Game theoretic analysis of supply chain based on mean-variance approach under cap-and-trade policy

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**ABSTRACT**

In recent years, carbon emission problem occurred by carbon dioxide as one of the main greenhouse gases, has become the focus due to its great influence on human life. With the increase of consumers’ low-carbon consciousness, this paper studies the supply chain which consists of a single supplier and a single manufacturer in presence of market low-carbon preference. First, we establish the mean-variance analysis model. Second, we study the optimal decisions of channel members considering the risk factor in three situations: traditional supply chain without emission reduction, individual emission reduction by manufacturer and supply chain collaborative emission reduction. Finally, the equilibrium results are demonstrated by numerical studies. The results show that chain members’ profits are not only affected by their own risk-aversion level, but also by other chain member’s risk aversion level. More important is that there exist the optimal carbon emission reduction level and profits in system collaborative emission reduction. The research makes operation mechanism of low-carbon supply chain clearer and provides a theoretical reference for supply chain members on pricing and investment strategy of emission reduction.

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1. Introduction

In recent years, the quick boosting of industry economy has brought many environmental problems, such as the greenhouse effect that threatens human health seriously. In this regard, it is universally accepted that massive carbon emission is the first dominant factor resulting in greenhouse effect Dinan [1], so many countries attempt to use scientific approaches to control carbon emissions. In Kyoto Protocol, the idea of ‘cap-and-trade’ policy was therefore designed to control the emission of greenhouse gases, which is a significant step to mitigate the negative effect of carbon emission. It is noteworthy that controlling carbon emission in the production process is the key tache of managing low-carbon supply chain. Thereby, a lot of manufacturing enterprises pay attention to low-carbon production and implement effective emission reduction measures. Generally, controlling carbon emission is of great significance to promote the sustainable development of environment.

A number of researches on low-carbon supply chain have focused on performance evaluation and management, such as, Yin et al. [2], Chen [3]. For in-depth research, the studies on low carbon supply chain can be divided into three categories: carbon footprint distribution, e.g. Wang et al. [4], Yang et al. [5], production and operation decision, e.g. Wang et al. [6], Nie et al. [7], Zhao
et al. [8]; supply chain structure under carbon emission constraints, e.g. Cholette et al. [9], Du et al. [10]. For ones interested in low-carbon supply chain, please refer to Jharkharia [11] to learn more progress.

One of main mathematical concepts to characterize our study is the use of mean variance. Chen et al. [12] studied the basic inventory model based on the newsvendor model by applying mean variance model. Wu et al. [13] researched newsvendor model with mean-variance analysis considering stockout cost, and the results reveal that the risk-averse newsvendor may even order more products than the risk-neutral. Ray et al. [14] conducted the research on purchasing strategy by mean-variance analysis considering disruption risk, which is the first time applying this approach to manage supply chain under disruption. Considering different risk attitudes, Choi et al. [15] investigated the newsvendor problem by utilizing mean-variance approach. Similarly, Yamaguchi et al. [16] developed mean-variance model, discussed the optimal order quantity following the three risk attitudes: risk-neutral attitude, risk-averse attitude and risk-prone attitude. Zhuo et al. [17] explored the two-echelon supply chain with option contracts under the mean-variance framework. More information about supply chain with mean-variance, please see Chiu et al. [18].

There is an increasing body of literature on cap-and-trade policy, which is relevant to the present study. In 1997 the cap-and-trade system was put forward in the Kyoto Protocol, the cap is proposed by government, and green organization has become a supplier of carbon emission. Different from traditional supply chain, the carbon emission permit can be traded via carbon market so that the carbon emission is utilized and controlled effectively. Considering stochastic demand, Zhang et al. [19] addressed the optimal production strategy of manufacturer in cap-and-trade system. Furthermore, they investigated purification efficiency in the case of multi-time. Du et al. [20] explored the low carbon supply chain consisting of one manufacturer who results in massive carbon emission and single emission permit supplier in a single period. Du et al. [21] discussed the carbon emission policy by utilizing the Stackelberg game in cap-and-trade system, furthermore, investigated the impact of cap on manufacturer, supplier and supply chain. Du et al. [22] constructed optimal production model for manufacturer considering environmental and preference with fixed cap and price of emission permit. According to the sizes of trade price and purification cost, the manufacturer decides whether to invest purification cost or trade emission permits via market for maximizing the profit. Zhao et al. [23] researched the problem of coordination mechanism and design of supply chain consisting of a single manufacturer and a single retailer considering the cap of carbon emission in a low carbon environment. Furthermore, Yuan et al. [24] studied the coordination of supply chain with revenue sharing contract in cap-and-trade system. In view of quick response strategy under newsvendor setting, Lee et al. [25] investigated the optimal pricing decisions of the company and policy maker who proposed the cap and trading price.

Our study is also related to consumers’ preference to low-carbon product, such as Ma et al. [26] and Wang et al. [27]. Similarly, Zhang [28] assumed that demand of products is a linear function with product prices and carbon emission and explored the optimal decisions of channel members in three situations: traditional supply chain without emission reduction, manufacturer’s individual emission reduction and collaborative emission reduction. Similar researches are also conducted in He et al. [29] and He et al. [30]. However, the risk factor of channel members wasn’t considered in the literature. Choi et al. [15] discussed maximum profit of supply chain under risk free constraint and risk constraint, and the result showed that supply chain risk actively affects the decision-making of supply chain members, so it cannot be ignored. Differently, this paper simultaneously considers the mean and variance to investigate the problem of low carbon supply chain, which makes the research more consistent with the reality, and ensure rigor of the research problem.

Comprehensively, the extant literature on low carbon supply chain mainly focus on optimization in firm and supply chain level but without incorporating risk concerns, which is exactly the gap we attempt to fill. The Mean-variance model considers both mean and variance to capture risk concerns, which is close to the reality. However, the existing literature on low carbon supply chain solely takes mean into account. In response to the enhancement of people’s awareness of
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low carbon, countries are also actively taking action to reduce carbon emission, such as, the designing cap-and-trading policy. To echo above problems, this paper exploits the mean-variance analysis to construct the game of supply chain under cap-and-trade policy, then the optimal decisions of chain members are explored and some numerical examples are provided to illustrate the models.

2. Preliminaries and notation
The necessary assumptions used in this paper are listed as follows:

- The market information is symmetrical and complete, and the market can completely be cleared. Thereby, there is no excess supply or excess demand, and the Pareto optimal of equilibrium price is ensured.
- Assume a unit material from the supplier can produce a unit product by the manufacturer.
- According to the study of Abdallah et al. [31], we assume that the government has implemented a more market-oriented economic means, namely, cap-and-trade system. In order to implement the strategy fairer and effectiveness, the government sets the carbon cap of per unit product which is assumed as \( \lambda_g \). According to the international practice and combined with China’s national conditions, the carbon trading price \( p_c \) is allowed in the carbon emission trading market.
- Assume the demand is linearly decreasing in retail price but increasing in the low-carbon preference with random interruption. We therefore give the linear function \( D = \tilde{\alpha} - \beta p + \gamma e \), where \( \alpha \) is intrinsic demand, \( \xi \) the random variable, \( e \) the emission reduction per unit product and \( \tilde{\alpha} = \alpha + \xi. \beta \) and \( \gamma \) are the demand responsiveness in price and low-carbon preference, respectively. For the ease of computation but without loss of generality, we assume that \( \xi \) follows normal distribution \( N(0, \sigma^2) \). Avoiding the trivial case, let \( \alpha > \beta (c_s + c_m) \).
- According to the standard hypothesis in classical model, namely, there is a quadratic function relationship between cost and R & D investment), we use \( C(e) = \theta e^2 / 2 \) to represent emission reduction cost, where \( \theta \) is the parameter of emission reduction cost. Zhang [28] and Raz et al. [32] also use the function to represent the relationship between carbon emission reduction level and emission reduction cost.

The notations used in this paper are summarized in Table 1.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Meaning</th>
</tr>
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<tbody>
<tr>
<td>( p )</td>
<td>Unit retail price</td>
</tr>
<tr>
<td>( p_c )</td>
<td>Unit trading price of carbon emission</td>
</tr>
<tr>
<td>( D )</td>
<td>The demand of product</td>
</tr>
<tr>
<td>( c_m )</td>
<td>Unit cost of manufacturer</td>
</tr>
<tr>
<td>( c_s )</td>
<td>Unit cost of supplier</td>
</tr>
<tr>
<td>( w )</td>
<td>Unit wholesale price</td>
</tr>
<tr>
<td>( e_0 )</td>
<td>Initial carbon emission of unit product</td>
</tr>
<tr>
<td>( \lambda_g )</td>
<td>Carbon emission cap of unit product</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Parameter of emission reduction cost</td>
</tr>
<tr>
<td>( A_m )</td>
<td>The risk-averse level of manufacturer</td>
</tr>
<tr>
<td>( A_s )</td>
<td>The risk-averse level of supplier</td>
</tr>
</tbody>
</table>

3. Optimal decision of supply chain considering risk attitude
3.1 Optimal decision-making in traditional supply chain without emission reduction
In cap-and-trade system, if the manufacturer want to keep his traditional production without emission reduction, he must purchase carbon permits to meet the cap set by the government. In this situation, the manufacturer’s profit is:
\[ \pi_m = (p - w - c_m)(\alpha + \xi - \beta p) + p_c(\lambda_g - e_0)(\alpha + \xi - \beta p) \]

The first term in the right hand side (RHS) of above function represents the sales revenue of manufacturer, and the second denotes the cost for purchasing carbon emission permit. In addition, we assume that the cap proposed by the government is less than the initial carbon emission of per unit product \( \lambda_g < e_0 \).

Accordingly, the supplier’s problem is:
\[ \pi_s = (w - c_s)(\alpha + \xi - \beta p) \]

The expected profits of chain members are expressed as follows
\[ E\pi_m = (p - w - c_m)(\alpha - \beta p) + p_c(\lambda_g - e_0)(\alpha - \beta p) \quad (1) \]
\[ E\pi_s = (w - c_s)(\alpha - \beta p) \quad (2) \]

Then, the variance profit functions of manufacturer and supplier are shown as follows
\[ Var_{\pi_m} = E[\pi_m - E\pi_m]^2 = (p - w - c_m + p_c(\lambda_g - e_0))^2\sigma^2 \quad (3) \]
\[ Var_{\pi_s} = E[\pi_s - E\pi_s]^2\sigma^2 \quad (4) \]

If the supply chain members want to get the optimal profits, which means that the mean and variance of profit must be simultaneously considered and maximize the mean of profit while minimize the variance. Ray et al. [14] established the mean-variance analysis model to study the disruption risk of supply chain. Moreover, the risk of supply chain was introduced into the model, which ensured the scientific of the research problem. Similarly, Wu et al. [13] introduced the mean-variance model into the newsvendor model considering shortage cost, and drew the different conclusion compared with the previous studies. Therefore, we establish the mean-variance model based on \( MV: E\pi - A\sqrt{Var} \), where \( A(0 < A \leq 1) \) represents the risk aversion level of supply chain members. Finally, we get the utility function of the manufacturer as follows
\[ MV: U_m = [p - w - c_m + p_c(\lambda_g - e_0)](\alpha - \beta p) - A_m[p - w - c_m + p_c(\lambda_g - e_0)]\sigma \quad (5) \]

The utility of manufacturer consists of two parts, i.e. fixed utility and risk utility. Let the revenue of per unit product multiply by the demand show the fixed utility of manufacturer. The second part represents the risk utility of manufacturer, where \( A_m \) denotes the risk aversion level of the manufacturer and \( \sigma \) shows the variation of demand.

Accordingly, the utility function of supplier is expressed as follow
\[ MV: U_s = (w - c_s)(\alpha - \beta p) - A_s(w - c_s)\sigma \quad (6) \]

Similarly, the first term in the RHS of above function shows the fixed utility of supplier by letting the revenue of per material multiply by demand. The second term expresses the risk utility of supplier, where \( A_s \) represents the risk aversion level of the supplier.

According to the Eqs. 5 and 6, we find that the profits of chain members in risk averse supply chain are less than that in risk-neutral supply chain.

The second partial derivative of Eq. 5 with the retail price takes the form:
\[ \frac{d^2U_m}{dp^2} = -2\beta < 0 \]

Let the first partial derivative of Eq. 5 with respect to the retail price equal zero, then we can get the optimal response function of \( p \) as follow
\[ p = \frac{\alpha - A_m\sigma}{2\beta} + \frac{\omega + c_m - p_c(\lambda_g - e_0)}{2} \quad (7) \]

Observing Eq. 7, we find that the retail price is decreasing in manufacturer’s risk aversion level. The reason is that a low retail price of the product will attract some of the lower price sensitive consumers, and promote consumption, then risk of fluctuations in the demand is reduced. Finally, the manufacturer’s profits have been ensured.
Substituting Eq. 7 into the utility function of supplier Eq. 6, then we get the second partial derivative in terms of $w$ as follows:

$$
\frac{d^2 U_s}{dw^2} = -\beta < 0
$$

Let the first partial derivative in $w$ equal zero, we can get the optimal wholesale price as follows:

$$
w^{1*} = \frac{\alpha + A_m \sigma - 2 A_s \sigma - c_m - c_s - p_c (\lambda_g - e_0)}{2\beta}
$$

(8)

In Eq. 8, the wholesale price decreases in supplier’s risk aversion level. The reason why this phenomenon happens can be explained as the risk-averse supplier reduces the wholesale price to attract manufacturer to order more material for maximizing her profit. Meanwhile, the wholesale price is increasing in manufacturer’s risk attitude in that the risk-averse manufacturer reduces the retail price in order to stimulate demand and accordingly needs to increase order quantity from supplier, which in turn encourage supplier charging manufacturer higher wholesale price.

Now we can rewrite the function of $p$ as follows:

$$
p^{1*} = \frac{3\alpha - A_m \sigma - 2 A_s \sigma + \beta [c_m + c_s - p_c (\lambda_g - e_0)]}{4\beta}
$$

(9)

### 3.2 Optimal decision-making in emission reduction by manufacturer

In this section we consider the fact that emission permit purchasing cost is considerable in the traditional production mode in cap-and-trade system, which seriously impact on the profit of manufacturer. In addition, consumers are willing to pay high price for low-carbon products with low-carbon preference. Considering above situation, manufacturer tries to change the traditional production mode and starts to conduct low-carbon production. Suppose that one unit product reduces $e_1$ units carbon emission after manufacturer applies low-carbon effort and the surplus carbon emission can’t be used in the next period. Hence, conducting low-carbon production results in associated cost $\frac{1}{2} \theta e_1^2$. Manufacturer’s profit can be expressed as follows:

$$
\pi_m = (p - w - c_m)(\alpha + \xi - \beta p + ye_1) + p_c(\lambda_g - e_o + e_1)(\alpha + \xi - \beta p + ye_1) - \frac{1}{2} \theta e_1^2
$$

The first term represents sales revenue, the second term the carbon trading cost (if buying permits) or revenue (if selling permits) and the last term carbon emission reduction cost.

Accordingly, the supplier’s profit can be described as below:

$$
\pi_s = (w - c_s)(\alpha + \xi - \beta p + ye_1)
$$

The expected profits of chain members are expressed as follows:

$$
E \pi_m = (p - w - c_m)(\alpha - \beta p + ye_1) + p_c(\lambda_g - e_o + e_1)(\alpha - \beta p + ye_1) - \frac{1}{2} \theta e_1^2
$$

(10)

$$
E \pi_s = (w - c_s)(\alpha - \beta p + ye_1)
$$

(11)

According to the Eqs. 3 and 4, we get the utility functions of chain members as follows:

$$
MV: U_m = (\alpha - \beta p + ye_1 - A_m \sigma)[p - w - c_m + p_c (\lambda_g - e_o + e_1)] - \frac{1}{2} \theta e_1^2
$$

(12)

$$
MV: U_s = (\alpha - \beta p + ye_1 - A_s \sigma)(w - c_s)
$$

(13)

The first partial derivatives of $U_m$ with respect to the retail price and carbon emission reduction level as follows:

$$
\frac{\partial U_m}{\partial p} = -2\beta p + \beta [w + c_m - p_c (\lambda_g - e_o + e_1)] + \alpha + ye_1 - A_m \sigma
$$

(14)
\[
\frac{\partial U_m}{\partial e_1} = \gamma \left[ p - w - c_m + p_c(\lambda_g - e_o + e_1) \right] + p_c(\alpha - \beta p + \gamma e_1 - A_m \sigma) - \theta e_1
\]

Solving the Hessian matrix of the Eq. 12 in terms of the retail price and carbon emission reduction level yields

\[
H^1 = \begin{bmatrix}
-2\beta & \gamma - \beta p_c \\
\gamma - \beta p_c & 2p_c \gamma - \theta
\end{bmatrix}
\]

When \(2\beta(\theta - 2p_c \gamma) > (\gamma - \beta p_c)^2\), there are the optimal solutions of Eq. 12. Let the Eqs. 14 and 15 equal zero, then the optimal decisions of manufacturer can be expressed as follows

\[
p = \frac{[\theta - p_c(\gamma + \beta p_c)](\alpha - A_m \sigma) - (\gamma^2 + \beta p_c \gamma - \theta)(w + c_m - p_c(\lambda_g - e_o))}{2\beta(\theta - 2p_c \gamma) - (\gamma - \beta p_c)^2}
\]

\[
e_1 = \frac{(\gamma + \beta p_c)[\alpha - A_m \sigma - \beta(w + c_m - p_c(\lambda_g - e_o))]}{2\beta(\theta - 2p_c \gamma) - (\gamma - \beta p_c)^2}
\]

Substituting the Eqs. 16 and 17 into the Eq. 13, we can rewrite the utility function of supplier as below

\[
U_s = \frac{w - c_s}{2\beta(\theta - 2p_c \gamma) - (\gamma - \beta p_c)^2} \{ \beta[\alpha - \beta(w + c_m - p_c(\lambda_g - e_o))] + [\beta \theta - (\gamma + \beta p_c)^2]A_m \sigma \\
- [2\beta(\theta - 2p_c \gamma) - (\gamma - \beta p_c)^2]A_s \sigma \}
\]

Solving the first derivative and second derivative with wholesale price of Eq. 18, we can obtain

\[
\frac{\partial U_s}{\partial w} = \frac{w - c_s}{2\beta(\theta - 2p_c \gamma) - (\gamma - \beta p_c)^2} \{ \alpha - \beta(w + c_m - p_c(\lambda_g - e_o)) + [\beta \theta - (\gamma + \beta p_c)^2]A_m \sigma \\
- [2\beta(\theta - 2p_c \gamma) - (\gamma - \beta p_c)^2]A_s \sigma \}
\]

\[
\frac{\partial^2 U_s}{\partial w^2} = -\frac{2\beta^2 \theta}{2\beta(\theta - 2p_c \gamma) - (\gamma - \beta p_c)^2} < 0
\]

Let Eq. 19 equal zero, the optimal wholesale price is expressed as below

\[
w^2 = \frac{\alpha}{2\beta} - \frac{c_m - c_s - p_c(\lambda_g - e_o)}{2} + [\beta \theta - (\gamma + \beta p_c)^2]A_m \sigma - \frac{[2\beta(\theta - 2p_c \gamma) - (\gamma - \beta p_c)^2]A_s \sigma}{2\beta^2 \theta}
\]

Finally, substituting the Eq. 20 into Eqs. 16 and 17, we rewrite the optimal solutions as below

\[
p^{2*} = -\frac{\gamma(\gamma + \beta p_c) - \beta \theta}{2\beta^2 \theta[2\beta(\theta - 2p_c \gamma) - (\gamma - \beta p_c)^2]} \{ \beta[\alpha + \beta(c_m + c_s - p_c(\lambda_g - e_o))] + [\alpha + \beta(c_m + c_s - p_c(2\beta(\theta - 2p_c \gamma) - (\gamma - \beta p_c)^2))]A_m \sigma \\
- [2\beta(\theta - 2p_c \gamma) - (\gamma - \beta p_c)^2]A_s \sigma \}
\]

\[
e_1^{2*} = \frac{\gamma + \beta p_c}{2\beta(\theta - 2p_c \gamma) - (\gamma - \beta p_c)^2} \{ \alpha - \beta(c_m + c_s - p_c(2\beta(\theta - 2p_c \gamma) - (\gamma - \beta p_c)^2)) \\
- [(3\beta \theta - (\gamma + \beta p_c)^2)]A_m \sigma - \frac{[2\beta(\theta - 2p_c \gamma) - (\gamma - \beta p_c)^2]A_s \sigma}{2\beta \theta} \}
\]

### 3.3 Optimal decision-making in channel collaborative emission reduction

In order to improve the market competitiveness and get more profits, manufacturer conducts low carbon production considering consumers’ preference of low carbon products. Then the market demand has been promoted with the behavior of manufacturers’ emission reduction. Finally, the profit of supplier has been improved with more materials ordered by manufacturer. Thereby, the rational supplier proposes that he is willing to cooperate with manufacturer and
bear a part of carbon emission reduction cost. Under the mode of cooperative emission reduction, what is the changes in decision-making between supply chain members? Does the collaborative emission reduction between the supply chain members maximize the economic benefits of each other? This section analyzes these problems.

First, we get the chain members’ profits as follows.

\[
\pi_m = (p - w - c_m)D + p_c(\lambda_g - e_0 + e)D - \frac{1}{2}(1 - \delta)e^2
\]

\[
\pi_s = (w - c_s)D - \frac{1}{2}\delta e^2
\]

Where the parameter \(\delta\) denotes the emission reduction cost sharing proportion of supplier.

The expected profits of chain members are expressed as follows

\[
E\pi_m = [p - w - c_m + p_c(\lambda_g - e_0 + e)](\alpha - \beta p + ye - A_m\sigma) - \frac{1}{2}(1 - \delta)e^2
\]  \hspace{1cm} (23)

\[
E\pi_s = (w - c_s)(\alpha - \beta p + ye - A_s\sigma) - \frac{1}{2}\delta e^2
\]  \hspace{1cm} (24)

The first term of RHS in Eq. 23 represents the revenues consisting of both sales revenue and carbon emission trading revenue or cost and the second term the carbon emission reduction cost undertaken by manufacturer. Similarly, the first term of Eq. 24 expresses the sales revenue, and the last term the carbon emission reduction cost undertaken by supplier.

Then we can get the utility functions of chain members as follows

\[
MV: U_m = [p - w - c_m + p_c(\lambda_g - e_0 + e)](\alpha - \beta p + ye - A_m\sigma) - \frac{1}{2}(1 - \delta)e^2
\]  \hspace{1cm} (25)

\[
MV: U_s = (w - c_s)(\alpha - \beta p + ye - A_s\sigma) - \frac{1}{2}\delta e^2
\]  \hspace{1cm} (26)

Similarly, we get the first partial derivatives of \(U_m\) with respect to the retail price and carbon emission reduction level as follows

\[
\frac{\partial U_m}{\partial p} = -2\beta p + \beta[w + c_m - p_c(\lambda_g - e_o)] + \alpha + ye_1 - A_m\sigma
\]  \hspace{1cm} (27)

\[
\frac{\partial U_m}{\partial e} = \gamma[p - w - c_m + p_c(\lambda_g - e_0 + e)] + p_c(\alpha - \beta p + ye - A_m\sigma) - (1 - \delta)e
\]  \hspace{1cm} (28)

Solving the Hessian matrix of the Eq. 25 in terms of the retail price and carbon emission reduction level, we get

\[
H^2 = \begin{bmatrix}
-2\beta & \gamma - \beta p_c \\
\gamma - \beta p_c & 2\beta y - (1 - \delta)e
\end{bmatrix}
\]

When \(2\beta[(1 - \delta)e - 2p_c\gamma] > (\gamma - \beta p_c)^2\), there are the optimal solutions of Eq. 25. Let the Eqs. 27 and 28 equal zero, the optimal decisions of manufacturer are expressed as follows

\[
p = \frac{[(1 - \delta)e - p_c(\gamma + \beta p_c)](\alpha - A_m\sigma) - [\gamma^2 + 2\beta p_c\gamma - \beta(1 - \delta)e][\lambda_g - e_0 + e]}{2\beta[(1 - \delta)e - 2p_c\gamma] - (\gamma - \beta p_c)^2}
\]  \hspace{1cm} (29)

\[
e = \frac{(\gamma + \beta p_c)[\alpha - A_m\sigma - \beta(w + c_m - p_c(\lambda_g - e_0))]}{2\beta[(1 - \delta)e - 2p_c\gamma] - (\gamma - \beta p_c)^2}
\]  \hspace{1cm} (30)

Substituting the Eqs. 29 and 30 into the Eq. 26, we can rewrite the utility function of supplier as below

\[
U_s = \frac{w - c_s}{2\beta(\theta - 2p_c\gamma) - (\gamma - \beta p_c)^2} \left[ \beta\theta(1 - \delta)[\lambda_g - e_0] + \beta\theta(1 - \delta)(\gamma + \beta p_c)^2 A_m\sigma \right]
\]

\[
\left[ \beta\theta(1 - \delta) - (\gamma + \beta p_c)^2 A_m\sigma \right] - \frac{1}{2} \delta e^2 = \frac{(\gamma + \beta p_c)[\alpha - A_m\sigma - \beta(w + c_m - p_c(\lambda_g - e_0))]}{2\beta(\theta - 2p_c\gamma) - (\gamma - \beta p_c)^2} \left[ (\gamma + \beta p_c)[\alpha - A_m\sigma - \beta(w + c_m - p_c(\lambda_g - e_0))] \right]^2
\]  \hspace{1cm} (31)

Solving the second derivative with wholesale price of Eq. 31, we can get
\[
\frac{d^2 U_s}{dw^2} = - \frac{2\beta^2 \theta (1 - \delta)}{\beta^2 \delta (y + \beta p_c)^2} - \frac{2\beta^2 \theta (1 - \delta)}{\beta^2 \delta (y + \beta p_c)^2} < 0
\]

Therefore, the optimal wholesale price is generated by letting the first derivative with wholesale price of Eq. 31 equal zero

\[
w^*_s = \frac{\alpha}{2\beta} - \frac{c_m - c_s - p_c(\lambda_g - e_o)}{2} + \frac{[\theta \beta (y + \beta p_c)^2]A_m \sigma - [2\beta \theta (1 - \delta) - 2p_c \gamma] - (y - \beta p_c)^2]}{2\beta^2 \theta [2(\beta \theta (1 - \delta) - 2p_c \gamma) - (y - \beta p_c)^2] + 2\beta^2 \theta \delta (1 - \delta) A_m \sigma}
\]

Finally, substituting the Eq. 32 into the Eqs. 29 and 30, we get

\[
p^*_s = \frac{(1 - \delta) \theta - p_c(y + \beta p_c)(\alpha - A_m \sigma) - [\theta (y^2 + \beta p_c \gamma - \beta (1 - \delta) \theta)][w^*_s + c_m - p_c(\lambda_g - e_o)]}{2\beta[(1 - \delta) \theta - 2p_c \gamma] - (y - \beta p_c)^2}
\]

\[
e = \frac{(y + \beta p_c)[(\alpha - A_m \sigma - \beta (w^*_s + c_m - p_c(\lambda_g - e_o))]}{2\beta[(1 - \delta) \theta - 2p_c \gamma] - (y - \beta p_c)^2}
\]

It is difficult to elegantly express the \(p^*_s\) and \(e\) in explicit format. Thereby, we substitute \(w^*_s\) into Eqs. 29 and 30 to represent the equilibrium results. Further research on the problem will be carried out in the numerical analysis section.

4. Computational study and discussion

In this section, we conduct a series of numerical computation to demonstrate previous models and obtain some managerial insights.

**Proposition 1:** The carbon emission reduction level in chain members’ reduction cooperation is greater than that in emission reduction by manufacturer solely.

The result reveals that the reduction cooperation of chain members can not only meet the low carbon preference of consumers, but promote the environment sustainable development. Additionally, the carbon emission reduction level is increasing in supplier’s risk-averse level, while decreasing in manufacturer’s risk-averse level, as shown in Fig. 1 and Fig. 2, respectively. The reason resulting in this phenomenon can be explained like that the risk-averse manufacturer wouldn’t like to input more costs to conduct low-carbon production, while the risk-averse supplier is just opposite.

**Proposition 2:** The retail price is decreasing in risk aversion level of supply chain members.

As shown in Figs. 3 and 4, we can explain the phenomenon in this way. The manufacturer reduces retailer price to attract price sensitive consumers for getting high profit. Furthermore, observing Fig. 3, we find there are different relationships among the three retail prices with the
variation of supplier’s risk aversion level. While the retail price in collaborative emission reduction is the lowest price no matter how manufacturer’s risk aversion level changes, as shown in Fig. 4.

**Proposition 3:** The wholesale price is closely related to the risk aversion level of chain members.

It is decreasing in supplier’s risk aversion level, while it shows opposite result in manufacturer’s risk aversion level, as shown in Figs. 5 and 6. The reason resulting in this phenomenon is that the risk-averse supplier reduces the wholesale price in order to promote the order of manufacturer. For risk-averse manufacturer, as we described aforesaid, he reduces retail price aim to stimulate consumption with more order occurred. Finally, the upstream supplier takes this chance to improve wholesale price.

**Proposition 4:** The profits of chain members are decreasing in their own risk aversion levels. In addition, there are always the optimal profits of chain members in emission reduction cooperation when supplier keeps the certain risk aversion level.

Observing Figs. 7, 8, 9, and 10, the chain members all get the optimal profits in collaborative emission reduction when the risk aversion level of supplier is less than 0.4. Additionally, risk-averse chain members’ profits are increasing in their own risk aversion levels.

the cause of this kind of appearance is that risk-averse chain members decreases the prices (wholesale price or retail price) to stimulate consumption for maximizing the profits, nevertheless, the profits are still decreasing.
Proposition 5: The profits of chain members are increasing in the cap imposed by government. The cap proposed by the government directly impacts on the economic benefits of the supply chain members in the risk-neutral supply chain. With increase of the cap, the economic benefits of supply chain members have been improved, and it is more beneficial to the chain members to cooperate with each other about carbon emission. The result also reveals another information that the cap provided by the government can be directly converted into economic income for the chain members. As shown in Table 2.

The parameters are summarized in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>α</td>
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</tr>
<tr>
<td>β</td>
<td>0.3</td>
</tr>
<tr>
<td>γ</td>
<td>0.4</td>
</tr>
<tr>
<td>c_m</td>
<td>10</td>
</tr>
<tr>
<td>c_s</td>
<td>8</td>
</tr>
<tr>
<td>θ</td>
<td>80</td>
</tr>
<tr>
<td>σ</td>
<td>10</td>
</tr>
<tr>
<td>p_c</td>
<td>3</td>
</tr>
<tr>
<td>e_0</td>
<td>10</td>
</tr>
<tr>
<td>δ</td>
<td>0.4</td>
</tr>
<tr>
<td>λ_g</td>
<td>5</td>
</tr>
</tbody>
</table>

5. Conclusion

Considering consumers’ low-carbon preference and the risk concerns of chain members, this study exploits mean-variance approach to explore the optimal decisions of supply chain agents. Three different cases have been discussed: traditional supply chain without emission reduction, emission reduction by manufacturer individually and chain members’ collaborative emission reduction. Some meaningful conclusions are obtained through analytical analysis and computa-
tional study. The results show the emission reduction level in collaborative emission reduction is greater than that by manufacturer individually. Moreover, there is an optimal profits of supply chain members in collaborative emission reduction, which indicates that in the cap-and-trade system, collaborative emission reduction of chain members can not only increases the profits of chain members, but also improves the emission reduction level. Additionally, the optimal decisions are affected by both manufacturer’s risk level and supplier's risk level. The research provides a cooperative emission-reduction way for chain members constrained by low-carbon policy.

The significance of this paper is reflected in three aspects. First, considering the chain member's risk, we explore the equilibrium results in three cases as described aforesaid, and figure out the operational mechanism for a risk-aversion supply chain in the cap-and-trade policy. Second, we study the influence of risk attitude on the profits of chain members, which reveals that the level of risk aversion plays an important role in the decision-making of supply chain members. The research compensates the defects of existing literature that assumes supply chain members are risk-natural. Finally, we consider the risk of chain members and construct mean-variance analysis model to let the research close to the reality.

The proposed models can be extended in several ways. For instance, extending the model to information asymmetry, which is common and important in reality. Furthermore, the study may be extended to more-than-two-tier supply chain. Additionally, studying the low carbon supply chain in multiple periods may be also a potential direction in the future.

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Reference


