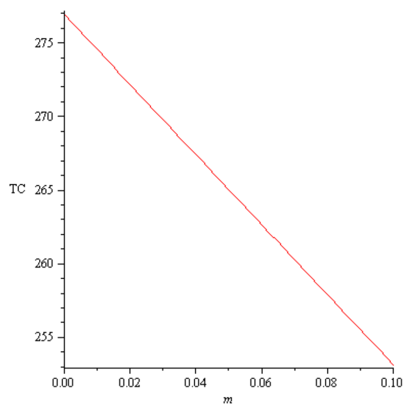

Fig. 2 Total cost vs. m

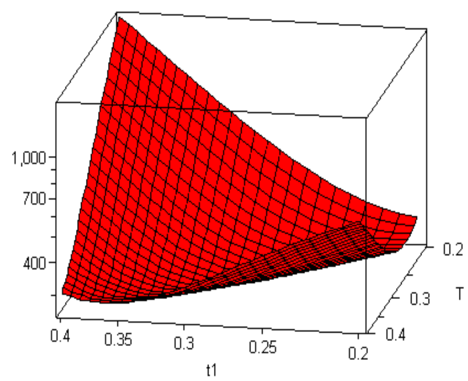

Fig. 3 Total cost vs. t_1 and t_2

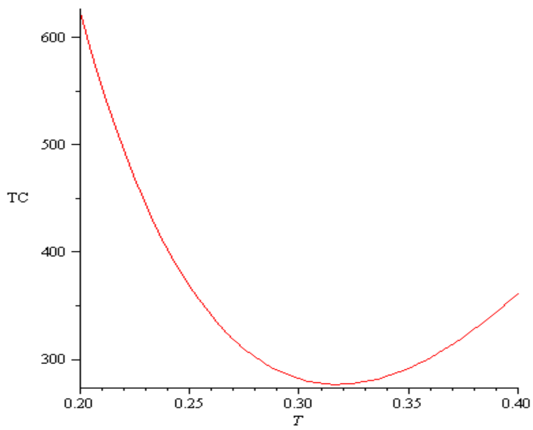

Fig. 4 Total cost vs. T at $t_1 = 0.29$

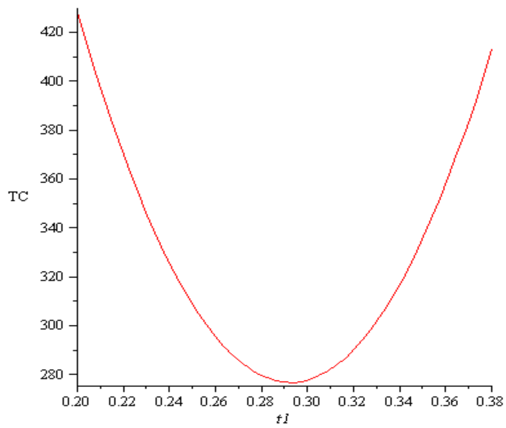

Fig. 5 Total cost vs. t_1 at $T = 0.32$

6. Conclusion

Traditional inventory models are developed mainly without using the concept of preservation technology. The main emphasis of this paper is on cost reduction by making effective capital investment in preservation technology. The products with high deterioration rate are always crucible to the retailer's business. In real markets, the retailer can reduce the deterioration rate of product by making effective capital investment in storehouse equipment. In this study we developed a deteriorating inventory model in which the deterioration rate is time dependent and to reduce the deterioration rate retailer invested in the of preservation technology cost, and deteriorating items incorporated with salvage value and shortages. The analytical solution of the model has given that minimizes the total inventory cost.

The model is very practical for the retailers who use the preservation technology in their warehouses to reduce the deterioration rate under other assumptions of this model. We have graphically shown that not only the optimal replenishment schedule exists uniquely, but also the total cost associated with the inventory system is a convex function. The numerical and graphical analysis of the model shows that the solution of the model is quite stable. For future research, the research presented in this paper can be extended in several ways. For instance, this model can further be extended by taking more realistic assumptions such as finite replenishment rate, time dependent demand, probabilistic demand, time dependent holding cost etc.

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