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A hybrid grey cuckoo search algorithm for job-shop scheduling problems under fuzzy conditions

Yang, F.[a,b,*], **Ye, C.M.**[a], **Shi, M.H.**[a]

[a]Business School, University of Shanghai for Science and Technology, Shanghai, P.R. China
[b]College of Management, Henan University of Traditional Chinese Medicine, Zhengzhou, Henan, P.R. China

ABSTRACT

This paper aims to acquire the precise makespan or delivery period in job-shop scheduling (JSP) under fuzzy conditions. To this end, the author designed a grey scheduling model and a hybrid grey cuckoo search (HGCS) algorithm in the following steps. Firstly, three- and four-parameter interval grey numbers were introduced to depict the fuzzy makespan and delivery period, respectively; then, the possibility measure and necessity measure were defined, and the tardiness credibility index was proposed to estimate the probability of job tardiness. After that, a grey mixed integer programming model was developed to minimize the mean tardiness credibility, and the HGCS was proposed to solve the model. Finally, simulations were conducted on the classical example of $6(3) \times 6$. The results show that the proposed algorithm outperformed the basic cuckoo search. The research findings shed new light on the JSP under fuzzy conditions.

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**Corresponding author:*
yangfeng1126@126.com
(Yang, F.)

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1. Introduction

Job-shop scheduling (JSP) is a common NP-hard scheduling problem in modern industry. The problem is about the processing of a number of jobs on several machines. Each of these jobs needs to go through multiple processes and can run on several machines. The constraints of the JSP are as follows: the jobs must be treated in a feasible sequence of processes and follow the same order of priority; the same job should be processed only once on the same machine; the same machine can only process one job at a time. Under the above constraints, the JSP aims to optimize performance indices like the minimum makespan or the minimum tardiness through rational arrangement of the process sequence and start time. Many algorithms have been developed to solve classical JSPs. For instance, Zhang *et al.* [1] proposed a hybrid meta-heuristics algorithm based on shuffled frog leaping algorithm (SFLA), intelligent water drops algorithm (IWD) and path relinking (PR) algorithm. Modrák *et al*. [2] converted an *m*-machine problem into a 2-machine problem, and created a Johnson algorithm to solve it. Chaudhry *et al*. [3] presented a genetic algorithm with process planning and scheduling. Considering sequence flexibility, this algorithm solves classical JSPs using the spreadsheet of domain independence. Huang *et al*. [4] suggested solving classical JSPs by sequence flexibility and minimized the makespan with an improved genetic algorithm.

In classical JSPs, the production is scheduled under the ideal condition, that is, all factors are determined. In real-world production scheduling, however, both makespan and delivery period are highly indeterminate owing to uncertainties like machine breakdown, unfavourable environment or wrong decisions. To reflect the exact production situation, it is necessary to treat the makespan and delivery period as fuzzy variables.

Recent years has seen much attention being paid to the JSP under fuzzy conditions. The most common solution is fuzzy scheduling, that is, describing the fuzzy makespan and delivery period with triangular and trapezoidal fuzzy numbers. For instance, Ishii *et al*. [5] proposed the concept of fuzzy delivery period after exploring various jobs, and then investigated single- and double-machine open-shop scheduling under fuzzy delivery period. Noori-Darvish *et al*. [6] created a two-objective fuzzy programming model for open-shop scheduling, whose sequence relies on the job preparation time, fuzzy makespan and fuzzy delivery period. Focusing on two-machine scheduling, Gharehgozli *et al*. [7] put forward a fuzzy mixed integer programming model for scheduling sequence dependent on the job preparation time and fuzzy delivery period. Simeunović *et al*. [8] built up a workforce scheduling model based on artificial neural network.

For triangular and trapezoidal fuzzy numbers, both sides are monotonous linear functions. Therefore, linear, uniform variations can be expected for the probability function of the fuzzy number value on both sides of the most likely value (i.e. the window of the most likely makespan and the most likely delivery period). Nevertheless, the probability functions of make-span and delivery period are not necessarily linear in actual production. The possible linear and nonlinear functions of the two indices are shown in Fig. 1.

Considering the fuzziness of makespan and delivery period, a scheduling constraint should be expressed as an interval rather than a fixed value. Moreover, the ideal makespan and delivery period in the classical JSPs are often the most likely values in fuzzy situations; the probability functions for the deviation from the most likely values are not necessarily linear ones. These features are consistent with those of interval grey numbers.

In grey system theory, an interval grey number has different probabilities in taking different values. This unique feature has been widely discussed in the academia. For example, reference [9] introduces the interval grey number into multi-criterion optimization, with the aim to solve various fuzzy decision-making problems in the real world. Considering the impact of fuzzy grey number on prediction results, reference [10] converts the interval grey number sequence into a real number sequence and establishes a prediction model of interval grey number. Based on the definition of the basic interval number, references [11-18] put forward the three-parameter interval number and a multi-attribute decision-making method. Among them, reference [11] discusses the grey target decision-making with known maximum probability of attribute values: the index weight was determined through subjective or objective weighting, a comprehensive optimization model was established for index weight, and the attribute values were identified by three-parameter interval numbers.

In light of the above, this paper establishes a grey JSP model, which treats fuzzy makespan and delivery period as three- and four-parameter interval grey numbers, respectively, and defines the tardiness credibility index of jobs based on the possibility measure and necessity measure. Then, the fuzzy tardiness was determined as the optimization target and a grey mixed integer programming model was established. Finally, the proposed model was solved by an intelligent algorithm.

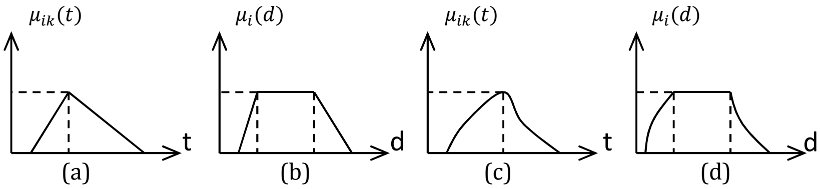

Fig. 1 (a) and (b): Linear probability functions, (c) and (d): Nonlinear probability functions

2. Grey job shop scheduling problem

2.1 Problem description

In actual scheduling, the job processing is more or less affected by some fuzzy factors. Thus, the scheduling process should be measured by probabilities. Here, the fuzzy makespan and delivery period are expressed as three- and four-parameter interval grey numbers, and the scheduling target is to optimize the mean tardiness credibility of the job. To realize the goal, it is necessary to review the definition of grey number.

Definition 1 [20] A number whose value falls in an interval rather than at a fixed point is known as the grey number and can be denoted as $\otimes$. A three-parameter interval grey number can be expressed as $a(\otimes) \in [\alpha^-, \alpha^*, \alpha^+]$ $(0 \leq \alpha^- \leq \alpha^* \leq \alpha^+)$ with α^-, α^+ and α^* being the lower limit, the upper limit and the most likely value (centre of gravity), respectively.

Remark: The three-parameter interval will degenerate into a real number if $\alpha^- = \alpha^* = \alpha^+$ and an interval grey number if $\alpha^- = \alpha^*$ or $\alpha^* = \alpha^+$. The three-parameter interval number satisfies $\int_{\alpha^-}^{\alpha^+} f(x)dx = 1$, $f_{max} = f(\alpha^*)$, and the value curve on either side of the ideal value is not necessarily linear. Hence, the membership function of the three-parameter interval number can be expressed as:

$$\mu_T(x) = \begin{cases} [x^{\frac{1}{2}} - (\alpha^-)^{\frac{1}{2}}] \cdot [(\alpha^*)^{\frac{1}{2}} - (\alpha^-)^{\frac{1}{2}}]^{-1}, & \alpha^- < x \leq \alpha^* \\ [x^{\frac{1}{2}} - (\alpha^+)^{\frac{1}{2}}] \cdot [(\alpha^*)^{\frac{1}{2}} - (\alpha^+)^{\frac{1}{2}}]^{-1}, & \alpha^* < x \leq \alpha^+ \\ 0, & x \leq \alpha^* \text{ or } x > \alpha^+ \end{cases} \tag{1}$$

Definition 2 [19] A four-parameter interval grey number can be expressed as $b(\otimes) \in [\alpha^-, \alpha^{*-}, \alpha^{*+}, \alpha^+]$ $(0 \leq \alpha^- \leq \alpha^{*-} \leq \alpha^{*+} \leq \alpha^+)$, with α^- and α^+ being the lower limit and the upper limit, respectively, and α^{*-} and α^{*+} being the most likely values of the lower limit and that of the upper limit, respectively. The two most likely values are considered as the dual centres of the interval. The four-parameter interval number satisfies $\int_{\alpha^-}^{\alpha^+} f(x)dx = 1$, $f_{max} = f(\alpha^{*-})$ or $f_{max} = f(\alpha^{*+})$.

Remark: The four-parameter interval will degenerate into a three-parameter interval grey number if $\alpha^{*-} = \alpha^{*+}$. In this case, the membership function of the four-parameter interval number can be expressed as:

$$\mu_D(x) = \begin{cases} \left[x^{\frac{1}{2}} - (\alpha^-)^{\frac{1}{2}}\right] \cdot [(\alpha^{*-})^{\frac{1}{2}} - (\alpha^-)^{\frac{1}{2}}]^{-1}, & \alpha^- < x \leq \alpha^{*-} \\ 1, & \alpha^{*-} < x \leq \alpha^{*+} \\ \left[x^{\frac{1}{2}} - (\alpha^+)^{\frac{1}{2}}\right] \cdot [(\alpha^{*+})^{\frac{1}{2}} - (\alpha^+)^{\frac{1}{2}}]^{-1}, & \alpha^{*+} < x \leq \alpha^+ \\ 0, & x \leq \alpha^- \text{ or } x > \alpha^+ \end{cases} \tag{2}$$

According to the above description, the grey JSP can be described as:

- Let $P = \{p_1, p_2, \ldots, p_n\}$ be a set of n jobs, with p_i being the i-th job $(i = 1,2, \ldots, n)$.
- Let $M = \{m_1, m_2, \ldots, m_m\}$ be a set of m machines, with m_j being the j-th machine $(j = 1,2, \ldots, m)$.
- Let $OP = \{op_1, op_2, \ldots, op_n\}$ be a set of process sequences, with $op_i = \{op_{i1}, op_{i2}, \ldots, op_{im}\}$ being the process sequence of job p_i, and op_{ik} being the machine number of the i-th job in the k-th process. If $op_{ik} = 0$, the k-th process of the i-th job is not implemented $(k = 1,2, \ldots, m)$.
- Let $T = \{a_{i1}(\otimes), a_{i2}(\otimes), \ldots, a_{im}(\otimes)\}$ be the set of makespans of job p_i, with $a_{ik}(\otimes)$ being the three-parameter interval grey number indicating the makespan of the i-th job in the k-th process. $a_{ik}(\otimes) \in [T_{ik}^-, T_{ik}^*, T_{ik}^+]$, where$T_{ik}^*$, $T_{ik}^-$and T_{ik}^+ are respectively the most likely, the shortest and the longest makespans of the k-th process. The function $\mu_{ik}(t)$ describes the possibility of completing k-th process of the i-th job at time t.

- Let $D_i(\otimes) = (d_i^-, d_i^{*-}, d_i^{*+}, d_i^+)$ be a four-parameter interval grey number indicating the delivery period of the *i-th* job. The interval (d_i^{*-}, d_i^{*+}) is the most likely delivery period of the i-th job.

The other constraints are the same with those in the classical JSP. Because the delivery period can accurately reflect the market changes, the ultimate target of our scheduling is to find a feasible scheduling plan that minimizes the mean tardiness of the job.

2.2 Grey number operator

Suppose all jobs to the processed on the k-th machine are allocated into one sequence, in which the i-th job is preceded by the job sequence 1,2,…,i-1. Thus, the makespan of the i-th job equals the total makespan of all jobs in the preceding sequence. To obtain the total makespan, the addition of two three-interval grey numbers should be defined as follows [21].

Definition 3 Let $a_{ik}(\otimes) = [\alpha^-, \alpha^*, \alpha^+]$ and $a_{jk}(\otimes) = [\beta^-, \beta^*, \beta^+]$. Then, we have:

$$a_{ik}(\otimes) + a_{jk}(\otimes) = [\alpha^- + \beta^-, \alpha^* + \beta^*, \alpha^+ + \beta^+]$$

According to Definition 3, the grey makespan $a_i(\otimes)$ of the i-th job can be obtained as:

$$a_i(\otimes) = \sum_{q=1}^{i} a_{qk}(\otimes), \quad q = 1,2,\dots,n \tag{3}$$

Where $a_{qk}(\otimes)$ is the grey makespan of the q-th job on the k-th machine. Before obtaining the job tardiness, it is also necessary to define the reduction operator of four-parameter interval grey numbers.

Theorem 1 For the membership function (2) of four-parameter interval number, there exist $x_1 \in [\alpha_i^-, \alpha_i^{*-}]$, $x_2 \in [\alpha_j^{*+}, \alpha_j^+]$ and $x_3 = (x_1^{\frac{1}{2}} - x_2^{\frac{1}{2}})^2$ that satisfy:

$$\mu_{D_i}(x_1) = \mu_{D_j}(x_2) = \frac{(\alpha_i^-)^{\frac{1}{2}} - (\alpha_j^+)^{\frac{1}{2}} - x_3^{\frac{1}{2}}}{(\alpha_i^-)^{\frac{1}{2}} - (\alpha_j^+)^{\frac{1}{2}} - (\alpha_i^{*-})^{\frac{1}{2}} + (\alpha_j^{*+})^{\frac{1}{2}}}$$

Proof: If $\mu_{D_i}(x_1) = \mu_{D_j}(x_2)$, then

$$\left[x_1^{\frac{1}{2}} - (\alpha_i^-)^{\frac{1}{2}}\right] \cdot \left[(\alpha_i^{*-})^{\frac{1}{2}} - (\alpha_i^-)^{\frac{1}{2}}\right]^{-1} = \left[x_2^{\frac{1}{2}} - (\alpha_j^+)^{\frac{1}{2}}\right] \cdot \left[(\alpha_j^{*+})^{\frac{1}{2}} - (\alpha_j^+)^{\frac{1}{2}}\right]^{-1}.$$

Solving this equation, we have: $x_2 = (\left[(\alpha_j^{*+})^{\frac{1}{2}} - (\alpha_j^+)^{\frac{1}{2}}\right] \cdot \frac{(\alpha_i^-)^{\frac{1}{2}} - x_1^{\frac{1}{2}}}{(\alpha_i^-)^{\frac{1}{2}} - (\alpha_i^{*-})^{\frac{1}{2}}} + (\alpha_j^+)^{\frac{1}{2}})^2$.

Hence, $\frac{(\alpha_i^-)^{\frac{1}{2}} - (\alpha_j^+)^{\frac{1}{2}} - x_3^{\frac{1}{2}}}{(\alpha_i^-)^{\frac{1}{2}} - (\alpha_j^+)^{\frac{1}{2}} - (\alpha_i^{*-})^{\frac{1}{2}} + (\alpha_j^{*+})^{\frac{1}{2}}} = \frac{\left[(\alpha_i^-)^{\frac{1}{2}} - x_1^{\frac{1}{2}}\right] \cdot \frac{1 + [(\alpha_j^{*+})^{\frac{1}{2}} - (\alpha_j^+)^{\frac{1}{2}}]}{(\alpha_i^-)^{\frac{1}{2}} - (\alpha_i^{*-})^{\frac{1}{2}}}}{(\alpha_i^-)^{\frac{1}{2}} - (\alpha_j^+)^{\frac{1}{2}} - (\alpha_i^{*-})^{\frac{1}{2}} + (\alpha_j^{*+})^{\frac{1}{2}}} = \frac{(\alpha_i^-)^{\frac{1}{2}} - x_1^{\frac{1}{2}}}{(\alpha_i^-)^{\frac{1}{2}} - (\alpha_i^{*-})^{\frac{1}{2}}}$

Hence, there exist x_1, x_2 and $x_3 = (x_1^{\frac{1}{2}} - x_2^{\frac{1}{2}})^2$ that satisfy:

$$\mu_{D_i}(x_1) = \mu_{D_j}(x_2) = \frac{(\alpha_i^-)^{\frac{1}{2}} - (\alpha_j^+)^{\frac{1}{2}} - x_3^{\frac{1}{2}}}{(\alpha_i^-)^{\frac{1}{2}} - (\alpha_j^+)^{\frac{1}{2}} - (\alpha_i^{*-})^{\frac{1}{2}} + (\alpha_j^{*+})^{\frac{1}{2}}}$$

Q.E.D.

Definition 4 Let $b_i(\otimes) = [\alpha_i^-, \alpha_i^{*-}, \alpha_i^{*+}, \alpha_i^+]$ and $b_j(\otimes) = [\alpha_j^-, \alpha_j^{*-}, \alpha_j^{*+}, \alpha_j^+]$ be two four-parameter interval grey numbers. Through reduction operation, another four-parameter interval grey number can be obtained as $b_i(\otimes) - b_j(\otimes) = [\alpha_i^- - \alpha_j^-, \alpha_i^{*-} - \alpha_j^{*-}, \alpha_i^{*+} - \alpha_j^{*+}, \alpha_i^+ - \alpha_j^+]$.

Whereas makespan and delivery period are respectively three- and four-parameter interval grey numbers, the three-parameter interval grey number should be treated as a special type of four-parameter interval grey number, that is, $a_{ik}(\otimes) \in [a_{ik}^{-}, a_{ik}^{*}, a_{ik}^{+}] \Leftrightarrow a_{ik}(\otimes) \in [a_{ik}^{-}, a_{ik}^{*}, a_{ik}^{*}, a_{ik}^{+}]$. By definition 4, the grey tardiness of the i-th job in the k-th machine can be obtained as $T_i(\otimes) = a_i(\otimes) - b_i(\otimes) = (\alpha_{ik}^{-} - \alpha_i^{+}, \alpha_{ik}^{*-} - \alpha_i^{*+}, \alpha_{ik}^{*-} - \alpha_i^{*-}, \alpha_{ik}^{*+} - \alpha_i^{-})$.

Definition 5 Let $b_i(\otimes) = [\alpha_i^{-}, \alpha_i^{*-}, \alpha_i^{*+}, \alpha_i^{+}]$ and $b_j(\otimes) = [\alpha_j^{-}, \alpha_j^{*-}, \alpha_j^{*+}, \alpha_j^{+}]$ be two four-parameter interval grey numbers. If $\alpha_i^{-} \geq \alpha_j^{-}, \alpha_i^{*-} \geq \alpha_j^{*-}, \alpha_i^{*+} \geq \alpha_j^{*+}$ and $\alpha_i^{+} \geq \alpha_j^{+}$, then $b_i(\otimes) \geq b_j(\otimes)$. Similarly, if $\alpha_i^{-} \leq \alpha_j^{-}, \alpha_i^{*-} \leq \alpha_j^{*-}, \alpha_i^{*+} \leq \alpha_j^{*+}$ and $\alpha_i^{+} \leq \alpha_j^{+}$, then $b_i(\otimes) \leq b_j(\otimes)$.

If the fuzzy tardiness $b_i(\otimes)$ of the i-th job is greater than zero, then the job production must be delayed; otherwise, the production is not delayed. Hence, the fuzzy tardiness of the job can be described by the possibility measure and the necessity measure.

2.3 Objective function

Definition 6 For the set $A, B \in F(X)(x \in X)$, suppose μ_A and μ_B are the membership functions of A and B, respectively. If $Pos_A(B) = \sup_{x \in X} min\{\mu_A(x), \mu_B(x)\}$ and $Nec_A(B) = \inf_{x \in X} max\{1 - \mu_A(x), \mu_B(x)\}$, then $Pos_A(B)$ and $Nec_A(B)$ are the possibility measures and necessity measures of B under the condition of A.

Obviously, $Pos(x \geq 0)$ and $Nec(x \geq 0)$ respectively stand for the possibility and necessity of $x \geq 0$.

Definition 7 Let $T_i(\otimes) = (\alpha_i^{-}, \alpha_i^{*-}, \alpha_i^{*+}, \alpha_i^{+})$ be the grey tardiness of the i-th job. Then, the tardiness credibility of the i-th job can be defined as the weighted sum of possibility measures and necessity measures and denoted as $Con_i(x \geq 0)$. Therefore, $Con_i(x \geq 0) = \delta Pos(x \geq 0) + (1 - \delta)Nec(x \geq 0)(0 \leq \delta \leq 1)$.

If $0 < \delta < 1$, the following can be derived from Definition 7: If tardiness credibility of the i-th job is $Con_i(x \geq 0) = 1$, then $Nec(x \geq 0) = 1$, i.e. the job production must be delayed; If $Con_i(x \geq 0) = 0$, then $Pos(x \geq 0) = 0$, i.e. the production of the i-th job cannot be delayed.

The coefficient δ is determined by various factors, such as the decision-maker and the extent of the tardiness. Its value is negatively correlated with the conservativeness of the decision. The following properties can be obtained according to the definition of credibility:

Property 1 If the grey tardiness of the i-th job is $T_i(\otimes) = (\alpha_i^{-}, \alpha_i^{*-}, \alpha_i^{*+}, \alpha_i^{+})(\alpha_i^{-} < \alpha_i^{*-} \leq \alpha_i^{*+} < \alpha_i^{+})$, then the tardiness credibility $Con_i(x \geq 0)$ of the job can be expressed as:

$$Con_i(x \geq 0) = \begin{cases} 0, & \alpha_i^{+} \leq 0 \\ \frac{\delta \alpha_i^{+}}{\alpha_i^{+} - \alpha_i^{*+}}, & \alpha_i^{+} > 0, \alpha_i^{*+} \leq 0 \\ \delta, & \alpha_i^{*+} > 0, \alpha_i^{*-} \leq 0 \\ \frac{\alpha_i^{*-} - \delta \alpha_i^{-}}{\alpha_i^{*-} - \alpha_i^{-}}, & \alpha_i^{*-} > 0, \alpha_i^{-} \leq 0 \\ 1, & \alpha_i^{-} > 0 \end{cases} \tag{4}$$

Let $a_{ik}(\otimes)$,$D_i(\otimes)$ and $T_i(\otimes)$ be the grey makespan, the grey delivery period and the grey tardiness of the i-th job in the JSP, respectively. Suppose the feasible scheduling plans of all jobs are allocated to the set FS and the objective function is denoted as $f(x)$. For a known scheduling plan $x \in FS$, it is possible to establish the following grey mixed integer programming model:

$$\min_{x \in S} f(x) = \sum_{i=1}^{n} \sum_{k=1}^{m} \left\{ \omega_1 - \frac{\delta \alpha_i^{+}}{\alpha_i^{+} - \alpha_i^{*+}} + \omega_2 \delta + \frac{\omega_3(\alpha_i^{*-} - \delta \alpha_i^{-})}{\alpha_i^{*-} - \alpha_i^{-}} + \omega_4 \right\} m_{ijk}/n \tag{5}$$

$$s.t.\ \sum_{k=1}^{m} \sum_{i=1}^{n} m_{ikj} = 1, \ \ i = 1,2,\dots,n \tag{6}$$

$$\sum_{i=1}^{n} m_{ikj} \leq 1, \ \ k = 1,2,\dots,m; j = 1,2,\dots,n \tag{7}$$

$$a_{ikj_i}(\otimes) = \sum_{i=1}^{n} \sum_{j=1}^{j_i} a_{jk}(\otimes) m_{ijk} \quad i, j_i = 1,2,\dots,n; k = 1,2,\dots,m \tag{8}$$

$$T_i(\otimes) = a_{ikj_i}(\otimes) - D_i(\otimes) \ \ i, j_i = 1,2,\dots,n \tag{9}$$

$$m_{ikj} = 0{,}1 \ \ i,j = 1,2,\dots,n; k = 1,2,\dots,m \tag{10}$$

$$\omega_q = 0{,}1 \ \ q \in \{1,2,3,4\} \tag{11}$$

The goal of this model is to obtain the optimal scheduling plan that minimizes the value of the objective function $f(x)$, i.e. the mean tardiness credibility of all jobs. Eq. 6 specifies that each job must occupy only one position in the process sequence on each machine; Eq. 7 stipulates that each position in the process sequence on each machine can only be occupied by one job; Eq. 8 defines $a_{ikj_i}(\otimes)$, the grey makespan of the i-th job at the j_i-th position of the process sequence on the k-th machine; Eq. 9 defines the grey tardiness $T_i(\otimes)$ of the i-th job; Eq. 10 defines the indicator variable m_{ikj} ($m_{ikj} = 1$ if the i-th job occupies the j-th position in the process sequence on the k-th machine, and $m_{ikj} = 0$ if otherwise); Eq. 11 ensures that $\omega_q = 1$ if $\omega_q \leq 0$ and $\omega_q = 0$ if otherwise.

Theorem 2 If the grey tardiness of the i-th job is $T_i(\otimes) = (\alpha_i^-, \alpha_i^{*-}, \alpha_i^{*+}, \alpha_i^+)(\alpha_i^- < \alpha_i^{*-} \leq \alpha_i^{*+} < \alpha_i^+)$, with $a_j(\otimes) = (\alpha_j^-, \alpha_j^{*-}, \alpha_j^{*+}, \alpha_j^+)$ being a four-parameter interval grey number, and the makespan is nonzero, then its tardiness credibility is $Con_{T_i(\otimes)} \geq Con_{T_i(\otimes)-a_j(\otimes)}$.

Proof: Since the tardiness credibility coefficient $\delta \in [0,1]$, it is clear that $0 \leq \delta\alpha_i^+/(\alpha_i^+ - \alpha_i^{*+}) \leq \delta \leq (\alpha_i^{*-} - \delta\alpha_i^-)/(\alpha_i^{*-} - \alpha_i^-) \leq 1$ for the piecewise function of Eq. 4.

Thus, we have $0 \leq \delta\alpha_i^+/(\alpha_i^+ - \alpha_i^{*+}) \leq \delta \leq (\alpha_i^{*-} - \delta\alpha_i^-)/(\alpha_i^{*-} - \alpha_i^-) \leq 1$.

It can be seen that $T_i(\otimes) - a_j(\otimes) = (\alpha_i^- - \alpha_j^+, \alpha_i^{*-} - \alpha_j^{*+}, \alpha_i^{*+} - \alpha_j^{*-}, \alpha_i^+ - \alpha_j^-)$ $(\alpha_i^- - \alpha_j^+ \leq \alpha_i^-, \alpha_i^{*-} - \alpha_j^{*+} \leq \alpha_i^{*-}, \alpha_i^{*+} - \alpha_j^{*-} \leq \alpha_i^{*+}$ and $\alpha_i^+ - \alpha_j^- \leq \alpha_i^+)$.

Hence, $Con_{T_i(\otimes)} \geq Con_{T_i(\otimes)-a_j(\otimes)}$.

Q.E.D.

Theorem 3 In a scheduling sequence S, if several jobs exist in the process sequence of the k-th machine, the grey makespan and delivery period of the i-th job are $a_{ik}(\otimes)$ and $D_i(\otimes)$, respectively, and the grey makespan and delivery period of the j-th are $a_{jk}(\otimes)$ and $D_j(\otimes)$, respectively. Note that $a_{ik}(\otimes) = (a_{ik}^-, a_{ik}^{*-}, a_{ik}^{*+}, a_{ik}^+)$, $a_{jk}(\otimes) = (a_{jk}^-, a_{jk}^{*-}, a_{jk}^{*+}, a_{jk}^+)$, $i,j = 1,2,\dots,r$ and $k = 1,2,\dots,m$; r is the number of jobs in the process sequence of the k-th machine; i and j are respectively the positions of the i-th and j-th jobs in the process sequence of the k-th machine. If $D_i(\otimes) < D_j(\otimes)$ and $a_{ik}(\otimes) - D_i(\otimes) < a_{jk}(\otimes) - D_j(\otimes)$, then the j-th job must be processed after the i-th job.

Proof: Suppose the makespan is greater than 0 in a scheduling sequence S. Then, the mean tardiness credibility $f_k(S)$ of all jobs in the process sequence of the k-th machine can be expressed as:

$$f_k(S) = \frac{1}{r}\left[\sum_{w=1}^{i-1} Con_{T_w(\otimes)} + Con_{T_i(\otimes)} + \sum_{v=i+1}^{j-1} Con_{T_v(\otimes)} + Con_{T_j(\otimes)} + \sum_{u=j+1}^{r} Con_{T_u(\otimes)}\right] \tag{12}$$

If the j-th job is processed before the i-th job, the mean tardiness credibility $f_k(S')$ of all jobs in the process sequence of the k-th machine for the new optimal scheduling sequence S' can be expressed as:

$$f_k(S') = \frac{1}{r}\left[\sum_{w=1}^{i-1} Con_{T'_w(\otimes)} + Con_{T'_i(\otimes)} + \sum_{v=i+1}^{j-1} Con_{T'_v(\otimes)} + Con_{T'_j(\otimes)} + \sum_{u=j+1}^{r} Con_{T'_u(\otimes)}\right] \tag{13}$$

Only the i-th and the j-th jobs are interchanged, while the other jobs in the process sequence of the k-th machine are in the same positions. Thus, the grey tardiness of the u-th, v-th and w-th jobs can be expressed as: $T_u(\otimes) = T'_u(\otimes)$, $T_v(\otimes) = T'_v(\otimes)$ and $T_w(\otimes) = T'_w(\otimes)$, where $u \in [j+1, r]$, $v \in [i+1, j-1]$, $w \in [1, i-1]$, and

$$T'_j(\otimes) = \sum_{w=1}^{i-1} a_{wk}(\otimes) + a_{jk}(\otimes) - D_j(\otimes) \tag{14}$$

$$T_i(\otimes) = \sum_{w=1}^{i-1} a_{wk}(\otimes) + a_{ik}(\otimes) - D_j(\otimes) \tag{15}$$

Since $a_{ik}(\otimes) - D_j(\otimes) < a_{jk}(\otimes) - D_j(\otimes)$, we have $T_j'(\otimes) > T_i(\otimes)$. It can be seen from Theorem 2 that the tardiness credibility $Con_{T_j'(\otimes)} > Con_{T_i(\otimes)}$. Then, the grey tardiness $T_i'(\otimes)$ of the i-th job in scheduling sequence S' and that $T_j(\otimes)$ of the j-th job in scheduling sequence S can be expressed as:

$$T_i'(\otimes) = \sum_{w=1}^{i-1} a_{wk}(\otimes) + a_{jk}(\otimes) + \sum_{v=i+1}^{j-1} a_{vk}(\otimes) + a_{ik}(\otimes) - D_i(\otimes) \quad (16)$$

$$T_j(\otimes) = \sum_{w=1}^{i-1} a_{wk}(\otimes) + a_{ik}(\otimes) + \sum_{v=i+1}^{j-1} a_{vk}(\otimes) + a_{jk}(\otimes) - D_j(\otimes) \quad (17)$$

Since $D_j(\otimes) > D_i(\otimes)$, we have $T_i'(\otimes) > T_j(\otimes)$ according to Eqs. 16 and 17. It can be seen from Theorem 2 that the tardiness credibility $Con_{T_i'(\otimes)} > Con_{T_j(\otimes)}$. Therefore, Eqs. 12 and 13 show that the sum of tardiness credibility of the process sequence of the k-th machine is $f_k(S') > f_k(S)$. This means the scheduling sequence after exchanging the i-th job and the j-th job is no better than that before the exchange. Thus, the j-th job must be processed after the i-th job.
Q.E.D.

3. Design of hybrid grey cuckoo search

3.1 Cuckoo search

Artificial intelligence algorithm is usually used to solve job shop scheduling models. Artificial intelligence algorithms include swarm intelligence optimization algorithm, decision support algorithm [22] [23], data fusion algorithm [24] and machine learning algorithm [25], etc. Xu *et al*. [26] proposed a bat algorithm to solve the problem of two-flexible job shop scheduling. Cuckoo search (CS) [27] is a heuristic search algorithm developed by Yang and Deb in 2009. It was inspired by the obligate brood parasitism of some cuckoo species by laying their eggs in the nests of other host birds. The cuckoo algorithm has better performance than particle swarm optimization [28, 29]. Since it was established, the CS has been extensively studied by scholars around the world. In 2010, Yang and Deb applied the CS in the multi-objective solution problem [30]. In 2011, Valian *et al*. improved the CS based on feedforward neural network [31]. In the same year, Walton *et al*. proposed an improved Brown free gradient CS [32]. The basic idea of the CS comes from the breeding behaviour of cuckoo and the Lévy flight mode of birds. The Lévy flight is a random walk in which the step-lengths have a probability distribution that is heavy-tailed. When defined as a walk in a space of dimension greater than one, the steps made are in isotropic random directions. Considering its ability to avoid the local optimum trap and achieve good global optimization, the CS is adopted to solve the grey mixed integer programming model in this paper.

The CS is suitable for small scale experiments rather than optimization problems with heavy computing load. Relying solely on random walk, the search for optimal solution consumes lots of computing power, and cannot guarantee the fast convergence of the algorithm. To improve the computing performance, it is necessary to integrate the features of the problem into the CS in a particular situation. Below are the steps to create the hybrid grey cuckoo search (HGCS).

3.2 Code design

The CS is not directly applicable to the coding problem of discrete processes. Thus, the coding rule here is based on continuous processes. Suppose the position of each nest represents a feasible scheduling solution. For n jobs and m machines, the code of the feasible solution can be described by the locations of $n \times m$ nests. The code stands for a process sequence, in which each job must appear exactly m times. Taking a 4-job 3-machine problem as an example, if the position code is 132143123442, the corresponding job processing sequence should be ($J_{1,1}$, $J_{3,1}$, $J_{2,1}$, $J_{1,2}$, $J_{4,1}$, $J_{3,2}$, $J_{1,3}$, $J_{2,2}$, $J_{3,3}$, $J_{4,2}$, $J_{4,3}$, $J_{2,3}$), with $J_{i,j}$ being the j-th process of the i. In other words, the first job and the third job should be processed successively on the first machine; then, the second job should be processed on the first machine, the first job should be processed on the second machine, and so on; Finally, the second job should be processed on the third machine.

3.3 Generation of initial population

In the CS, the nest location must be initialized at the start. However, the initial population is of poor quality because it is generated randomly without using any knowledge related to the problem to be solved. To improve the quality, the priority rules for minimum makespan and earliest delivery period in the JSP were introduced to develop a grey heuristic algorithm for initializing the locations of some nests. The specific steps are as follows.

For the four-parameter interval grey number $w(\otimes) = (\alpha^-, \alpha^{*-}, \alpha^{*+}, \alpha^+)$, let $sign\big(w(\otimes)\big) = \delta(\alpha^- + \alpha^{*-}) + (1-\delta)(\alpha^{*+} + \alpha^+)$ be the grey mark of $w(\otimes)$, and δ be the tardiness credibility coefficient. Then, $sign(a_{ik}(\otimes)) = \delta(a_{ik}^- + a_{ik}^*) + (1-\delta)(a_{ik}^* + a_{ik}^+)$ holds for the grey makespan $a_{ik}(\otimes) = (a_{ik}^-, a_{ik}^*, a_{ik}^+)$, and $sign(D_i(\otimes)) = \delta(d_i^- + d_i^{*-}) + (1-\delta)(d_i^{*+} + d_i^+)$ holds for the grey delivery period $D_i(\otimes) = (d_i^-, d_i^{*-}, d_i^{*+}, d_i^+)$. Thus, the minimum makespan and earliest delivery period can be expressed as $Min\{sign(a_{ik}(\otimes))\}$ and $Min\{sign(D_i(\otimes))\}$, respectively. Then, the said grey heuristic can be established through the following steps:

Step 1: Set up a set of grey markers for jobs, machines and delivery period:

$P_k = \{sign\big(a_{1k}(\otimes)\big), sign\big(a_{2k}(\otimes)\big), \ldots, sign(a_{nk}(\otimes))\}(k \in [1,m])$,
$M_l = \{sign\big(a_{l1}(\otimes)\big), sign\big(a_{l2}(\otimes)\big), \ldots, sign\big(a_{lm}(\otimes)\big)\}(l \in [1,n])$
and $D_q = \{sign\big(D_1(\otimes)\big), sign\big(D_2(\otimes)\big), \ldots, sign(D_n(\otimes))\}$.
$M_l = M_l - sign\big(a_{lk'}(\otimes)\big)$

Step 2: Let $sign\left(D_q'(\otimes)\right) = Min(D_q)$, $sign(a_{ik'}(\otimes)) = Min(M_l)$ and $sign(a_{l'k}(\otimes)) = Min(P_l)$, that is, assign the i'-th job to the k'-th machine. Then, we have $P_k = P_k - sign\big(a_{l'k}(\otimes)\big)$, $D_q = D_q - sign\big(D_{l'}(\otimes)\big)$, with $i = 1,2,\ldots,n$ and $k = 1,2,\ldots,m$.

Step 3: Repeat Step 2 until the jobs are assigned to each machine.

To diversify nest locations in the initial population, a third of the entire population was generated through the above steps, while the remaining parts were still generated randomly. The basic parameters of the CS were configured as: the dimension of the search space is m, the number of nests is n, the probability that the host bird discovers the egg laid by the cuckoo is P_a, and the maximum number of iterations is $MaxT$. After that, the initial nest locations were converted into process sequence through process coding, the value of objective function was computed corresponding to each nest location, and the optimal nest location was determined. Here, the goal is to calculate $minf(x), x \in S$, i.e. the minimum mean tardiness credibility of all jobs in the fuzzy JSP.

3.4 Generation of candidate population

First, the optimal nest location of the previous generation was preserved, and the fitness function $fitness = Q - f(x), x \in S$ of each nest location was calculated, with Q being a suitable positive number and $f(x)$ being the objective function. Then, the candidate population was generated by exploring paths and locations through Lévy flight. The direction of Lévy flight is arbitrary, and the step length obeys a heavy-tailed probability distribution. To sum up, the candidate population can be generated by:

$$x_i^{t+1} = x_i^t + \alpha \oplus Levy(\lambda), \quad i = 1,2, \ldots n \tag{18}$$

where x_i^t is the location of the i-th nest in the t-th generation; α is the step length control coefficient, which is usually 1; $\oplus$ is the point-to-point multiplication; $Levy(\lambda)$ is the path of the random search of Lévy flight. The direction of Lévy flight obeys uniform distribution.

3.5 Optimal selection

The locations of all nests in the current population were improved by Theorem 3, that is, the delivery periods of the nest locations were compared. If the grey delivery period $D_i(\otimes) < D_j(\otimes)$ and $a_{ik}(\otimes) - D_i(\otimes) < a_{ik}(\otimes) - D_j(\otimes)$, the j-th job must be processed after the i-th job. Then, the selection operator of the CS was employed to compare the fitness between parent and candidate generations, and the individuals with high fitness were retained to the next generation. In other words, the algorithm calculates the maximum fitness $fitness_{max}$ of nest in the current population; if the maximum fitness of the previous generation is greater than the $fitness_{max}$, then the location of the nest with the maximum fitness in the current population must be replaced with that in the previous generation:

$$x_i^{t+1} = \begin{cases} y_i^t, f(y_i^t) \leq f(x_i^t) \\ x_i^t, f(y_i^t) > f(x_i^t) \end{cases} \tag{19}$$

The operation selection adopts the greedy strategy, which can track the optimal solution found in the evolution. This strategy prevents the degradation in the iterative process and accelerates the convergence.

3.6 Random migration

Random migration is similar to the mutation operation of the genetic algorithm. In the basic CS, the cuckoo will look for a new nest randomly after its egg was discovered by the host bird. This type of random search may fall into the local optimum trap. To solve the problem, the new nest location could be searched for based on the undiscovered nests, i.e. the crossover operation of the genetic algorithm. The crossover operation goes like this: sort N host nests randomly and save the result; repeat the operation; get a crossover step length by subtracting the results of the previous two steps; find new host nest based on the crossover step length.

Suppose the probability that the host bird discovers the cuckoo egg is P_a. If P_a is smaller than a random number obeying uniform distribution $\gamma \in [0,1]$, then the crossover operation was implemented to find new host nests, and the cross-border check was performed on the locations of the nests. The nest locations should be either retained or replaced by locations with better fitness, forming a new set of better locations. Through random migration, the relatively good nests were retained, the poor individuals were eliminated and the population was diversified.

3.7 Termination conditions

After optimal selection and random migration, the optimal nest position and the best fitness should be examined. If the iteration termination condition is satisfied (reaching the maximum number of iterations $MaxT$) or the required accuracy is attained, the global optimal value and the corresponding global optimal position should be outputted; Otherwise, the optimal selection and random migration should be performed again. Finally, the required scheduling plan should be obtained by decoding the global optimal location.

4. Results and discussion

To verify the proposed HGCS algorithm, a simulation was carried out on the classical example of $6(3) \times 6$. The example was proposed by Sakawa to simulate the scheduling problem with fuzzy makespan and delivery period. In our experiment, the HGCS and CS parameters were configured as: the number of nests was 30, the probability that the host bird discovers the cuckoo egg $P_a = 0.25$, the maximum number of iterations $MaxT = 1{,}000$. The simulation was carried out on Matlab 2014a (OS: Windows 7; CPU: Intel® Core™ i3-2350M 2.0GHz; Memory: 2GB).

For the $6(3) \times 6$ grey JSP, the minimum mean tardiness credibility was taken as the objective function. The grey makespan and delivery period are given in Table 1, and the process sequence is listed in Table 2.

Table 1 Grey makespan and grey delivery period

Job	Processing time a						Delivery D	Weight
	M1	M2	M3	M4	M5	M6		
J1	(5,6,13)	(1,2,3)	(2,3,4)	(2,3,4)	(3,4,5)	(3,4,5)	(20,25,35,40)	0.15
J2	(3,4,5)	(2,4,5)	(1,3,5)	(5,6,7)	(6,7,8)	(4,5,6)	(20,25,35,40)	0.15
J3	(1,2,3)	(1,2,3)	(1,2,3)	(3,4,5)	(4,5,6)	(5,6,7)	(20,28,32,40)	0.25
J4	(3,4,6)	(2,3,5)	(3,4,5)	(2,3,4)	(1,2,3)	(2,3,4)	(20,28,32,40)	0.25
J5	(2,3,4)	(4,5,6)	(2,3,4)	(1,2,3)	(2,3,4)	(3,4,5)	(30,35,40,45)	0.10
J6	(2,3,4)	(3,4,5)	(2,3,4)	(1,2,3)	(6,7,8)	(4,5,6)	(30,35,40,45)	0.10

Table 2 Process sequence

Job	Processing order
J1	1-5-2-6-4-3
J2	1-2-3-6-4-5
J3	3-6-5-4-2-1
J4	6-5-4-2-1-3
J5	6-5-4-3-2-1
J6	5-6-1-2-3-4

Table 3 Minimum mean tardiness credibility of the HGCS and the CS

δ	Algo-rithm	$f(x)$										mean value
		1	2	3	4	5	6	7	8	9	10	
0.1	CS	0.0491	0.0486	0.0480	0.0499	0.0483	0.0492	0.0488	0.0484	0.0493	0.0497	0.0489
	HGCS	0.0481	0.0485	0.0472	0.0480	0.0477	0.0476	0.0480	0.0479	0.0483	0.0470	0.0478
0.2	CS	0.0723	0.0786	0.0655	0.0689	0.0822	0.0755	0.0623	0.0741	0.0777	0.0814	0.0739
	HGCS	0.0564	0.0654	0.0423	0.0568	0.0601	0.0656	0.0564	0.0589	0.0641	0.0532	0.0579
0.5	CS	0.1895	0.1876	0.1910	0.1747	0.1885	0.1841	0.1678	0.1708	0.1933	0.1850	0.1832
	HGCS	0.1331	0.1230	0.1445	0.1211	0.1391	0.1509	0.1436	0.1280	0.1472	0.1370	0.1368
0.7	CS	0.2135	0.2213	0.2331	0.2210	0.2178	0.2311	0.2258	0.2361	0.2283	0.2209	0.2249
	HGCS	0.1954	0.2031	0.1986	0.1870	0.1986	0.2015	0.2364	0.2013	0.1845	0.1922	0.1999
0.8	CS	0.2548	0.2631	0.2745	0.2610	0.2511	0.2537	0.2610	0.2503	0.2587	0.2611	0.2589
	HGCS	0.2456	0.2315	0.2498	0.2356	0.2475	0.2501	0.2468	0.2468	0.2511	0.2432	0.2448

The tardiness credibility coefficient was set to 0.1, 0.2, 0.5, 0.7 and 0.8, respectively. The target value is the minimum mean tardiness credibility. The program was ran 10 times at random to obtain the mean value. The experimental results are recorded in Table 3.

As shown in Table 3, the mean target value increased with the tardiness credibility cient δ. This is mainly because the growth in the coefficient led to a significant increase in the weight of the probability measure.

With δ = 0.2, 0.5, 0.2 and 0.5, simulations were conducted respectively by the CS and the HGCS. For each value of δ, the convergence curve of the mean tardiness credibility was plotted according to one of the simulations (e.g., Fig. 2, 3, 4 and 5).

According to Table 3, the HGCS always obtained a smaller mean target value than the CS, whichever the tardiness coefficient δ. Figs. 2 to 4 reveal that the HGCS obtained better optimal target value and realized faster convergence than the CS when the curve tended to global convergence.

The objective of this paper is to minimize the average value of tardy reliability of all jobs. As is shown in Table 3, the running results are obtained by the 10 operations, whatever CS algorithm or HGCS algorithm. For example, when the operation has run six times, the value of CS algorithm is 0.1841 at δ=0.5, HGCS 0.1509. Tardiness credibility represents the weighting sum of tardiness of probability measure and certainty measure. By definition 3, we can see that the uncertainty tardiness of jobs can be well embodied by possibility measures and necessity measures. The experimental values of the algorithm are 0.1841 and 0.1509, which fully reflect the tardiness of jobs.

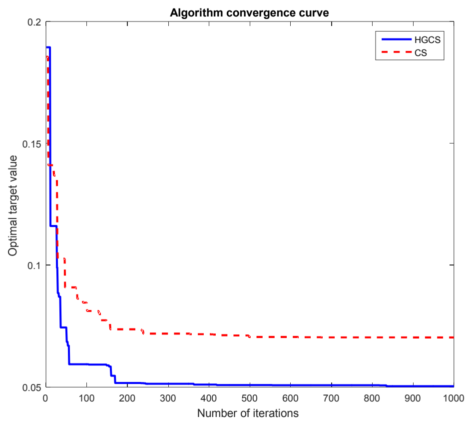

Fig. 2 Optimal target value convergence curves (δ = 0.2)

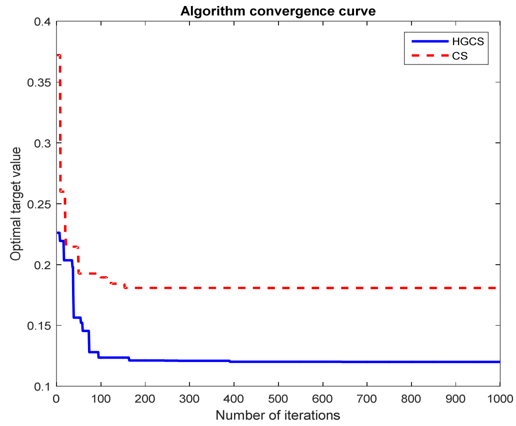

Fig. 3 Optimal target value convergence curves (δ = 0.5)

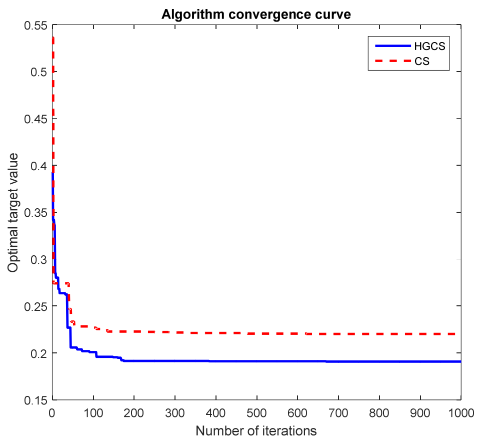

Fig. 4 Optimal target value convergence curves (δ = 0.7)

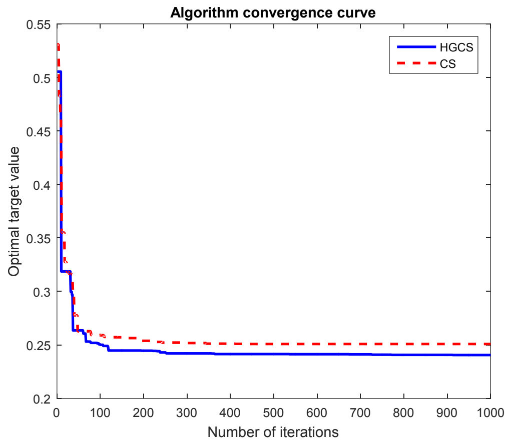

Fig. 5 Optimal target value convergence curves (δ = 0.8)

5. Conclusion

This paper proposes a grey scheduling model for the fuzzy makespan and delivery period in the JSP, which expresses the fuzzy makespan as three-parameter interval grey numbers and the fuzzy delivery period as four-parameter interval grey numbers. This approach is more generic than expressing fuzzy scheduling factors with triangular and trapezoid fuzzy numbers. On this basis, some features of optimal grey scheduling were obtained from the properties of grey numbers. These features were used to improve the basic CS into the HGCS. Then, simulations were performed on the classical example of 6(3) × 6. The results show that the HGCS can solve the job tardiness problem in fuzzy JSP with fuzzy makespan and delivery period. Compared to the basic CS, the HGCS achieved an excellent scheduling result. Of course, the solution presented here only applies to single objective optimization problem. The future research will tackle multi-objective optimization problems under resource constraints.

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References

[1] Zhang, H., Liu, S., Moraca, S., Ojstersek, R. (2017). An effective use of hybrid metaheuristics algorithm for job shop scheduling problem, *International Journal of Simulation Modelling*, Vol. 16, No. 4, 644-657, doi: 10.2507/IJSIMM16(4)7.400.

[2] Modrák, V., Pandian, R.S. (2010). Flow shop scheduling algorithm to minimize completion time for *n*-jobs *m*-machines problem, *Tehnički vjesnik – Technical Gazette*, Vol. 17, No. 3, 273-278.

[3] Chaudhry, I.A., Usman, M. (2017). Integrated process planning and scheduling using genetic algorithms, *Tehnički vjesnik – Technical Gazette*, Vol. 24, No. 5, 1401-1409, doi: 10.17559/TV-20151121212910.

[4] Huang, X.W., Zhao, X.Y., Ma, X.L. (2014). An improved genetic algorithm for job-shop scheduling problem with process sequence flexibility, *International Journal of Simulation Modelling*, Vol. 13, No. 4, 510-522, doi: 10.2507/IJSIMM13(4)CO20.

[5] Ishii, H., Tada, M., Masuda, T. (1992). Two scheduling problems with fuzzy due-dates, *Fuzzy Sets and Systems*, Vol. 46, No. 3, 339-347, doi: 10.1016/0165-0114(92)90372-B.

[6] Noori-Darvish, S., Mahdavi, I., Mahdavi-Amiri, N. (2012). A bi-objective possibilistic programming model for open shop scheduling problems with sequence-dependent setup times, fuzzy processing times, and fuzzy due dates, *Applied Soft Computing*, Vol. 12, No. 4, 1399-1416, doi: 10.1016/j.asoc.2011.11.019.

[7] Gharehgozli, A.H., Tavakkoli-Moghaddam, R., Zaerpour, N. (2009). A fuzzy-mixed-integer goal programming model for a parallel-machine scheduling problem with sequence-dependent setup times and release dates, *Robotics and Computer-Integrated Manufacturing*, Vol. 25, No. 4-5, 853-859, doi: 10.1016/j.rcim.2008.12.005.

[8] Simeunović, N., Kamenko, I., Bugarski, V., Jovanović, M., Lalić, B. (2017). Improving workforce scheduling using artificial neural networks model, *Advances in Production Engineering & Management*, Vol. 12, No. 4, 337-352, doi: 10.14743/apem2017.4.262.

[9] Stanujkic, D., Magdalinovic, N., Jovanovic, R., Stojanovic, S. (2012). An objective multi-criteria approach to optimization using MOORA method and interval grey numbers, *Technological and Economic Development of Economy*, Vol. 18, No. 2, 331-363, doi: 10.3846/20294913.2012.676996.

[10] Zeng, B., Liu, S.-F., Xie, N.-M., Cui, J. (2010). Prediction model for interval grey number based on grey band and grey layer, Control and Decision, Vol. 25, No. 10, 1585-1588, doi: 10.13195/j.cd.2010.10.148.zengb.002.

[11] Luo, D., Wang, X. (2012). The multi-attribute grey target decision method for attribute value within three-parameter interval grey number, *Applied Mathematical Modelling*, Vol. 36, No. 5, 1957-1963, doi: 10.1016/j.apm.2011.07.074.

[12] Luo, D. (2009). Decision-making methods with three-parameter interval grey number, *Systems Engineering – Theory & Practice*, Vol. 29, No. 1, 124-130, doi: 10.1016/s1874-8651(10)60033-6.

[13] Jin, F., Liu, P., Zhang, X. (2013). The multi-attribute group decision making method based on the interval grey linguistic variables weighted harmonic aggregation operators, *Technological and Economic Development of Economy*, Vol. 19, No. 3, 409-430, doi: 10.3846/20294913.2013.821685.

[14] Kamfiroozi, M.H., Aliahmadi, A., Jafari-Eskandari, M. (2012). Application of three parameter interval grey numbers in enterprise resource planning selection, *International Journal of Information, Security and Systems Management*, Vol. 1, No. 2, 72-77.

[15] Hu, Q., Zhang, W., Yu, L. (2007).The research and application of interval numbers of three parameters, *Engineering Science*, Vol. 9, No. 3, 47-51, doi: 10.3969/j.issn.1009-1742.2007.03.008.

[16] Sahu, N.K., Datta, S., Mahapatra, S.S. (2013). Decision making for selecting 3PL service provider using three parameter interval grey numbers, *International Journal of Logistics Systems and Management*, Vol. 14, No. 3, 261-297, doi: 10.1504/ijlsm.2013.052061.

[17] Yang, B., Zhao, J. (2013). Correlation coefficients of hesitant three-parameter interval grey number and their applications to clustering analysis, *Journal of Grey System*, Vol. 25, No. 2, 139-147.

[18] Song, S. (2018). Application of gray prediction and linear programming model in economic management, *Mathematical Modelling of Engineering Problems*, Vol. 5, No. 1, 46-50, doi: 10.18280/mmep.050107.

[19] Luo, D. (2009). Decision-making methods with three-parameter interval grey number, *Systems Engineering – Theory & Practice*, Vol. 29, No. 1, 124-130, doi: 10.1016/s1874-8651(10)60033-6.

[20] Hu, J., Lin, Z. (2013). Multi-criteria decision making method based on interval numbers of four parameters, *Operations Research and Management Science*, Vol. 6, 84-91, doi: 10.3969/j.issn.1007-3221.2013.06.014.

[21] Wang, N., Hu, L.-P., Li, B.-J. (2014). Study on the distance entropy model based on three-parameter interval grey numbers and its application, *Journal of Henan Agricultural University*, Vol. 48, No. 3, 386-390.

[22] Zaccone, R., Sacile, R., Fossa, M. (2017). Energy modelling and decision support algorithm for the exploitation of biomass resources in industrial districts, *International Journal of Heat and Technology*, Vol. 35, No. S1 (Special Issue), S322-S329, doi: 10.18280/ijht.35Sp0144.

[23] Dai, Y., Zhu, X., Zhou, H., Mao, Z., Wu, W. (2018) Trajectory tracking control for seafloor tracked vehicle by adaptive neural-fuzzy inference system algorithm, *International Journal of Computers Communications & Control*, Vol. 13, No. 4, 465-476, doi: 10.15837/ijccc.2018.4.3267.

[24] Wang, T.C., Xie, Y.Z. (2016). BP-GA data fusion algorithm studies oriented to smart home, *Mathematical Modelling of Engineering Problems*, Vol. 3, No. 3, 135-140.

[25] Tian, Y., Chen, W., Li, L., Wang, X., Liu, Z. (2018). Gait recognition via coalitional game-based feature selection and extreme learning machine, *NeuroQuantology*, Vol. 16, No. 2, 32-39, doi: 10.14704/nq.2018.16.2.1173.

[26] Xu, H., Bao, Z.R., Zhang, T. (2017). Solving dual flexible job-shop scheduling problem using a Bat Algorithm, *Advances in Production Engineering & Management*, Vol. 12, No. 1, 5-16, doi: 10.14743/apem2017.1.235.

[27] Yang, X.-S., Deb, S. (2010). Cuckoo search via Lévy flights, In: *2009 World Congress on Nature & Biologically Inspired Computing (NaBIC)*, Coimbatore, India, 210-214, doi: 10.1109/NABIC.2009.5393690.
[28] Liang, C.H., Zeng, S., Li, Z.X., Yang, D.G., Sherif, S.A. (2016). Optimal design of plate-fin heat sink under natural convection using a particle swarm optimization algorithm, *International Journal of Heat & Technology*, Vol. 34, No. 2, 275-280, doi: 10.18280/ijht.340217.
[29] Sen, G.D., Sharma, J., Goyal, G.R., Singh, A.K. (2017). A multi-objective PSO (MOPSO) algorithm for optimal active power dispatch with pollution control, *Mathematical Modelling of Engineering Problems*, Vol. 4, No. 3, 113-119, doi: 10.18280/mmep.040301.
[30] Yang, X.-S., Deb, S. (2013). Multiobjective cuckoo search for design optimization, *Computers & Operations Research*, Vol. 40, No. 6, 1616-1624, doi: 10.1016/j.cor.2011.09.026.
[31] Valian, E., Mohanna, S., Tavakoli, S. (2011). Improved cuckoo search algorithm for feedforward neural network training, *International Journal of Artificial Intelligence & Applications*, Vol. 2, No. 3, 36-43, doi: 10.5121/ijaia.2011.2304.
[32] Walton, S., Hassan, O., Morgan, K., Brown, M.R. (2011). Modified cuckoo search: A new gradient free optimisation algorithm, *Chaos, Solitons & Fractals*, Vol. 44, No. 9, 710-718, doi: 10.1016/j.chaos.2011.06.004.