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Revenue sharing or profit sharing? An internet production perspective

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A B S T R A C T

The booming of Internet economics brings new opportunities for small- and medium-sized product and service providers in e-platforms. Usually, the Internet platform (thereafter "platform") and Internet providers (thereafter "providers") operate under a consignment revenue sharing production model. Another production model is profit sharing, under which the platform undertakes part of the providers' operational costs. Intuitively, common sense conjectures that the platform prefers the revenue sharing model while the providers may prefer the profit sharing scheme. However, this is not the case. In this paper, we compare these two forms of emerging production models with a theoretical framework and investigate both market participants' performance under different schemes. Starting from the single provider case, we find that the provider has less incentive to operate under the revenue sharing contract when compared with the profit sharing contract. Counter intuitively, we identify the threshold of the cutting ratio above which it is more beneficial for the platform to choose the profit sharing mode. Our results are proved to be robust when the number of the providers increases. A numerical study is provided to illustrate this effect.

A R T I C L E I N F O

Keywords:
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1. Introduction

With the ascending of Internet economics, more product (service) providers are cooperating with platforms such as Airbnb, Amazon, Uber, and eBay. According to a survey by JP Morgan & Chase, 4.3 % of adults earned income from the Internet production economy by 2016 [1]. An increasing number of workforces are seeking job opportunities through the platform production eco-system. Homeowners coordinate through the online rental platform, Airbnb, to provide cheaper and more convenient houses [2]. On the other side, tenants are able to find more households through the Internet with less transaction and searching costs than ever before. Another example comes with Internet producers such as Amazon and Alibaba, through which the sellers can sell to more potential end customers. Due to the convenience and cost saving of FBA (fulfillment by Amazon) service, more small business sellers are doing business through this powerful Internet production. Apart from that, thousands of active Amazon prime members are another important impetus to sell on the platform [3]. The sellers on the platform retain the authority to decide what kind of product to sell, how much to stock, and even the selling price [4].

In platform production eco-systems, the product (service) providers and the e-platform constitute a complex social system. Usually, the platform and the product (service) providers cooperate under a consignment revenue sharing (thereafter "R-S") arrangement [5], under which the sales revenue is split between the providers and the platform for each successful sale. Under such a production model, the platform bears no operational costs. However, the product (service) provider undertakes much of the over-stock or under-stock losses, which in turn reduces the provider's operational incentive. In comparison with the prevailing consignment R-S mode, profit sharing (thereafter "P-S") mechanisms can coordinate the interest between the platform and the providers at the cost of the platform taking on more risk. We realize that some core enterprises in a supply chain system have started to build up a P-S mechanism by holding the shares of their main suppliers in recent years, such as the Li & Fung Group, an HK based company [6]. The supply chain has achieved very good performance by taking the upstream suppliers' interest into account.

Based on this Internet production phenomenon, some interesting research questions arise: 1) How does the production model type impact both market participants' operational decisions? 2) Which model is more profitable for the platform, the R-S or the P-S mode? 3) How will the system performance vary with different production models? 4) Will the competition landscape alter the results above? To answer these research questions, we construct a theoretical model by considering a powerful platform hosting several products (service) providers. We start from the single provider case, which means there is only one provider on the digital platform. We show that the R-S production model lessens the provider's incentive to operate. The system performance decreases in cutting ratio. Intuitively, the platform prefers the R-S mode while the providers may prefer the P-S from a risk-sharing perspective. However, we do not identify a threshold of the cutting ratio above which the platform is better off by selecting the P-S mode. We then extend the basic model to cases where multiple providers exist. The results derived from the single provider model prove to be robust.

The remainder of the paper proceeds as follows. In Section 2, we review the literature that has the closest relationship with this paper. Section 3 follows with the model preliminaries. We introduce the time sequence and the notation conventions. We start with the single provider case in Section 4, which means there is only one provider selling through the platform. Our discussion is divided into two parts depending on the production model variations studied in this paper, namely, R-S and P-S. Section 5 extends the basic model with a single provider to multiple providers. We summarize our main findings and conclusions in the last part.

2. Literature review

Two streams of literature relate to our research; these streams are R-S and P-S in the supply chain and horizontal competitions in the supply chain.

The first stream lies in R-S and P-S in a supply chain. Dana and Spier [7] make an early attempt to investigate the revenue-sharing contract and vertical control in a video rental market. Wang *et al*. [5] study the channel performance under a consignment revenue-sharing contract by taking the e-retailing guru, Amazon, as a prototype, and finding that both the overall channel performance and the performance of individual firm depend on demand price elasticity as well as the retailer's share of the channel cost. Gong *et al*. [8] examine the production coordination problem from the perspective of asymmetric information: how a manufacturer coordinates the relationships with its subsidiary firm(s). Giannoccaro and Pontrandolfo [9] propose a model of a supply chain contract aimed at coordinating a three-stage supply chain, which is based on the revenue sharing mechanism. Gerchak and Wang [10] compare two distinct types of arrangements between an assembler/retailer and its suppliers. One scheme is a vendor-managed inventory with revenue sharing, and the other is a wholesale-price driven contract. Cachon and Lariviere [11] demonstrate that revenue sharing coordinates a supply chain with a single retailer and arbitrarily allocates the total supply chain profit. They compare this contract with other contract forms such as buy-back contracts, price-discount contracts, and quantity-flexibility contracts. For more application of revenue sharing contracts in operations management literature, please see the in-depth review by Cachon [12]. Chen and Gupta [13] study a problem where the budget constraint supplier and retailer operates under a consignment revenue sharing contract in the presence of external financing. On the other hand, some literature highlights the advantages of P-S within a supply chain coordination framework. Çanakoğlu and Bilgic [14] analyze a two-stage telecommunication supply chain consisting of one operator and one vendor under a multiple period setting. They suggest a P-S contract where firms share both the revenue and operating costs. Wei and Choi [15] provide an industry practice of P-S in the apparel industry. Then, they explore the use of a wholesale pricing and profit sharing scheme for supply chain coordination under the mean-variance criteria. We differ significantly from the existing literature because we compare two different emerging production models, the R-S and P-S, within the framework of an online platform.

The second stream of research that relates to our research is on horizontal supply chain competition. Please see Singh and Vives [16], and Kreps and Scheinkman [17] for further comparison between quantity and pricing competitions. Gong *et al*. [18] investigate resources sharing's impact on the supply chain revenue. Van Meighem and Dada [19] summarize the value of strategic postponement for two firms competing in capacity and responsive pricing. Goyal and Netessine [20] evaluate the strategic value of manufacturing flexibility in an uncertain environment to understand whether the value of flexibility increases or decreases under competition. Anupindi and Jiang [21] consider duopoly models where firms make decisions on capacity, production, and price under demand uncertainty. In their model, flexible firms can postpone production decisions until the actual demand curve is observed, but inflexible firms cannot. Under general demand structures and cost functions, they characterize the equilibrium for symmetric duopoly and establish the strategic equivalence of price and quantity competitions when firms are flexible. In addition, Wen *et al*. [22, 23] also propose related methods for volatility of energy market, which provides support for this study. We apply the horizontal quantity competition in our model. We combine the production model selection with horizontal competition in an e-retailing supply chain framework.

3. Model preliminaries

Consider a powerful platform (thereafter "she"), hosting at least one product (service) provider (thereafter "he"), i.e., small- and medium-sized sellers selling through an e-retailing platform such as Amazon or Taobao, which is a Chinese based e-commerce platform owned by Alibaba. The platform and providers reach an agreement on the profit structure beforehand. In this paper, we consider two types of modes, namely, R-S and P-S. The former means that for each successful sale, the platform takes a certain percentage of the total sales revenue, which is known as a "referral fee" according to industrial practice. Usually, referral fees are different across different product categories. Note that under such a production model, the platform undertakes almost no operational risk since she refuses to share the production costs with the providers. However, the providers take on much of the inventory costs. Apart from the R-S mode, there is another production model, called P-S. In such a case, for each profit made, the platform takes a cut of the total profit. Therefore, the time sequence is as follows: the powerful digital platform negotiates with the providers on the production model type (R-S or P-S). Then, providers compete in quantities simultaneously. The market clearance price is determined then. The platform takes the revenue (profit) cut according to the agreement.

For notation conventions, we denote $q^i, i = 1,2, \dots, n$ as the quantity provided by the provider i, where the subscript states the number of providers. We apply a linear demand system where the market clearance price is determined by $p = a - \sum_{i=1}^{n} q^i$. Intercept a denotes the basic market demand. For simplicity without any loss of generality, there is no uncertainty considered in our model. γ $(0 \leq \gamma \leq 1)$ is the revenue (profit) cutting ratio, which is negotiated between the platform and the providers ex ante. Therefore, we assume the ratio is exogenously given according to industry practice. Let $\pi^i, i = 1,2, \dots, n$ stand for the providers' profit and Π denote the platform's profit.

4. Single provider case

We start from a case where only one provider is selling through the platform, which can be regarded as a benchmark for the competing scenario. We divide the discussion into two parts depending on the production model chosen. Then, we compare both agents' performance under these two production models.

Before we investigate the details of these two production models, we show the "first-best" solution when $\gamma = 0$ for both cases. In such case, the provider's profit is:

$$\pi(q;\gamma) = (a-q)q - cq \tag{1}$$

Therefore, the corresponding first-best quantity decision is the same in the monopoly case as $q_{FB} = \frac{a-c}{2}$. Then, we start from the R-S model.

4.1 R-S model

Under the R-S model, the provider's profit is:

$$\pi(q;\gamma) = (1-\gamma)(a-q)q - cq \tag{2}$$

We can solve the optimal quantity decision of the provider as$q^*_{RS}(\gamma) = \frac{a}{2} - \frac{c}{2(1-\gamma)} \leq q_{FB} = \frac{a-c}{2}$. $q^*_{RS}(\gamma)$ is determined when the marginal revenue is equal to the marginal cost. Accordingly, by substituting $q^*_{RS}(\gamma)$ into the provider's profit. We have $\pi_{RS}\left(q^*_{RS}(\gamma)\right) = \frac{(a(1-\gamma)-c)^2}{4(1-\gamma)}$.

Proposition 1: *With only one product (service) provider, the provider's quantity decision and profit decreases in γ under R-S.*

Proposition 1 shows that the revenue cut ratio reduces the provider's incentive to operate. As γ increases, the provider produces less and receives less profit. That is, because the platform takes a larger part of the total sales revenue as γ increases. On the other side, the provider's quantity decision deviates from the first-best solution due to a decrease in operational incentive.

In such a case, the platform's profit is:

$$\Pi_{RS}\left(q^*_{RS}(\gamma)\right) = \frac{(a(1-\gamma)-c)\gamma(a(1-\gamma)+c)}{4(1-\gamma)^2} = \frac{\gamma(a^2(1-\gamma)^2 - c^2)}{4(1-\gamma)^2} \tag{3}$$

Proposition 2: *With only one product (service) provider, the platform's profit is concave in* γ. *The optimal referral fee* γ^* *can be derived with* $\frac{\partial \Pi_{RS}(q^*_{RS}(\gamma))}{\partial \gamma} = 0$ *under R-S.*

When there is only one product (service) provider on the platform, the platform's profit is not always increasing in γ for the following reasons. Even though the total percentage of sales revenue increases as γ increases, the provider also reduces the output amount as γ increases. Consider an extreme case such that when the referral fee is extremely large, providers may shut down production, which leaves both market participants with 0 profit.

Denote the system profit as:

$$\begin{aligned} W_{RS}\left(q^*_{RS}(\gamma)\right) &= \pi_{RS}\left(q^*_{RS}(\gamma)\right) + \Pi_{RS}\left(q^*_{RS}(\gamma)\right) \\ &= c^2(1-2\gamma) + a^2(1-\gamma)^2 - 2ac(1-\gamma)^2 4(1-\gamma)^2 \end{aligned} \tag{4}$$

By checking the properties of $W_{RS}\left(q^*_{RS}(\gamma)\right)$, we have Proposition 3 below:

Proposition 3: *With only one product (service) provider, the system profit decreases in* γ*under R-S.*

Proposition 3 shows that the sum of the platform and providers' profit decreases in γ. When $\gamma = 0$, the revenue sharing production model degenerates to the first best model. As γ increases, the interest conflict between the platform and provider deepens because the platform is not burdened by the production cost carried by the provider. To eliminate the incentive mismatch, they can operate under a P-S model, as noted by Wei and Choi [14]. Then, we analyze the case when the P-S model is present.

4.2 P-S model

Under a P-S model, the platform will take a cut of the total profit gained. Thus, the provider's profit is:

$$\pi_{PS}(q;\gamma) = (1-\gamma)(a-c-q)q \tag{5}$$

where γ is the cut ratio under a P-S model determined ex ante. We can find from the expression that the platform helps to bear the production cost under such a case.

We can solve the optimal quantity decision for the provider as $q^*_{PS}(\gamma) = \frac{a-c}{2} = q^*_{RS}(0) = q_{FB} = \frac{a-c}{2}$. Additionally, $q^*_{PS}(\gamma)$ is determined when the marginal revenue is equal to the marginal cost. Observe that the optimal quantity is independent of γunder P-S, which is the same as the first-best solution. The provider's profit is $\pi_{PS}\left(q^*_{PS}(\gamma)\right) = \frac{(1-\gamma)(a-c)^2}{4}$, which is decreasing in γ.

Proposition 4: *With only one product (service) provider, the provider's profit decreases in γ under P-S. The optimal quantity decision is independent of γ and achieves the first-best solution.*

Proposition 4 shows that the provider achieves the first-best solution under P-S because at this time the platform takes on the operational risks together with the provider. Unlike the revenue sharing scenario, the quantity decision is independent of γ.

In such a case, the platform's profit is:

$$\Pi_{PS}\big(q^*_{PS}(\gamma)\big) = \frac{\gamma(a-c)^2}{4} \tag{6}$$

which is increasing in γ. As γ increases, the platform takes a larger part of the total profit. The system profit is:

$$W_{PS}\big(q^*_{PS}(\gamma)\big) = \pi_{PS}\big(q^*_{PS}(\gamma)\big) + \Pi_{PS}\big(q^*_{PS}(\gamma)\big) = \frac{(a-c)^2}{4} \tag{7}$$

As we can find from the system profit, it equals the first-best profit. Under the P-S model, the interest conflicts between both market participants have been eliminated. We then need to compare their performance under different production models.

4.3 A comparison between R-S and P-S

We summarize both parties' profit in Table 1, where "R-S" is the revenue sharing and "P-S" is the profit sharing.

We then compare both players' profit under R-S and P-S for a given γ. We start from the provider's side.

Denote $\Delta\pi_{PS}(\gamma) = \pi_{PS}\left(q^*_{PS}(\gamma)\right) - \pi_{RS}\left(q^*_{RS}(\gamma)\right)$. We have

$$\Delta\pi_{PS}(\gamma) = \frac{(a(1-\gamma)-c)^2}{4(1-\gamma)} - \frac{(1-\gamma)(a-c)^2}{4} = \frac{c\big(c(-2+\gamma)-2a(-1+\gamma)\big)\gamma}{4(-1+\gamma)} \tag{8}$$

$\Delta\pi_{PS}(0) = 0$ holds.

$$\frac{\partial\Delta\pi_{PS}(\gamma)}{\partial\gamma} = \frac{c(-2a(-1+\gamma)^2 + c(2-2\gamma+\gamma^2))}{4(-1+\gamma)^2} \tag{9}$$

Table 1 Profits with a single provider

	Provider	Platform	System (Provider + Platform)
R-S	$\frac{(a(1-\gamma)-c)^2}{4(1-\gamma)}$	$\frac{\gamma(a^2(1-\gamma)^2-c^2)}{4(1-\gamma)^2}$	$\frac{c^2(1-2\gamma)+a^2(1-\gamma)^2-2ac(1-\gamma)^2}{4(1-\gamma)^2}$
P-S	$\frac{(1-\gamma)(a-c)^2}{4}$	$\frac{\gamma(a-c)^2}{4}$	$\frac{(a-c)^2}{4}$

$$\frac{\partial^2 \Delta\pi_{PS}(\gamma)}{\partial\gamma^2} = \frac{c^2}{2(1-\gamma)^3} > 0 \tag{10}$$

By solving $\Delta\pi_{PS}(\gamma) = 0$, we have

$$\gamma^\pi = \frac{2(a-c)}{2a-c} = 1 - \frac{c}{2a-c}, \tag{11}$$

which we call the "R-S or P-S" threshold for the provider. Recall that to avoid triviality, we must have $(1-\gamma)a > c$, which is equivalent to $\gamma < 1 - \frac{c}{a} < \frac{c}{2a-c}$. Therefore, $\Delta\pi_{PS}(\gamma)$ is decreasing in the interval. In other words, $\pi_{PS}\left(q^*{}_{PS}(\gamma)\right) \leq \pi_{PS}\left(q^*{}_{RS}(\gamma)\right)$ for $\gamma \in [0,1-\frac{c}{a}]$.

We then compare the platform's profit under R-S and P-S.

Denote $\Delta\Pi_{PS}(\gamma) = \Pi_{PS}\left(q^*{}_{PS}(\gamma)\right) - \Pi_{RS}\left(q^*{}_{RS}(\gamma)\right)$. We have

$$\Delta\Pi_{PS}(\gamma) = \frac{\gamma(a^2(1-\gamma)^2 - c^2)}{4(1-\gamma)^2} - \frac{\gamma(a-c)^2}{4} = \frac{c\gamma\left(2a(1-\gamma)^2 - c(1+(1-\gamma)^2)\right)}{4(1-\gamma)^2} \tag{12}$$

$\Delta\Pi_{PS}(0) = 0$ holds.

$$\frac{\partial\Delta\Pi_{PS}(\gamma)}{\partial\gamma} = \frac{c(2a(-1+\gamma)^3 - c(-2+2\gamma-3\gamma^2+\gamma^3))}{4(-1+\gamma)^3} \tag{13}$$

$$\frac{\partial^2\Delta\Pi_{PS}(\gamma)}{\partial\gamma^2} = -\frac{c^2(2+\gamma)}{2(1-\gamma)^4} < 0 \tag{14}$$

Therefore, there is another root γ^Π to make $\Delta\Pi_{PS}(\gamma^\Pi) = 0$ hold. $\gamma^\Pi = 1 - \frac{\sqrt{2ac-c^2}}{2a-c}$, which we call the "R-S or P-S" threshold for the platform. Then, we compare $\frac{\sqrt{2ac-c^2}}{2a-c}$ with $\frac{c}{a}$. When $a > c$, we have $\frac{\sqrt{2ac-c^2}}{2a-c} > \frac{c}{a}$. Therefore, $1 - \frac{\sqrt{2ac-c^2}}{2a-c} < 1 - \frac{c}{a}$.

In other words, $\Pi_{PS}\left(q^*{}_{PS}(\gamma)\right) \geq \Pi_{RS}\left(q^*{}_{RS}(\gamma)\right)$ for $\gamma \in [0,1-\frac{\sqrt{2ac-c^2}}{2a-c}]$ and $\Pi_{PS}\left(q^*{}_{PS}(\gamma)\right) < \Pi_{RS}\left(q^*{}_{RS}(\gamma)\right)$ for $\gamma \in (1-\frac{\sqrt{2ac-c^2}}{2a-c}, 1-\frac{c}{a}]$. However, from the platform's side, this does not always hold for $\Pi_{PS}\left(q^*{}_{PS}(\gamma)\right) > \Pi_{RS}\left(q^*{}_{RS}(\gamma)\right)$. Theorem 1 below summarizes the above results:

Theorem 1: *When $a(1-\gamma) > c$, the powerful platform selects a revenue sharing contract for $\gamma \in [0,1-\frac{\sqrt{2ac-c^2}}{2a-c}]$, and selects a profit sharing contract for $\gamma \in (1-\frac{\sqrt{2ac-c^2}}{2a-c}, 1-\frac{c}{a}]$. The platform and the product (service) provider achieve interest alignment when $\gamma \in [1-\frac{\sqrt{2ac-c^2}}{2a-c}, 1-\frac{c}{a}]$.*

Theorem 1 seems slightly counterintuitive since common sense conjectures that the platform will always prefer the R-S model since she will not bear any operational costs (take no additional operational risks). The provider seems to always prefer the P-S model as long as the platform is willing to share risks with him. However, we identify some conditions under which the platform may be better off by providing the P-S model, which means she must have no hesitation with undertaking more operational risks when γ is relatively high. If not, the provider loses the desire to operate. We then try to figure out how the system profit varies under different production models. We have Proposition 5 below.

Proposition 5: *The system profit under R-S is no more than the profit with P-S. The system achieves first-best performance with P-S.*

The platform and product (service) provider's interest conflicts under the R-S because the platform is unwilling to undertake more operational costs. However, with a P-S alignment, the system achieves the *first-best* solution. It seems that P-S is the best mechanism from a systematic view. However, in reality, the powerful platform may control the game and selects the best scheme for herself, which in turn harms the social welfare.

How will the results change when more than one provider is present in the market? Will the competition landscape alter our main results? We extend our basic model to a case where multiple providers exist.

5. Competing providers case

Usually, the product (service) providers on the platform always face cutthroat competition. The competition landscape may have an impact on the performance of both players. We consider that n providers are selling through the platform. As we have introduced in the model preliminaries section, the market clearance price is determined by $p = a - \sum_{i=1}^{n} q_i$. Similar to the single provider case, we start from the R-S model.

5.1 R-S model

The provider i's profit function is

$$\pi^{i}{}_{RS}\left(q^i; q^{-i}\right) = (1-\gamma)\left(a - q^i - \sum_{j \neq i} q^j\right) q^i - c q^i \tag{15}$$

By solving the Nash equilibrium (NE) with the responsive functions, the unique NE is derived as $q^{i}{}_{RS}(\gamma; n) = \frac{2}{n+1}\left(\frac{a}{2} - \frac{c}{2(1-\gamma)}\right) = \frac{2}{n+1} q^{*}{}_{RS}(\gamma)$. We can find that with competition each provider's willingness to operate decreases as the number of providers increases. Additionally, the equilibrium quantity is decreasing in γ. The total providers' profit is

$$\pi_{PS}\left(q^{i}{}_{RS}(\gamma; n), n\right) = \frac{n(c + a(-1+\gamma))^2}{(1+n)^2(1-\gamma)} \tag{16}$$

Proposition 6: *With n product (service) provider, the provider's profit is decreases innand γ under R-S.*

From Proposition 6, we know that as competition becomes more cutting-throat, the provider's profit drops due to a diminishing marginal sales revenue.

We can derive the platform's profit

$$\Pi_{RS}\left(q^{i}{}_{RS}(\gamma; n), n\right) = \frac{n(a(1-\gamma) - c)\gamma(a + cn - a\gamma)}{(1+n)^2(1-\gamma)^2} \tag{17}$$

Proposition 7: *With n product (service) provider, the provider's profit decreases in n under P-S.*

As the product market becomes more competitive, the total output in the market increases dramatically, which leads to a decrease in the market clearance price. Therefore, the total sales revenue decreases. Accordingly, the platform gets less from the revenue cut.

Denote the system profit as:

$$\begin{aligned} W_{RS}\left(q^{i}{}_{RS}(\gamma; n), n\right) &= \pi_{RS}\left(q^i(\gamma; n), n\right) + \Pi_{RS}\left(q^i(\gamma; n), n\right) \\ &= \frac{n(a(1-\gamma) - c)(a - a\gamma + c(-1 + \gamma + n\gamma))}{(1+n)^2(1-\gamma)^2} \end{aligned} \tag{18}$$

We then analyze the P-S model with n symmetric providers.

5.2 P-S model

We then consider the P-S model, which means the platform will take a profit cut from each provider. In such a case, the provider i's profit function is:

$$\pi^i{}_{PS}\left(q^i;q^{-i}\right) = (1-\gamma)\left(a-c-q^i-\sum_{j\neq i} q^j\right)q^i \tag{19}$$

By solving the problem above, we can derive the equilibrium quantity as $q^i{}_{PS}(\gamma;n) = \frac{a-c}{n+1} > \frac{2}{n+1}\left(\frac{a}{2}-\frac{c}{2(1-\gamma)}\right) = q^i{}_{RS}(\gamma;n)$. It shows that the equilibrium quantity is independent of the revenue cut parameter γ. The provider under the revenue sharing contract has less incentive to produce. We take the equilibrium quantity into the provider's profit function:

$$\pi^i{}_{PS}\left(q^i{}_{PS}(\gamma;n),n\right) = \frac{(1-\gamma)n(a-c)^2}{(n+1)^2} \tag{20}$$

The platform's profit is

$$\Pi_{PS}\left(q^i{}_{PS}(\gamma;n),n\right) = \frac{\gamma n(a-c)^2}{(n+1)^2} \tag{21}$$

Similar to Section 4, we summarize the market players' profit function in Table 2.

We then compare both market agents' profit under R-S and P-S for a given γ.

Denote $\Delta\pi_{PS}(\gamma,n) = \pi_{PS}\left(q^i{}_{PS}(\gamma;n),n\right) - \pi_{RS}\left(q^i{}_{RS}(\gamma;n),n\right)$, we have

$$\Delta\pi_{PS}(\gamma,n) = \frac{cn(c(-2+\gamma)-2a(-1+\gamma))\gamma}{(1+n)^2(-1+\gamma)} \tag{22}$$

We can check that the SOC of $\Delta\pi_{PS}(\gamma,n)$ w.r.t. γ is greater than 0. To make $\Delta\pi_{PS}(\gamma,n) = 0$, we have $\gamma^\pi(n) = 0, \gamma^\pi(n) = \frac{2(a-c)}{2a-c} = 1-\frac{c}{2a-c}$.Recall that we call $\gamma^\pi(n)$ as the "R-S or P-S" threshold for the provider with n symmetric providers. To avoid triviality, we must have $(1-\gamma)a > c$. Therefore, $\gamma < 1-\frac{c}{a} < \frac{c}{2a-c}$. Thus, $\pi_{PS}\left(q^i{}_{PS}(\gamma;n),n\right) \leq \pi_{RS}\left(q^i{}_{RS}(\gamma;n),n\right)$ for $\gamma \in [0,1-\frac{c}{a}]$. Note that the "R-S or P-S" threshold has nothing to do with the number of providers. From the provider's perspective, it always holds for $\pi_{PS}\left(q^i{}_{PS}(\gamma;n),n\right) < \pi_{RS}\left(q^i{}_{RS}(\gamma;n),n\right)$ when $\gamma \in [0,1-\frac{c}{a}]$.

We then check the R-S or P-S threshold for the platform. Denote

$\Delta\Pi_{PS}(\gamma,n) = \Pi_{PS}\left(q^i{}_{PS}(\gamma;n),n\right) - \Pi_{RS}\left(q^i{}_{RS}(\gamma;n),n\right)$.

Therefore,

$$\Delta\Pi_{PS}(\gamma,n) = -\frac{cn\left(c(n+(-1+\gamma)^2)+a(1+n-2\gamma)(-1+\gamma)\right)\gamma}{(1+n)^2(-1+\gamma)^2} \tag{23}$$

$$\frac{\partial^2\Delta\Pi_{PS}(\gamma,n)}{\partial\gamma^2} = -\frac{2cn\left(a(-1+n)(-1+\gamma)+cn(2+\gamma)\right)}{(1+n)^2(-1+\gamma)^4} < 0 \tag{24}$$

$\Delta\Pi_{PS}(\gamma,n)$ is concave in γ. To make $\Delta\pi_{PS}(\gamma,n) = 0$, we have

$\gamma = 0$,

$\gamma^\Pi(n) = \frac{3a-2c+an-\sqrt{a^2-2a^2n+8acn-4c^2n+a^2n^2}}{2(2a-c)} = 1-\frac{\sqrt{a^2-2a^2n+8acn-4c^2n+a^2n^2}-a(n-1)}{2(2a-c)}$,

$\gamma(n) = 0$.

Table 2 Profits with n symmetric provider

	Provider	Platform	System (Provider + Platform)
R-S	$\frac{n(c+a(-1+\gamma))^2}{(1+n)^2(1-\gamma)}$	$\frac{n(a(1-\gamma)-c)\gamma(a+cn-a\gamma)}{(1+n)^2(1-\gamma)^2}$	$\frac{n(a(1-\gamma)-c)(a-a\gamma+c(-1+\gamma+n\gamma))}{(1+n)^2(1-\gamma)^2}$
P-S	$\frac{(1-\gamma)n(a-c)^2}{(n+1)^2}$	$\frac{\gamma n(a-c)^2}{(n+1)^2}$	$\frac{n(a-c)^2}{(n+1)^2}$

$\gamma^{\Pi}(n)$ is called the platform's "R-S or P-S" threshold with n symmetric providers.

$$\frac{\partial^2 \gamma^{\Pi}(n)}{\partial n^2} = -\frac{2(a-c)^2 c}{(a^2(-1+n)^2 + 8acn - 4c^2 n)^{3/2}} < 0 \tag{25}$$

Proposition 8: *The provider's "R-S or P-S" threshold is independent of the number of product (service) providers in the market n. However, the platform's "R-S or P-S" threshold is concave in n.*

Up until now, we have shown that most of the results remain robust when the number of providers increases. Several numerical studies are illustrated to show the effects.

6. Numerical study

From Fig. 1, the horizontal axis shows the cutting ratio γ, while the vertical axis denotes the profit. The blue line is the provider's profit with P-S, which is a linearly decreasing function in γ. The red line is the provider's profit with R-S, which is convex decreasing in γ. The spread, denoted in the purple line, is a uni-modal function in γ. It starts from the zero point, then decreases, which means the P-S is always better than the R-S for the provider. We then illustrate the platform's profit under different production models.

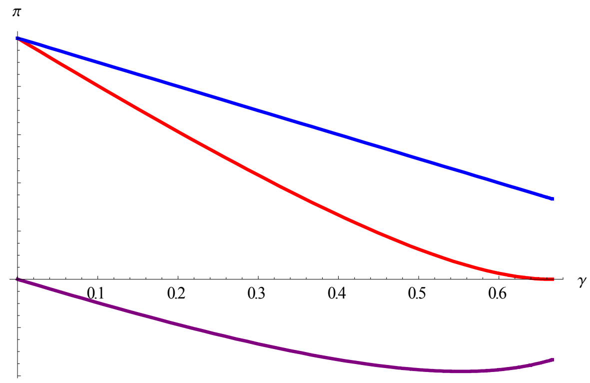

Fig. 1 Provider's profit

Fig. 2 shows the platform's profit as γ varies. The blue line denotes the profit under P-S, which is an increasing function in γ. The red line is the platform's profit under R-S, which is a concave function inγ. The purple line illustrates the spread between the R-S and the P-S. It starts from the zero point, then increases and finally decreases below 0. When γ is larger than a threshold, itis more profitable for the platform to select the P-S model.

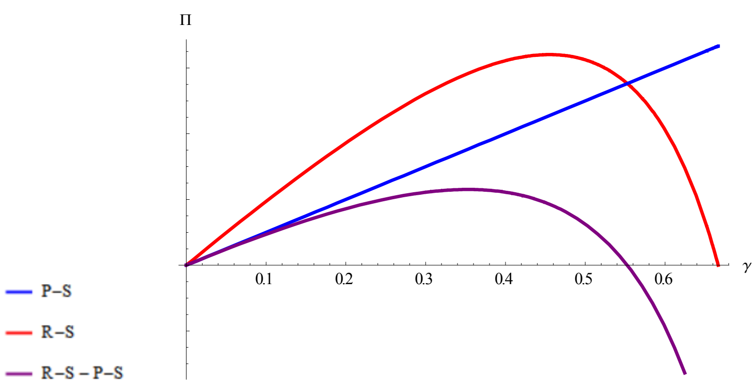

Fig. 2 Platform's profit

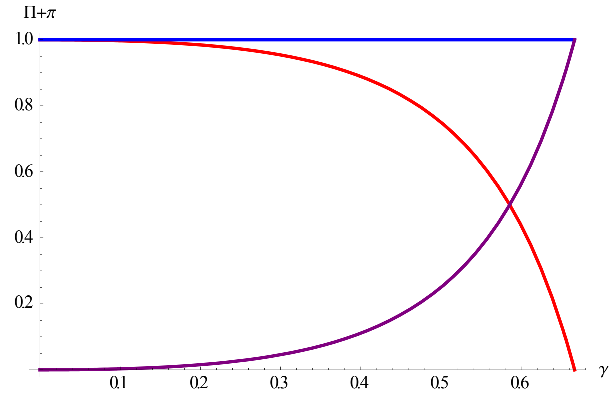

Fig. 3 System performance

Fig. 3 shows the system performance as γ varies. The blue line denotes the system performance under P-S, which remains stable as γ increases. The red line is the total profit under R-S, which is a decreasing function in γ. As γ increases, the mismatch between the provider and platform deepens. Therefore, the system performance under R-S is always below that under P-S. The purple line illustrates the spread between the P-S and the R-S, which is increasing as γ increases.

7. Discussion and conclusion

With the emergence of Internet economics, more product (service) providers are selling through e-platforms. In this paper, we compare two distinguishing production models, namely, the revenue sharing (R-S) model and the profit sharing (P-S) model. We find that the providers have less incentive to operate under the R-S model. The system performance decreases as the cutting ratio increases with R-S. Astonishingly, we show that it is not always beneficial for the platform to select the revenue sharing model, under which she seems to take no inventory risks by just taking a sales revenue cut. Additionally, the providers may be better off when the cutting ratio is larger than a threshold. We extend the basic model to a case when multiple providers exist. We find that the providers' "R-S or P-S" threshold remains stable as the number of players increases. However, the platform's "R-S or P-S" threshold is concave in the number of players in the market. Most of the results are proved to be robust.

There are several promising extensions along this theoretical framework. We ignore the randomness in our model. When the market is uncertain, the platform is exposed to more operational risks when she takes the P-S model. How will the results change in such a case? We assume perfect substitution in this paper, does it matter when these providers are selling partial substitute (complimentary) products or service via the same platform? Expanding the basic model into a dynamic setting would be another valuable study that could provide managerial insights for managers.

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References

[1] Farrell, D., Greig, F. (2017). *The online platform economy: Has growth peaked?,* JP Morgan Chase Institute, New York, USA, doi: 10.2139/ssrn.2911194.

[2] Zervas, G., Proserpio, D., Byers, J.W. (2017). The rise of the sharing economy: Estimating the impact of Airbnb on the hotel industry, *Journal of Marketing Research*, Vol. 54, No. 5, 687-705, doi: 10.1509/jmr.15.0204.

[3] Nunes, P.F., Bellin, J., Lee, I., Schunck, O. (2013). Converting the nonstop customer into a loyal customer, *Strategy & Leadership*, Vol. 41, No. 5, 48-53, doi: 10.1108/sl-05-2013-0035.

[4] Dong, L., Ren, L., Zhang, D. (2017) *Financing small and medium-sized sellers via E-commerce platform,* Working paper, Washington University, St. Louis.
[5] Wang, Y., Jiang, L., Shen, Z.-J. (2004). Channel performance under consignment contract with revenue sharing, *Management Science*, Vol. 50, No. 1, 34-47, doi: 10.1287/mnsc.1030.0168.
[6] Christopher, M., Towill, D. (2001). An integrated model for the design of agile supply chains, *International Journal of Physical Distribution & Logistics Management*, Vol. 31, No. 4, 235-246, doi: 10.1108/09600030110394914.
[7] Dana Jr., J.D., Spier, K.E. (2001). Revenue sharing and vertical control in the video rental industry, *The Journal of Industrial Economics*, Vol. 49, No. 3, 223-245, doi: 10.1111/1467-6451.00147.
[8] Gong, D., Tang, M., Liu, S., Li, Q. (2017). Reconsidering production coordination: A principal-agent theory-based analysis, *Advances in Production Engineering & Management*, Vol. 12, No. 1, 51-61, doi: 10.14743/apem2017.1.239.
[9] Giannoccaro, I., Pontrandolfo, P. (2004). Supply chain coordination by revenue sharing contracts, *International Journal of Production Economics*, Vol. 89, No. 2, 131-139, doi: 10.1016/s0925-5273(03)00047-1.
[10] Gerchak, Y., Wang, Y. (2004). Revenue-sharing vs. wholesale-price contracts in assembly systems with random demand, *Production and Operations Management*, Vol. 13, No. 1, 23-33, doi: 10.1111/j.1937-5956.2004.tb00142.x.
[11] Cachon, G.P., Lariviere, M.A. (2005). Supply chain coordination with revenue-sharing contracts: Strengths and limitations, *Management Science*, Vol. 51, No. 1, 30-44, doi: 10.1287/mnsc.1040.0215.
[12] Cachon, G.P. (2003). Supply chain coordination with contracts, *Handbooks in Operations Research and Management Science*, Vol. 11, No. 6, 227-339, doi: 10.1016/S0927-0507(03)11006-7.
[13] Chen, Y., Gupta, D. (2014). *Trade-finance contracts for small-production suppliers and the effect of third-party financing,* Working paper, University of Minnesota.
[14] Çanakoğlu, E., Bilgiç, T. (2007). Analysis of a two-stage telecommunication supply chain with technology dependent demand, *European Journal of Operational Research*, Vol. 177, No. 2, 995-1012, doi: 10.1016/j.ejor.2006.01.006.
[15] Wei, Y., Choi, T.-M. (2010). Mean-variance analysis of supply chains under wholesale pricing and profit sharing schemes, *European Journal of Operational Research*, Vol. 204, No. 2, 255-262, doi: 10.1016/j.ejor.2009.10.016.
[16] Singh, N., Vives, X. (1984). Price and quantity competition in a differentiated duopoly, *The RAND Journal of Economics*, Vol. 15, No. 4, 546-554, from *http://www.jstor.org/stable/2555525*, accessed February 19, 2018.
[17] Kreps, D.M., Scheinkman, J.A. (1983). Quantity precommitment and bertrand competition yield cournot outcomes, *The Bell Journal of Economics*, Vol. 14, No. 2, 326-337, doi: 10.2307/3003636.
[18] Gong, D., Liu, S., Lu, X. (2015). Modelling the impacts of resource sharing on supply chain efficiency, *International Journal of Simulation Modelling*, Vol. 14, No. 4, 744-755, doi: 10.2507/IJSIMM14(4)CO20.
[19] Van Mieghem, J.A., Dada, M. (1999). Price versus production postponement: Capacity and competition, *Management Science*, Vol. 45, No. 12, 1639-1649, doi: 10.1287/mnsc.45.12.1631.
[20] Goyal, M., Netessine, S. (2007). Strategic technology choice and capacity investment under demand uncertainty, *Management Science*, Vol. 53, No. 2, 192-207, doi: 10.1287/mnsc.1060.0611.
[21] Anupindi, R., Jiang, L. (2008). Capacity investment under postponement strategies, market competition, and demand uncertainty, *Management Science*, Vol. 54, No. 11, 1876-1890, doi: 10.1287/mnsc.1080.0940.
[22] Wen, F., Xiao, J., Huang, C., Xia, X. (2018). Interaction between oil and US dollar exchange rate: Nonlinear causality, time-varying influence and structural breaks in volatility, *Applied Economics*, Vol. 50, No. 3, 319-334, doi: 10.1080/00036846.2017.1321838.
[23] Wen, F., Gong, X., Cai, S. (2016). Forecasting the volatility of crude oil futures using HAR-type models with structural breaks, *Energy Economics*, Vol. 59, 400-413, doi: 10.1016/j.eneco.2016.07.014.

Appendix A

Proof of Proposition 1:

$$\frac{\partial \pi_{RS}(\gamma)}{\partial \gamma} = -\frac{(a(1-\gamma)-c)(a+c-a\gamma)}{4(1-\gamma)^2} \leq 0$$

Proof of Proposition 2:

$\frac{\partial \Pi_{RS}\left(q^*{}_{RS}(\gamma)\right)}{\partial \gamma} = \frac{a^2(-1+\gamma)^3+c^2(1+\gamma)}{4(-1+\gamma)^3} = \frac{a^2}{4} - \frac{c^2(1+\gamma)}{4(1-\gamma)^3}, \frac{\partial^2 \Pi_{RS}\left(q^*{}_{RS}(\gamma)\right)}{\partial \gamma^2} = -\frac{c^2(2+\gamma)}{2(1-\gamma)^4} < 0$. By solving the FOC, we have $\gamma^* = 1 - \frac{c^2}{3^{1/3}\left(-9a^4c^2+\sqrt{3}\sqrt{27a^8c^4+a^6c^6}\right)^{1/3}} + \frac{\left(-9a^4c^2+\sqrt{3}\sqrt{27a^8c^4+a^6c^6}\right)^{1/3}}{3^{2/3}a^2}$

Proof of Proposition 3:

$$\frac{\partial W_{RS}\left(q^*{}_{RS}(\gamma)\right)}{\partial \gamma} = \frac{c^2\gamma}{2(1-\gamma)^3} < 0$$

Proof of Proposition 5:

$$\frac{c^2(1-2\gamma)+a^2(1-\gamma)^2-2ac(1-\gamma)^2}{4(1-\gamma)^2}-\frac{(a-c)^2}{4}=-\frac{c^2\gamma^2}{4(1-\gamma)^2}<0$$

Proof of Proposition 6:

$$\frac{\partial\pi_{PS}\left(q^i{}_{RS}(\gamma;n),n\right)}{\partial n}=-\frac{(-1+n)(c+a(-1+\gamma))^2}{(1+n)^3(1-\gamma)}<0$$

$$\frac{\partial\pi_{PS}\left(q^i{}_{RS}(\gamma;n),n\right)}{\partial\gamma}=\frac{n\left(c+a(-1+\gamma)\right)(a+c-a\gamma)}{(1+n)^2(1-\gamma)^2}$$

$$\frac{\partial^2\pi_{PS}\left(q^i{}_{RS}(\gamma;n),n\right)}{\partial\gamma^2}=\frac{2c^2n}{(1+n)^2(1-\gamma)^3}>0$$

Proof of Proposition 7:

$$\frac{\partial\Pi_{RS}\left(q^i{}_{RS}(\gamma;n),n\right)}{\partial n}=\frac{(a(1-\gamma)-c)(2cn+a(-1+n)(-1+\gamma))\gamma}{(1+n)^3(-1+\gamma)^2}<0$$