APEM journal
Advances in Production Engineering & Management
Volume 13 | Number 1 | March 2018 | pp 31–43
https://doi.org/10.14743/apem2018.1.271
ISSN 1854-6250
Journal home: apem-journal.org
Original scientific paper

Optimal production planning with capacity reservation and convex capacity costs

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A B S T R A C T

In this paper, we develop an analytical model for a multi-period production planning problem with dual supply sources of production capacity, where the supply price of one source (i.e., the spot market) is random and the supply capacity of the other source (i.e., the contract supplier) is limited. The purchasing cost of reserved capacity is assumed to be a convex function, rather than a linear function. We solve this problem by first characterizing the structure of the optimal production policy by employing a stochastic dynamic programming approach, and then determining the optimal capacity reservation level by applying a single-variable optimization method. For any given level of capacity reservation, the optimal periodic production policy is a quantity-dependent base stock policy with a threshold of production quantity that increases with the spot price. With this structure of the optimal production policy, the expected total discounted cost function is shown to be convex in the capacity reservation level. These results are also extended to the infinite-horizon case. A numerical study is conducted to examine the impacts of spot market characteristics on the optimal capacity reservation level and the corresponding optimal total cost.

A R T I C L E I N F O

Keywords:
Production planning;
Capacity reservation;
Stochastic programming;
Optimization;
Optimal policy;
Base-stock

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Article history:
Received 26 December 2017
Revised 22 February 2018
Accepted 24 February 2018

1. Introduction

Supply chain disruptions caused by natural disasters or man-made interruptions have drawn significant attention in the past decade [1]. To manage the risk of disruptions and to mitigate their impacts on the whole supply chain, many companies have adopted dual sourcing (backup supply) as an easy-to-implement but quite effective operational mitigation strategy. In the presence of a spot trading market, the manufacturer inclines to build a long-term relationship with the primary supplier through supply contracts while utilizing spot purchase as a backup resource. A typical example is the Chinese steel-makers' procurement practice for iron ore [2]. They receive their supply of iron ore from long-term contracts with major iron miners in the world. Besides the long-term contracts, they also purchase iron ore cargoes from the spot market with a volatile trading price.

There are numerous forms of supply contracts, among which quantity-flexible or options contracts such as capacity reservation are more preferred in industrial procurement. In a capacity reservation contract, the manufacturer first pays a reservation fee to set a capacity reservation quantity. After some uncertainties of the spot market price and consumer demand are resolved, the manufacturer has the right to use any desired portion of the reserved capacity according to the contract agreement. The manufacturer then pays an additional capacity cost for only the used capacity. This provides the manufacturer with flexibility in responding to changes in market demands and spot prices. In practice, such supply contracts are observed in apparel, power generation and high-tech industries such as semiconductors, electronics, and telecommunications equipment. For example, Apple's major chip suppliers were reported to reserve capacity for the second and third quarters of 2016 for iPhone 7 production [3].

There is a rich body of literature in operations research/management science that addresses various procurement problems with capacity reservation contracts. Early work mainly focused on deriving the optimal contract parameters to minimize the total cost of the supply chain system. Silver and Jain [4] and later Jain and Sliver [5] considered the case where the buyer pays a premium to the supplier for reserving a certain level of dedicated capacity in response to capacity supply uncertainty. They developed computational methods for the buyer to determine the capacity reservation level and the periodic replenishment quantity. Costa and Silver [6] extended the work of [4] to the case with discretely distributed demand and capacity supply. Brown and Lee [7] considered a pay-to-delay capacity reservation contract which is commonly seen in the semiconductor manufacturing. Peleg *et al*. [8] studied a two-period inventory model where the manufacturer (buyer) can procure from a long-term contract supplier with a fixed price and from the suppliers on the Internet with a random price. They discussed three alternative procurement strategies and identify the conditions under which each strategy is optimal. A comprehensive review of earlier literature on option contracts is provided by Kleindorfer and Wu [9].

Some research papers are devoted to the design of reservation contracts to achieve channel coordination. Barnes-Schuster *et al*. [10] studied channel coordination with option contracts in a two-period buyer-supplier model with correlated demand. They found that channel coordination can be achieved in case of piecewise linear option exercise prices. Moreover, they also showed that the backup agreement (Eppen and Iyer [11]), the quantity flexibility contract (Tsay and Lovejoy [12]), and the pay-to-delay capacity reservation contract are special cases of a general option contract they discussed. Erkoc and Wu [13] analysed a single-period supplier-buyer model with capacity reservation in the context of high-tech manufacturing. They proposed two reservation contracts under which channel coordination can be achieved and discussed the implications of them under different situations on contract compliance and demand uncertainty resolution. In a similar model setting, Jin and Wu [14] compared the deductible reservation contract with the take-or-pay contract in terms of channel coordination. They also extended their model to the multi-buyer case where one buyer with unsatisfied demand may use the unused capacity reserved by other buyers. In the presence of spot market trading, Pei *et al*. [15] studied the design of option contracts including exercise price and reservation price in a two-period seller-buyer model. They showed that the optimal contract can be either a sales contract or an options contract depending on the value of the optimal exercise price, while the optimal reservation price can be either fixed or volume-dependent (i.e., volume discounts or volume premia).

The research on capacity reservation in a multi-period setting arises in recent years. Serel *et al*. [16] first studied a multi-period buyer-supplier model with capacity reservation. They showed that the buyer's optimal sourcing decision is a base-stock type policy with two order-up-to levels corresponding to different supply strategies. Serel [17] extended the work of [16] to the case of considering supply uncertainty in the spot market. Inderfurth and Kelle [18] incorporated spot price uncertainty into the model discussed in [16], and provided a base stock policy for the dual sourcing problem which is suboptimal but easy to implement. For the same model, Inderfurth *et al*. [19] derived the optimal policy which contains three critical parameters: a capacity reservation level, a base stock level for sourcing from the long-term supplier, and a spot-price-dependent base stock level for sourcing from the spot market.

In addition to these above mentioned papers, many researchers have been working on the issues of production planning optimization and simulation [20-23], production coordination [24], cell production [25] and spot market price volatility [26-30]. In these related papers it is usually assumed that the reserved capacity is charged at a fixed unit capacity cost. However, as stated in [13], "capacity expansion demonstrates diseconomy of scale in this context (high-tech manufacturing), we (should) assume convex capacity costs". It is recognized that the reserved capacity price should not be linear for some industries such as high-tech industries, the apparel industry and the power generation industry. The presence of nonlinear purchasing cost of reserved capacity creates new challenges for manufacturers in making their production planning decisions. Previous research work mainly focus on the production planning problem with capacity reservation in the context of a linear capacity cost (see, e.g., [16-19]), very limited literature has been devoted to the production planning problem with nonlinear capacity costs.

In this paper, we consider a manufacturer who procures capacity to produce finished products to satisfy random market demands. During the planning horizon, the manufacturer pays a unit premium per period to the contract supplier for reserving a certain level of production capacity. In each period, the manufacturer can either procure any amount up to the capacity reservation level from the contract supplier or procure from the spot market. The purchasing cost for the reserved capacity is increasing and convex, which reflects the diseconomy of scale when the contract supplier builds capacity. The spot market price follows a Markov process, whose realization is revealed to the manufacturer before the periodic procurement decisions are made. If the reserved capacity is not fully utilized in a period, the unused capacity cannot be sold back to the spot market and has no value to the manufacturer. The manufacturer's objective is to make optimal capacity reservation level and production decisions to minimize the expected total discounted cost over the planning horizon.

This problem can be modelled as a cost minimization problem in which the long-term capacity reservation level and the periodic production decisions are determined sequentially. For any given capacity reservation level, we formulate the problem as a stochastic dynamic program and characterize the structure of the optimal production policy. Our analysis reveals that the optimal periodic production policy is a quantity-dependent base stock policy, characterized by a threshold increasing with the spot price. That is, if the production quantity is lower than the threshold, the manufacturer produces exclusively from the reserved capacity; otherwise he produces from both the reserved capacity and the purchased capacity. For the former case the optimal produce-up-to level is increasing with the initial inventory level, whereas for the latter case the optimal produce-up-to level is decreasing with the spot price. With this structure of the production policy, the expected total discounted cost function is shown to be convex in the capacity reservation level. These results are also extended to the infinite-horizon case. Numerical experiments are conducted to examine the impacts of spot market characteristics on the optimal capacity reservation decisions.

The main contributions of this paper are threefold. First, in the present of convex capacity costs we find that the optimal order-up-to level for purchasing from the long-term supplier is an increasing function of the initial inventory level, rather than a fixed value in the linear capacity cost case. Second, we show that with the increase of spot price variance, the optimal capacity reservation level may decrease while the corresponding optimal total cost increases, which implies that the capacity reservation contract might not be an effective operational hedging tool against spot market price volatility. Third, the optimal capacity reservation level is shown to be decreasing with the initial inventory level at the beginning of the planning horizon, which demonstrates the substitution effect in mitigating supply chain risks between the carrying inventory strategy and the sourcing strategy [31].

The remainder of this paper is organized as follows. In Section 2, we introduce the model and its mathematical formulation in detail. In Section 3, we study the structural properties of the model and analyse the manufacturer's optimal capacity reservation and production strategies. A numerical study is conducted in Section 4. We conclude the paper in Section 5.

2. The model

We consider a manufacturer who procures production capacity from upstream suppliers and supplies finished products to end-consumers. The planning horizon is finite and indexed by $t = 1, \ldots, T$. The random demands D_t for the finished products in each period are independently and identically distributed. Without loss of generality, we assume that the production lead time is negligible compared to the period length and one unit of capacity is required for producing one unit of finished product. Thus, D_t can also be regarded as the manufacturer's demand for production capacity in each period.

In a capacity reservation contract, the long-term supplier allows the manufacturer to pay a unit premium c_r per period for keeping a certain level of reserved capacity K throughout the planning horizon. In each period, the manufacturer has the right, but not the obligation, to produce Q $(0 \leq Q \leq K)$ units of products by the reserved capacity. The capacity cost charged by the long-term supplier, $R(Q)$, is a convex and increasing function of Q, where $R'(0) = 0$.

The manufacturer can also purchase capacity from the spot market – an outside market which is established by a group of homogeneous short-term suppliers. Let C_t be the spot price in period t and c_t be its realization. We assume that the spot prices $C_1,\ldots, C_T$ follow a Markov process. That is, the distribution of c_{t+1} is only determined by c_t. In addition, D_t and C_t are assumed to be independent since the spot market size is sufficiently large.

The unit production cost c_m is identical for both the reserved capacity and the purchased capacity. At the end of each period, excess inventory of finished products is carried over to the next period and unsatisfied demand is backlogged. Denote $L_t(y)$ as the expected holding/backlogging cost in period t if the inventory level after production is y, where

$$L_t(y) = hE[(y - D_t)^+] + bE[(D_t - y)^+] \tag{1}$$

where h and b are respectively the unit inventory holding cost and unit demand backlogging cost in one period.

The sequence of events is as follows. At the beginning of the planning horizon, the manufacturer determines the long-term capacity reservation level K. At the beginning of each period t, the manufacturer first observes his on-hand inventory level x_t and the realized spot price c_t. Then he decides on how many to produce from each capacity. We denote by y_{1t} and y_{2t} the inventory level after production from the reserved capacity and the inventory level after production from all the capacity, respectively. Hence, $y_{1t} - x_t$ is the production quantity from the reserved capacity, and $y_{2t} - y_{1t}$ is production quantity from the capacity purchased from the spot market. Finally, demand is realized and satisfied by the available inventory. All costs are incurred and calculated at the end of the period. The objective of the manufacturer is to make optimal capacity reservation and production decisions to minimize the expected total discounted cost over the planning horizon.

Let $V_t(x_t, c_t, K)$ be the optimal expected total discounted cost from period t to T, given the initial inventory level x_t, the realized spot price c_t, and the capacity reservation level K. Then the manufacturer's production planning problem can be formulated as the following dynamic program: for $t = 1, \ldots, T$,

$$V_t(x_t, c_t, K) = \min_{x_t + K \geq y_{1t} \geq x_t, y_{2t} \geq y_{1t}} \{c_r K + R(y_{1t} - x_t) - c_t y_{1t} - c_m x_t + G_t(y_{2t})\} \tag{2}$$

where

$$G_t(y_{2t}) = (c_t + c_m) y_{2t} + L_t(y_{2t}) + \alpha E_t[V_{t+1}(y_{2t} - D_t, C_{t+1}, K)] \tag{3}$$

The boundary condition is $V_{T+1}(x_{T+1}, c_{T+1}, K) = 0$. Here we use $E_t[\cdot]$ to denote $E_{D_t}\left[E_{C_{t+1}}[\cdot \,|C_t = c_t]\right]$, where the first expectation, E_{D_t}, is taken with respect to the random demand D_t, and the second expectation, $E_{C_{t+1}}[\cdot \,|C_t = c_t]$, is taken with respect to the random spot price in period $t + 1$, C_{t+1}, given the realized spot price in period t, c_t.

Our model includes the models in Henig *et al.* [32] and Inderfurth *et al.* [19] as special cases. Henig *et al.* [32] studied a multi-period inventory problem embedded in a transportation contract assuming that the unit transportation cost for the delivery amount beyond the contract volume is larger. In the presence of a spot market, we assume that the spot price is random and thus may be either smaller or larger than the contract price. Inderfurth *et al.* [19] studied the multi-period inventory problem with a linear reserved capacity cost. Here we generalize the model in Inderfurth *et al.* [19] to the case with convex reserved capacity costs.

3. Optimal policies

In this section, we first characterize the structure of the optimal production policies for a given capacity reservation level, and then determine the optimal capacity reservation level.

For notational convenience, we define

$$H_t(y) = c_m y + L_t(y) + \alpha E_t[V_{t+1}(y - D_t, C_{t+1}, K)] \tag{4}$$

Then, the optimality equation (2) with respect to initial inventory level x can be rewritten as

$$V_t(x) = \min_{x+K \geq y_{1t} \geq x} \left\{ R(y_{1t} - x) - c_t y_{1t} + \min_{y_{2t} \geq y_{1t}} [c_t y_{2t} + H_t(y_{2t})] \right\} + c_r K - c_m x \tag{5}$$

The convexity of the cost-to-go function is established in Lemma 1.

Lemma 1: *In period $t = 1, \ldots, T$, for any given K and c_t, $V_t(x)$ is convex in x.*

Proof: We prove by induction that, when $V_{t+1}(x)$ is convex in x, then $V_t(x)$ is convex. Since $V_{T+1}(x) = 0$ is clearly convex, we assume that the convexity of $V_{t+1}(x)$ holds for period $t + 1$ where $t \leq T$. Then it is easy to verify that $H_t(y)$ is convex in y. Thus, $c_t y_{2t} + H_t(y_{2t})$ is a convex function of y_{2t}. Note that the feasible region $y_{2t} \geq y_{1t}$ is a convex set of (y_{1t}, y_{2t}). Then it follows from Proposition B-4 in Heyman and Sobel [33] that, $\min_{y_{2t} \geq y_{1t}}[c_t y_{2t} + H_t(y_{2t})]$ is convex in y_{1t}. Given c_t, $R(y_{1t} - x) - c_t y_{1t} + \min_{y_{2t} \geq y_{1t}}[c_t y_{2t} + H_t(y_{2t})]$ is jointly convex in x and y_{1t}. For a given K, the feasible region $x + K \geq y_{1t} \geq x$ is a convex set of (x, y_{1t}). Then by [33] again we obtain that $V_t(x)$ is convex in x. ■

Define

$$S_H = \arg\min_y H_t(y) \tag{6}$$

$$S_R(x) = \arg\min_y R(y - x) + H_t(y) \tag{7}$$

$$S_F(c_t) = \arg\min_y c_t\, y + H_t(y) \tag{8}$$

$$S_K = \{y | R'(K) + H_t'(y) = 0\} \tag{9}$$

$$M_t = R'^{-1}(c_t) \tag{10}$$

and

$$c_K = R'(K) \tag{11}$$

where S_H is the minimizer of $H_t(y)$, $S_R(x)$ is the minimizer of $R(y - x) + H_t(y)$ for any given x, $S_F(c_t)$ is the minimizer of $c_t y + H_t(y)$ for any spot price realization c_t, S_K is the minimizer of $c_K y + H_t(y)$ for any given K, M_t is a threshold capacity level for any c_t, and c_K is a threshold price for any K. Some order relations and monotonic properties of these critical points are given in Lemma 2.

Lemma 2: *For any period $t = 1, \ldots, T$, we have the following results.*

(a) $S_K \leq S_F(c_t) < S_H$ if $c_t \leq c_K$; otherwise $S_F(c_t) < S_K < S_H$
(b) $S_R(S_H) = S_H$, $S_R(S_F(c_t) - M_t) = S_F(c_t)$, $S_F(0) = S_H$, $S_F(c_K) = S_K$
(c) $S_R(x)$ is increasing in x
(d) $S_R(x) - x$ is decreasing in x
(e) $S_F(c_t)$ is decreasing in c_t
(f) S_K is decreasing in K
(g) M_t is increasing in c_t
(h) If $c_t \leq c_K$, then $K \geq M_t$; otherwise $K < M_t$

Proof: Please see the Appendix A. ■

Based on these critical points, the optimal production policy is characterized as follows.

Theorem 1: *In period* t $(t = 1, \dots, T)$, *for a given state* (x, c_t, K) *the optimal production policy,* (y_1^*, y_2^*), *is*:

$$\text{if } c_t \le c_K,\ (y_1^*, y_2^*) = \begin{cases} \big(x + M_t, S_F(c_t)\big), & if\ x < S_F(c_t) - M_t \\ \big(S_R(x), S_R(x)\big), & if\ S_F(c_t) - M_t \le x < S_H \\ (x, x), & if\ S_H \le x \end{cases}$$

$$\text{otherwise, } (y_1^*, y_2^*) = \begin{cases} \big(x + K,\ S_F(c_t)\big), & if\ x < S_F(c_t) - K \\ (x + K, x + K), & if\ S_F(c_t) - K \le x < S_K - K \\ \big(S_R(x), S_R(x)\big), & if\ S_K - K \le x < S_H \\ (x, x), & if\ S_H \le x \end{cases}$$

Proof: It follows from the convexity of $R(x)$ that, the marginal reserved capacity cost $R'(x) \le c_t$ if $x \le M_t$; and $R'(x) > c_t$ otherwise. Thus, when the production quantity is smaller than M_t, it is optimal to produce exclusively from the reserved capacity (the exclusive case); otherwise it is optimal to produce from both types of capacity in which only M_t reserved capacity is used (the non-exclusive case).

When $c_t \le c_K$, the capacity reservation level K is larger than M_t. For the exclusive case, from the convexity of $R(y - x) + H_t(y)$, the optimal production policy is to make the inventory level after production from the reserved capacity, y_{1t}, as close to $S_R(x)$ as possible. If $S_F(c_t) - M_t \le x < S_H$, it is optimal to set $y_{2t} = y_{1t} = S_R(x)$ since the optimal production quantity satisfies that $0 < S_R(x) - x \le M_t$. Next, if $x \ge S_H$, it is optimal to set $y_{2t} = y_{1t} = x$ since for any $y \ge x \ge S_H$, $R(y - x) + H_t(y)$ is monotonically increasing in y. For the non-exclusive case, the optimal production policy is to make the inventory level after production from both capacity, y_{2t}, as close to $S_F(c_t)$ as possible. Then, if $x < S_F(c_t) - M_t$, it is optimal to set $y_{1t} = x + M_t$ and $y_{2t} = S_F(c_t)$.

When $c_t > c_K$, the reserved capacity level K is smaller than M_t. For the non-exclusive case, the optimal policy is modified by replacing M_t with K since only K reserved capacity can be used. For the exclusive case, following a similar proof to that of Theorem 1(a), it can be shown that it is optimal to set $y_{2t} = y_{1t} = S_R(x)$ if $S_K - K \le x < S_H$ and $y_{2t} = y_{1t} = x$ if $x \ge S_H$. Finally, if $S_F(c_t) - K \le x < S_K - K$, it is optimal to set $y_{1t} = x + K$ to make y_{1t} as close to $S_R(x)$ as possible, and set $y_{2t} = y_{1t}$ since the optimal inventory level after production from the reserved capacity, $x + K$, is larger than $S_F(c_t)$. ■

Fig. 1 illustrates the optimal policy in Theorem 1. For a given capacity reservation level K, the state space (x, c_t) is divided into different decision regions. If the state (x, c_t) falls into region I, it is optimal to raise the inventory level to $S_F(c_t)$ by producing an amount of $\min(M_t, K)$ from the reserved capacity and the additional amount over $\min(M_t, K)$ from the capacity purchased in the spot market. For (x, c_t) in region II, it is optimal to produce up to $S_R(x)$ exclusively from the reserved capacity. However, in region II′, the desired inventory level $S_R(x)$ cannot be reached and it is optimal to exploit all the reserved capacity to raise the inventory level as close to $S_R(x)$ as possible. Finally, for (x, c_t) in region III, it is optimal for the manufacturer to do nothing.

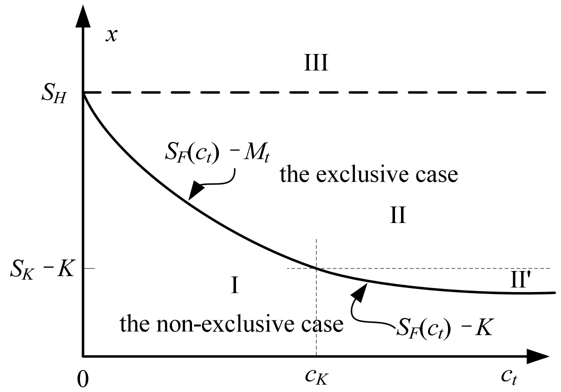

Fig. 1 The optimal policy for different x and c_t

The optimal production policy in Theorem 1 can be interpreted as a quantity-dependent base stock policy with dual production capacity supply sources. If the production quantity is lower than a threshold level, the manufacturer produces exclusively from the reserved capacity; otherwise he produces an amount equal to the threshold level from the reserved capacity and the additional amount over the threshold level from the capacity purchased from the spot market. The threshold level of production quantity, $\min(M_t, K)$, is non-decreasing in the spot price c_t. When the capacity reservation level is sufficient, the optimal produce-up-to level for the exclusive case is $S_R(x)$; otherwise, this desired inventory level cannot be reached and the optimal produce-up-to level is $x + K$. For the non-exclusive case the optimal produce-up-to level is $S_F(c_t)$, which is decreasing in c_t.

Corollary 1: *The threshold of x that triggers spot purchase is decreasing in c_t.*

Proof: It is clearly seen from Theorem 1 that spot purchase is used only when the initial inventory level x is low (i.e., $x < S_F(c_t) - \min(M_t, K)$). Since $S_F(c_t)$ is decreasing in c_t and $\min(M_t, K)$, is non-decreasing in c_t, the threshold that triggers spot purchase, $S_F(c_t) - \min(M_t, K)$, is decreasing in c_t. ■

Corollary 1 implies that with a higher spot market price, spot purchase will be used by the manufacturer at a lower initial inventory level.

It is shown in Theorem 1 that if the capacity reservation level K is insufficient, the manufacturer cannot extract the benefit of lower capacity cost from capacity reservation. However, if K is too large, some amount of the reserved capacity will never be used. In Lemma 3, given the initial inventory level at the beginning of the planning horizon, x_1, we characterize the convexity of the expected total discounted cost $V_1(x_1, K)$ in Eq. 2 with respect to the capacity reservation level.

Lemma 3: *Given the initial inventory level x_1, $V_1(x_1, K)$ is convex in K.*

Proof: The convexity of $V_1(x_1, K)$ can be shown by induction. The proof is similar with that of Lemma 1. ■

Lemma 3 guarantees that for a given x_1 the optimal capacity reservation level can be obtained by minimizing the expected total discounted cost $V_1(x_1, K)$ with respect to K.

Theorem 2: *When demands are stationary, all cost parameters are time homogeneous, and the discount factor $\alpha \in (0,1)$, the optimal policy for the infinite-horizon case (i.e., $T \to +\infty$) takes the same structure as that of the finite-horizon case.*

Proof: We prove this theorem by verifying that conditions (a)-(d) and (f) in Theorem 8-15 in Heyman and Sobel [33] is hold here. Consider a (non-optimal) production policy which selects $y_1 = x$, $y_2 = S_F(c_t)$ for all t, and let $v^\lambda(x)$ denote its total expected (infinite horizon) discounted cost when initial inventory level is x. Since the discount factor $\alpha \in (0,1)$, it is easy to prove that $v^\lambda(x)$ is bounded. From Theorem 8-13 in Heyman and Sobel [33], condition (a) holds. Since all costs are non-negative, the single-period cost function is increasing and condition (b) holds. Conditions (c) and (d) are immediate. Furthermore, for a given x, the total inventory level after production, y_2, is bounded by $\max(S_H, x)$ and thus Condition (f) is valid. □

Theorem 2 implies that for a given capacity reservation level the optimal policy for the infinite-horizon problem inherits the structure of the T-period problem. Moreover, the convexity of the expected total discounted cost function in Lemma 3 is still hold in the infinite-horizon case as the limit of the convex function $V_1(x_1, K)$ is convex as well.

4. A numerical study: Results and discussion

In this section, we conduct numerical experiments to examine the impacts of spot market characteristics on the optimal capacity reservation level and the corresponding optimal total cost. We consider a five-period example with the following cost parameters: $c_m = 10$, $c_r = 5$, $h = 8$, $b = 50$ and $\alpha = 0.95$. The reserved capacity cost is assumed to be $R(x) = 0.2x^2$. We assume that demand in each period follows a discrete uniform distribution on $[A, B]$, where $A = 1$ and

$B = 20$. We use a simple Markov chain with three possible states: $(c_1, c_2, c_3) = (12 - \Delta, 12, 12 + \Delta)$ to model the spot price process, where a larger value of Δ implies a higher level of spot price variance. The one-step transition probabilities $p_{ij} = pr(c_{t+1} = c_j | c_t = c_i)$ for $i, j = 1,2,3$ are given in the following matrix:

$$P = \begin{bmatrix} 0.80 & 0.15 & 0.05 \\ 0.15 & 0.70 & 0.15 \\ 0.15 & 0.25 & 0.60 \end{bmatrix}$$

Figs 2 and 3 illustrate the optimal capacity reservation level and the corresponding optimal total cost with respect to the initial inventory level at different level of Δ. It is shown that the optimal capacity reservation level is decreasing in the initial inventory level. The explanation is that with a larger initial inventory level, a fewer production quantity is needed and the manufacturer would like to keep a lower capacity reservation level. The optimal total cost is observed to be convex in the initial inventory level, which is in line with the preceding analytical results.

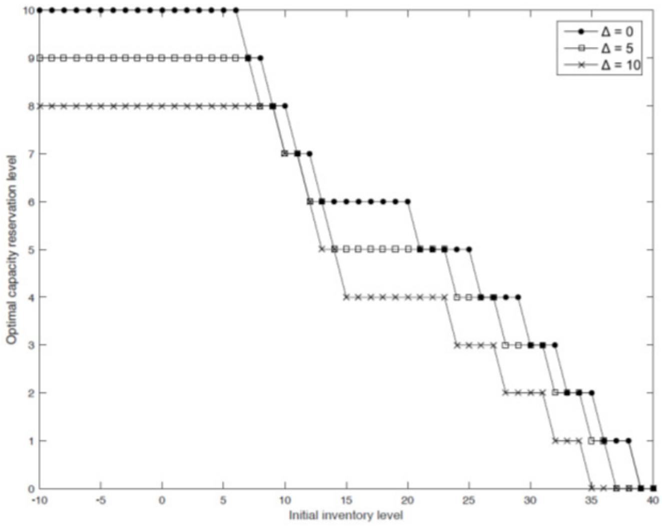

Fig. 2 Optimal capacity reservation level vs. initial inventory level at different level of Δ

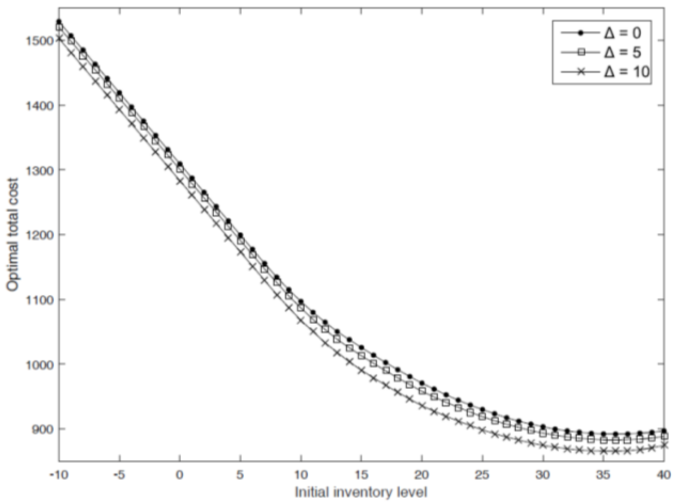

Fig. 3 Optimal total cost vs. initial inventory level at different level of Δ

Another observation is that both the optimal capacity reservation level and the corresponding optimal total cost are decreasing in Δ. This is because that as Δ increases, the likelihood that the manufacturer can purchase capacity at a lower price also increases, which directly benefits the manufacturer and makes the capacity reservation contracts less valuable. This observation implies that the more volatile the spot market price is, the less production capacity the manufacturer would like to reserve in advance.

Next, we examine the impacts of the expected spot price. We set the possible spot price realizations as $(c_1, c_2, c_3) = (S-2, S, S+2)$. Here, a larger value of S implies a higher level of expected spot price. Figs 4 and 5 show the optimal capacity reservation level and the corresponding optimal total cost with respect to the initial inventory level at different level of S. We find that both the optimal capacity reservation level and the corresponding optimal total cost are increasing in S. This is in line with the intuition that as the expected spot price increases the manufacturer becomes more dependent on supply contracts with the long-term supplier. However, when the expected spot price is at a lower level, the optimal capacity reservation level may drop to 0 more quickly as the initial inventory level increases. Under such spot market conditions, the manufacturer will refuse to sign the long-term reservation contract with the supplier and fully depends on the spot market. This is what the Chinese steel makers did in their iron ore procurement in 2014!

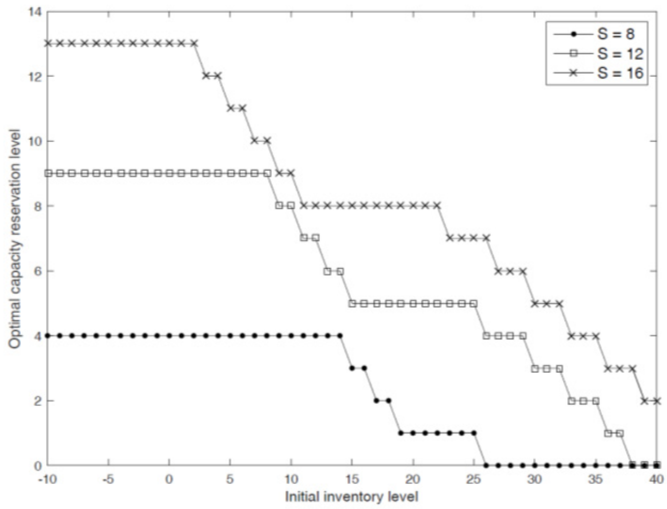

Fig. 4 Optimal capacity reservation level vs. initial inventory level at different level of S

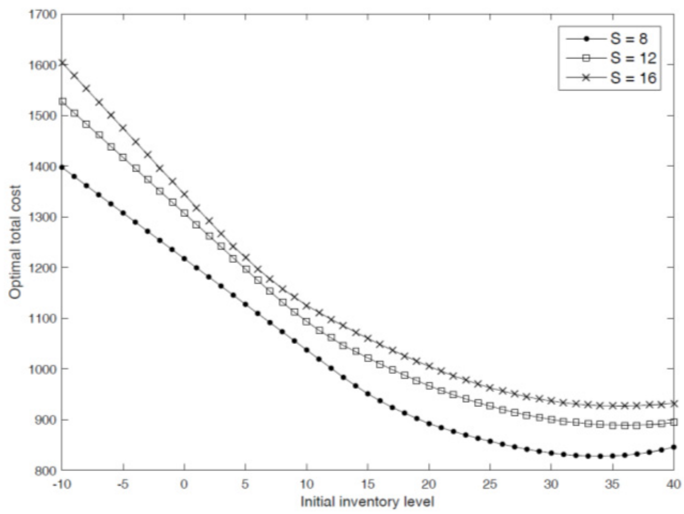

Fig. 5 Optimal total cost vs. initial inventory level at different level of S

For the final analysis, we investigate the impacts of spot price distribution. Denote the steady-state distribution of the spot prices as $p_i = pr(c_t = c_i)$ for $i = 1, 2, 3$. We set $(p_1, p_2, p_3) = (\beta, 0, 1 - \beta)$, where the value of $\beta \in [0, 1]$ affects the spot price distribution. It can be shown that the expected spot price, $12 + (1 - 2\beta)\Delta$, is decreasing in β and the spot price variance, $4\beta(1 - \beta)\Delta^2$, is convex in β.

The observation that both the optimal capacity reservation level and the corresponding optimal total cost are decreasing in Δ is still valid for the case $\beta > 0.5$. The interpretation is as follows. In this case, a larger Δ will lead to a lower expected spot price and a higher level of spot price variance, both of which result in a decreasing trend in the optimal capacity reservation level and the corresponding optimal total cost.

However, for the case $\beta < 0.5$, these observations on the impacts of Δ may be quite different. For example, as shown in Fig. 6, the optimal capacity reservation level is increasing with Δ when $\beta = 0.2$. The explanation is that when the spot price is more possible to take the high realization, a larger price difference Δ will increase the manufacturer's risk of purchasing at a higher spot price and thus makes the capacity reservation contract more favorable.

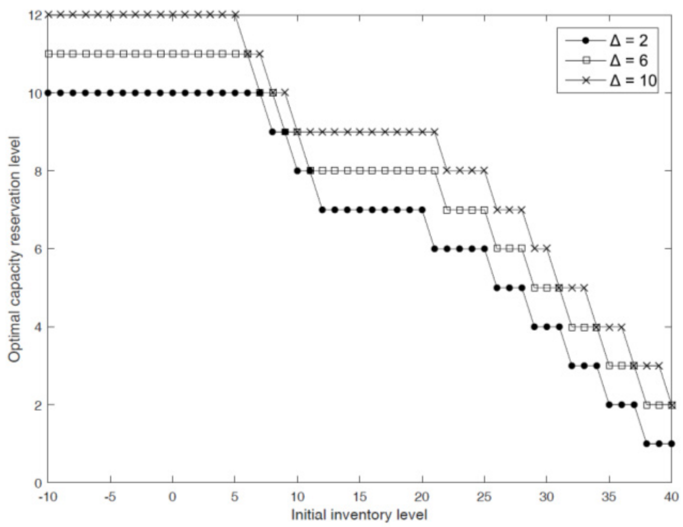

Fig. 6 The optimal capacity reservation level vs. initial inventory level at different level of Δ for the case β = 0.2

5. Conclusion

In this paper, we have studied how a manufacturer jointly uses a long-term supplier and the spot market to procure production capacity and produce finished products to satisfy random market demands. A multi-period production planning problem with a convex purchasing cost of reserved capacity is discussed. The manufacturer's optimal capacity reservation and production decisions that minimize the expected total discounted cost over the planning horizon are provided.

For a given capacity reservation level, the optimal production policy is a quantity-dependent base stock policy, characterized by a threshold of production quantity that increases with the spot price. That is, if the production quantity is lower than the threshold, the manufacturer produces exclusively from the reserved capacity; otherwise he produces an amount equal to the threshold level from the reserved capacity and the additional amount over the threshold level from the capacity purchased from the spot market. For the former case the optimal produce-up-to level is increasing with the initial inventory level, whereas for the latter case the optimal produce-up-to level is decreasing with the spot price. Compared with the linear reserved capacity purchasing cost case, we find that the optimal order-up-to level for purchasing from the long-term supplier is an increasing function of the initial inventory level, rather than a fixed value.

With this structure of periodic production policy, the expected total discounted cost function is shown to be convex in the capacity reservation level. These results are also extended to the infinite-horizon case. We have also examined the impacts of initial inventory level and spot characteristics on the capacity reservation decisions, and found that: (1) when the spot price is more possible to take the lower (higher) realization, the capacity reservation level may decrease (increase) with the price difference between different spot price realizations; (2) the capacity reservation level increases with the expected spot price; and (3) the capacity reservation level decreases with the initial inventory level at the beginning of the planning horizon.

For future research, there are several important and possible extensions to this study. First, we assume that the demand and spot price distributions are independent as most related models did in the literature. However, in some cases, the spot price of production capacity may influence the selling price of the finished product, which in turn affects the demand for the finished product as well as the demand for the production capacity. Hence, developing production planning models with considering the correlation between demand and spot price is an interesting direction for future research. Second, we assume that the spot market is reliable with an infinite supply capacity in this paper. Considering various supply conditions in the spot market such as supply capacity limits, supply uncertainty and capacity-dependent spot price will be another interesting direction. Finally, incorporating endogenous sales price of the finished products into our model and investigating the periodic pricing decisions in the production planning problem is also an interesting direction for future research.

Acknowledgement

This work was supported by Fundamental Research Funds of Beijing Jiaotong University (Special Fund Project of Humanities and Social Science, Grant No. B15JB00500).

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Appendix A

Proof of Lemma 2: By Lemma 1, it is straightforward to verify that $H_t(y)$, $R(y-x)+H_t(y)$ and $c_t y + H_t(y)$ are both convex in y. Then S_H, $S_R(x)$, $S_F(c_t)$ and S_K are given by the first order condition, where

$$H_t'(S_H) = 0 \tag{A.1}$$

$$R'(S_R(x)-x) + H_t'(S_R(x)) = 0 \tag{A.2}$$

$$H_t'(S_F(c_t)) + c_t = 0 \tag{A.3}$$

and

$$R'(K)+H_t'(S_K) = 0 \tag{A.4}$$

It follows from Eq. A.1, Eq. A.3 and Eq. A.4 that

$$H_t'(S_K) = -R'(K) \le H_t'\big(S_F(c_t)\big) = -c_t < H_t'(S_H) = 0, \text{when } c_t \le c_K, \tag{A.5}$$

and

$$H_t'\big(S_F(c_t)\big) = -c_t < H_t'(S_K) = -R'(K) < H_t'(S_H) = 0, \text{when } c_t > c_K. \tag{A.6}$$

Then, the result of Lemma 2(a) is obtained by the fact that $H_t'(y)$ is increasing in y.

From Eq. A.2, it is easy to show that:

- if $S_R(x) = S_H$, then x is given by $R'(S_H - x) + H_t'(S_H) = R'(S_H - x) = 0$;
- if $S_R(x) = S_F(c_t)$, then x is given by $R'(S_F(c_t) - x) + H_t'\big(S_F(c_t)\big) = R'(S_F(c_t) - x) - c_t = 0$.

From Eq. A.3, it is easy to show that

- if $c_t = 0$, then $S_F(0)$ is given by $H_t'\big(S_F(0)\big) = 0$.
- if $c_t = c_K$, then $S_F(c_K)$ is given by $H_t'\big(S_F(c_K)\big) = -c_K = -R'(K)$.

Then, the results of Lemma 2(b) are obtained by the convexity of $R(\cdot)$ and the definition of S_H, S_K and M_t.

For Lemma 2(c) and 2(d), taking the first derivative of $S_R(x)$ with respect to x, we have

$$\frac{dS_R(x)}{dx} = \frac{R''(S_R(x) - x)}{R''(S_R(x) - x) + H_t''(S_R(x))} > 0 \tag{A.7}$$

and

$$\frac{dS_R(x) - x}{dx} = \frac{dS_R(x)}{dx} - 1 \tag{A.8}$$

For Lemma 2(e), taking the first derivative of $S_F(c_t)$ with respect to c_t, we have

$$\frac{dS_F(c_t)}{dc_t} = -\frac{1}{H_t''\big(S_F(c_t)\big)} < 0 \tag{A.9}$$

For Lemma 2(f), taking the first derivative of S_K with respect to K, we have

$$\frac{dS_K}{dK} = -\frac{R''(K)}{H_t''(S_K)} < 0 \tag{A.10}$$

For Lemma 2(g), by the definition of M_t, we have

$$\frac{dM_t}{dc_t} = \frac{1}{R''(M_t)} > 0 \tag{A.11}$$

It follows from the convexity of $R(\cdot)$ and the definition of M_t and c_K that,

- if $c_t = R'(M_t) \le c_K = R'(K)$, then $K \ge M_t$;
- if $c_t = R'(M_t) > c_K = R'(K)$, then $K < M_t$.

This completes the proof of Lemma 2.