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An integrated optimization of quality control chart parameters and preventive maintenance using Markov chain

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A B S T R A C T

Manufacturing costs are reduced significantly with the integrated optimization of preventive maintenance and quality control. In this paper, a new mixed integer non-linear programming model is presented. This model determines the optimal preventive maintenance interval and the optimal parameters of the ($\bar{X}$) control chart, including the sampling interval and the sample size and the control limit. The production system is considered in the form of a continuous time Markov chain. Formulation of the production process of a machine in the form of a continuous time Markov chain is a breakthrough in the integrated modeling of repair and quality. The goal is to reduce costs per unit time. It is assumed that preventive maintenance can be carried out at several levels either perfect or imperfect. The duration of corrective and preventive maintenance is not negligible. Considering the length of time for maintenance, this model is closer to the real production environment. A numerical example is used to illustrate this new model. Sensitivity analysis was performed to determine the effect of the model parameters on optimal decisions. This analysis further shows the relationship between preventive maintenance and statistical quality control as well as the performance of the new model.

A R T I C L E I N F O

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1. Introduction

The performance of a production system essentially depends on the performance of the floor of the workshop. The operating policies of the floor of the workshop include maintenance scheduling, quality control and production scheduling. These three aspects of operational planning are mutually reinforcing. Therefore, their integrated optimization considerably improves the performance of the system [1].

Production planning, quality and maintenance are the main elements of a production system. Many researchers believe that models that optimize each of these elements independently do not provide an optimal global solution for the whole production system. Accordingly, the literature has grown in the field of integrated models [2].

Pandey *et al.* [3] review articles that optimize quality and maintenances simultaneously. Hadidi *et al.* [2] referred to the list of articles that optimize integrated the maintenances and quality control.

Pandey *et al.* [4] present an article on simultaneous optimization of maintenance planning, quality control and production planning. In this study, a model was first developed to integrate maintenance planning and decision-making related to quality control of the process. Then, with attention to the preventive maintenance interval, the sequence of production batches with minimization of the production schedule delay was performed.

In another paper, Pandey *et al*. [5] present an integrated model for optimizing the preventive maintenance interval and control chart parameters using the Taguchi loss function.

Liu *et al*. [6] consider a $\bar{X}$ control chart for a two-unit production system that operates in series. This system is described using the continuous time Markov chain. In this paper, it is assumed that the system is controlled by a $\bar{X}$ control chart to avoid cost of failure and an optimization model has been developed to obtain optimal control chart parameters for minimizing maintenance costs. When the control chart gives an out-of-control signal, a complete inspection is performed if this inspection indicates the partial failure of each system unit, it should immediately be replaced as a part of preventive maintenance, and if the system stops, that is, a unit of the system is in a state of failure. In this paper, the length of maintenance is considered negligible. Xiang [7] provide an article on determining the optimal parameters of the $\bar{\mathrm{X}}$ control chart and preventive maintenance in the form of a discrete time Markov chain. In this model, the length of time for preventive and corrective maintenance is negligible.

Zhang *et al*. [8] integrated a $\bar{\mathrm{X}}$ control chart with a repair plan. This paper proposes a delayed maintenance policy. This policy takes a delay time to detect after an alarm from the control chart.

Yin *et al*. [9] provide an integrated model for statistical quality control and maintenance decisions based on a delay control policy. This mathematical model is solved to minimize the expected cost.

Tambe and Kulkarni [1] have provided an article to optimize the maintenance and quality program with the constraint on schedule, availability, and repair time and detection time for a single-machine production system and a Simulated Annealing algorithm and a Genetic algorithm are used to solve the model. Bouslah *et al*. [10] propose an integrated production, preventive maintenance, and quality control system for a production system, which is subject to deterioration in quality and reliability. The main objective of this study is to optimize the production, inventory level, parameters of the sampling plan and the overall repair level, minimizing the total cost imposed simultaneously. Tambe and Kulkarni [11] have presented an approach to integrate planning of repairs, quality control and production planning. The purpose of this study is to examine the benefits of the integration of these three issues with regard to the overall expected cost of the system. Lu *et al*. [12] presented an integrated model in which the process improvement with PM decisions in a single-machine production system was performed simultaneously.

Nourelfath *et al*. [13] optimize production, maintenance and quality policies for a complete process in a multi-period multi-product production system with limited production size. Shrivastava *et al*. [14] presented an integrated model for optimizing preventive maintenance and quality control policies with CUSUM chart. Zhong *et al*. [15] provide an integrated model for optimizing control chart parameters and maintenance times in the supply chain.

Ardakan *et al*. [16] presented a hybrid model for combining control charts and preventive maintenance (PM) systems to quickly diagnose out-of-control modes and this model reduces system control costs. In this paper a multivariate control chart (MEWMA) is used to control process changes. Khrueasom and Pongpullponsak [17] provide an integrated model for determining the parameters of the control charts of EWMA and Kolmogorov-Smirnov, with regard to repair management. Salmasnia *et al*. [18] provide an integrated model for determining the size of economic production, statistical process control, and repair, in a system, with a number of reasonable causes for failure and Particle Mass Optimization algorithms are used to minimize the total cost expected for each production cycle, according to the limitations of statistical quality. Zhong and Ma [19] provide an integrated model for statistical process control and maintenance. This paper optimizes Shewhart individual-residual $(Z_x - Z_e)$ control chart and repair parameters for two-step dependent processes, with the goal of minimizing the total cost of repair, inspection and quality.

In an article, Beheshti Fakher *et al*. [20] propose integrated production planning, incomplete repairs and process inspection in a multi-machine system. Rasay *et al*. [21] presented an integrated model that coordinates the decisions on designing the chi-square chart and the planning of maintenance, and an independent maintenance model is also presented for assessing the integrated model, and the performance of these two the model is compared with each other.

The purpose of this paper is to consider a mixed integer nonlinear programming model for simultaneous optimization of preventive maintenance and quality policies in a jobshop system in the form of a continuous time Markov chain. In this model, the process has an in control state and several out of control modes which are invisible. In out of control states, the percentage of manufactured parts is inconsistent. The failure mode is directly visible and detected immediately. Preventive maintenance is carried out at several levels, which can be perfect such that the process is turned into a state of in control or can be performed imperfect, in which case the process is converted with a probability to a state that is not worse before, but corrective maintenance is perfect and the process is then turned into a state of in control. Different modes of machine and sampling and various levels of preventive maintenance and corrective maintenance and false alarm are considered as nodes of a continuous time Markov chain.

It is assumed that the duration of stay in various machine modes and the various levels of preventive maintenance and corrective maintenance and inspection for false alarm is an exponential random variable. However, the duration of stay in different modes of the machine until entering sampling mode and the duration of stay in sampling mode is a hyper exponential random variable. This model determines the optimal preventive maintenance interval and $\overline{\mathrm{X}}$ control chart parameters for each machine at the time of production of each product, so that the cost per unit time is minimized.

This paper is close to article [7]. In that paper, a discrete-time Markov chain is proposed for the integrated optimization of $\overline{\mathrm{X}}$ control chart and preventive maintenance. In [7], the length of time for preventive and corrective maintenance is negligible and, as stated in the article itself, such a hypothesis is not feasible in practical situations. In the present study, the length of time for corrective and preventive maintenance is considered, so that the proposed model is closer to the reality of production systems. According to review articles by Pandey *et al.* [3] and Hadidi *et al.* [2], as well as reviewing the literature presented in this paper and the search, the following points can be cited as the innovation of this research. (1) All process modes including in control mode and out-of-control modes, and sampling mode and preventive maintenance at various levels, and corrective maintenance and inspection for false alarm, are considered as a continuous time Markov chain. (2) The duration of preventive and corrective repairs is not zero and the duration of their execution is exponential random variable.

The rest of the article is presented as follows. Section 2 describes the problem and provides an integrated planning model for preventive maintenance and $\overline{\mathrm{X}}$ control chart. In section 3, a numerical example is solved and sensitivity analysis is performed. In the end, section 4 will present a summary of the paper and conclusions and future suggestions.

2. Description of proposed non-linear model

A jobshop system is considered. In this system, several machines work in series at the stations. In order for the production line machines to work together on balance, at some workstations several machines work in parallel to provide a specific production rate. The failure of each of the machines reduces a certain percentage of line production. A component of each machine is considered as one piece that must be preventive maintenance done on it. The length of time until the failure of each machine follows exponential distribution. Two failure modes are considered for each machine. The first one is that the machine breaks down, and the production of the same moment stops, and the machine cannot continue to work. The second is that machine failure can reduce the process quality of the machine, which is due to a change in the average of the process. Therefore, the cost of a breakdown of the first mode includes the cost of stopping the line, the cost of repair work and the fixed cost of repair and the cost of set up. The second failure mode affects the product in terms of quality and increases the production rate of the defective product until the failure has been discovered and the production is stopped, so the cost of defect product includes quality costs.

In this paper, it is assumed that the quality of each process can be assessed by measuring a qualitative key characteristic of the output of that process. It is also assumed that this qualitative characteristic is a random variable with a certain mean and standard deviation. When the process is in control, the average of this variable is within the control limit. This average can be out of limit due to machine failure or some other external causes such as environmental effects, operator error, use the wrong tool, etc. After this happens, the process is considered out of control. In this case, it is assumed that the inspection of the machine is carried out without stopping the process and the cause of the failure is determined. If the cause is due to machine failure, the machine will be stopped and repairs will be done.

From the above, it is obvious that machine failure and repair affects the quality of the process. Therefore, the optimization of preventive maintenance and the economic design of the control chart should be carried out simultaneously. The operation of each machine on each product is considered as a process. For each process, a controlled state (mode 1) and several $(\mathrm{f}-1)$ modes out of the control $i = 2, 3, .., f-1$ are considered. Mode f is a failure mode. A $\bar{X}$ control chart is used to evaluate and control the process. The distribution of the qualitative feature of the process is supposed to follow the normal distribution. When the process is in control, the mean of the process is $\mu = \mu_0$ and the standard deviation of the process is $\sigma = \sigma_0$.

The occurrence of the assignable cause causes the change in the mean of the process, but the process variance does not change. In this case $\mu_i = \mu_0 + \delta_i \sigma$ and $2 \leq i \leq f$. The average of the process increases as the machine worsens so that $0 < \delta_1 < \delta_2 < \cdots < \delta_{f-1}$.

In this paper, preventive maintenance can be carried out at several levels; either perfect or imperfect is considered. The imperfect preventive maintenance is defined as the condition of the machine that is being repaired is not worse than the previous one, but the machine may not be turned into in control state, but the perfect preventive maintenance turns the machine into in control state (mode 1). Different modes of the machine (process) include in control mode and several out of control modes, failure mode and sampling mode, and different levels of preventive maintenance and corrective maintenance and false alarm. These modes are nodes of a continuous time Markov chain.

We now describe the integrated model of $\bar{X}$ control chart and preventive maintenance. First, we introduce sets, indices, parameters, and variables, and then objective function and a set of constraints are introduced.

Sets:

M	Set of machines
P	Set of products
I	Set of machine states
L	Set of preventive maintenance levels

Indicators:

m	Machine
p	Product
i	Machine mode
s	Sampling mode
l	Preventive maintenance level mode
f	Corrective maintenance, failure mode
ins	False alarm mode

Parameters:

ais_{imp}	The probability of transferring the mode of the machine (process) m during the production of the product p from the state i to the state s, so that the mod s is the sampling mode.
asi_{imp}	The probability of transferring the machine mode (process) m when producing the product p from the state s to the state i.

asl_{lmp} The probability of transferring the mode of the machine (process) m during the production of the product p from s to state l such that the state l is a level of preventive maintenance.

ali_{limp} The probability of transferring the mode of the machine (process) m during the production of the product p from state l to state i so that the state l is a level of preventive maintenance.

λij_{ijmp} Machine arrival rate (process) m when producing product p from i mode to j mode so that j mode is worse than i.

λli_{limp} Machine arrival rate (process) m during production of product p from preventive maintenance of level l to mode i.

λins_{mp} Machine arrival rate (process) m during production of Product p from inspection mode of false alarm to in control mode.

λf_{mp} Machine arrival rate (process) m during production of product p from failure mode to in control state.

θs_{mp} The length of time to check a sample taken from the product p machine (process) m.

δ_{mp} The magnitude of quality shift of the process mean of the machine (process) m during the production of the product p .

ci_{imp} The cost of each unit time when the machine (process) m is in the state of i during the production of the product p.

cl_{lmp} The cost of each unit time when the machine (process) m is at the level l of the preventive maintenance during production of the product p.

ccm_{mp} The cost of each unit time when the machine (process) m is in the corrective maintenance mode when producing the product p.

$cstop_{mp}$ The cost of each unit time of stopping the machine (process) m during the production of the product p, which is imposed on the production line during both the preventive maintenance and corrective maintenance, which is, in fact, the cost of losing output per unit of time.

cf_{mp} The fixed cost of each sampling of the machine (process) m during the production of the product p.

cv_{mp} The variable cost of each sampling unit of the machine (process) m during the production of the product p.

$cins_{mp}$ The cost of each unit of time for inspection of the machine (process) m during the production of the product p due to the false alarm.

Variables:

α_{mp} Type 1 error for machine (process) m when producing product p.

πi_{imp} The percentage of production time that the machine (process) m is in state i when produces the product p.

πl_{lmp} The percentage of production time that the machine (process) m during production of the product p is in the level of l preventive maintenance.

πf_{mp} The percentage of production time that the machine (process) m is in the corrective maintenance mode when producing the product p.

πins_{mp} The percentage of production time that machine (process) m is in the inspection mode for false alarm when production of product p.

πs_{mp} The percentage of production time that the machine (process) m is in sampling mode when production of product p.

λIS_{pm} The machine arrival rate (Process) m from operating modes i (machine modes) to sampling mode s when producing product p.

n_{mp} The number of samples taken from the machine (process) m during the production of the product p at each sampling time.

k_{mp} The amount of standard deviations allowed for the machine (process) m during the production of the product p at each sampling time.

β_{mp} The probability of the second type of machine error (process) m during the production of the product p at each sampling time.

h_{mp} The sampling interval of the machine (process) m at during the production of the product p.

τ_{mp} The preventive maintenance interval of the machine (process) m at the time of production of the product p.

The objective function is of cost type, therefore, should be minimized.

$$
\begin{aligned}
Min\,Z = &\sum_{m=1}^{M}\sum_{p=1}^{P}\sum_{i=1}^{f-1} ci_{imp}\pi i_{imp} + \sum_{m=1}^{M}\sum_{p=1}^{P} cstop_{mp}\pi f_{mp} + \sum_{m=1}^{M}\sum_{p=1}^{P}\sum_{l=1}^{L} cstop_{mp}\pi l_{lmp} \\
&+ \sum_{m=1}^{M}\sum_{p=1}^{P}\sum_{l=1}^{L} cl_{lmp}\pi l_{lmp} + \sum_{m=1}^{M}\sum_{p=1}^{P} ccm_{mp}\pi f_{mp} + \sum_{m=1}^{M}\sum_{p=1}^{P} cins_{mp}\pi ins_{mp} \\
&+ \sum_{m=1}^{M}\sum_{p=1}^{P} cf_{mp}\pi s_{mp} + \sum_{m=1}^{M}\sum_{p=1}^{P} cv_{mp}n_{mp}\pi s_{mp}
\end{aligned} \tag{1}
$$

Constraints:

Equations of equilibrium are written for each node (state). Eq. 2 is the equilibrium equation of the node in-control (node one).

$$
\begin{aligned}
&\sum_{l=1}^{L} ali_{limp}\lambda li_{limp}\pi l_{lmp} + \lambda ins_{mp}\pi ins_{mp} + 2\left[\left((1-\alpha_{mp})\lambda IS_{mp}\right)\left(\frac{1}{\theta s_{mp}n_{mp}}\right)\right]\pi s_{mp} \\
&+\lambda f_{mp}\pi f_{mp} - \sum_{j=2}^{f} \lambda ij_{ijmp}\pi i_{imp} - (f-1)ais_{imp}\lambda IS_{mp} = 0 \qquad i=1, \qquad \forall m \in M, \forall p \in P
\end{aligned} \tag{2}
$$

Eq. 3 is equilibrium equations for out-of-control nodes other than failure mode.

$$
\begin{aligned}
&\sum_{l=1}^{L} ali_{limp}\lambda li_{limp}\pi l_{lmp} + 2asi_{imp}\left[\left(\beta_{mp}\lambda IS_{mp}\right) + \left(\frac{1}{\theta s_{mp}n_{mp}}\right)\right]\pi s_{mp} + \sum_{j=1}^{j=i-1} \lambda ij_{jimp}\pi i_{jmp} \\
&- \sum_{j=i+1}^{f} \lambda ij_{ijmp}\pi i_{imp} - (f-1)ais_{imp}\lambda IS_{mp} = 0 \qquad i=2,\dots,f-1, \qquad \forall m \in M, \forall p \in P
\end{aligned} \tag{3}
$$

Eq. 4 is equilibrium equation for failure node (corrective maintenance).

$$
\sum_{i=1}^{f-1} \lambda ij_{ijmp}\pi i_{imp} - \lambda f_{mp}\pi f_{mp} = 0 \qquad j=f, \qquad \forall m \in M, \forall p \in P \tag{4}
$$

Eq. 5 is equilibrium equation for sampling node.

$$
\begin{aligned}
&(f-1)\sum_{i=1}^{f-1} ais_{imp}\lambda IS_{mp}\pi i_{imp} - 2\left[\left((1-\alpha_{mp})\lambda IS_{mp}\right) + \left(\frac{1}{\theta s_{mp}n_{mp}}\right)\right]\pi s_{mp} - 2\left[(\alpha_{mp}\lambda IS_{mp})\right. \\
&\left. + \left(\frac{1}{\theta s_{mp}n_{mp}}\right)\right]\pi s_{mp} - 2\sum_{i=2}^{i=f-1} asi_{imp}\left[(\beta_{mp}\lambda IS_{mp}) + \left(\frac{1}{\theta s_{mp}n_{mp}}\right)\right]\pi s_{mp} - \\
&-2\sum_{l=1}^{L}(asl_{lmp})\left[\left((1-\beta_{mp})\lambda IS_{mp}\right) + \left(\frac{1}{\theta s_{mp}n_{mp}}\right)\right]\pi s_{mp} = 0 \qquad \forall m \in M, \quad \forall p \in P
\end{aligned} \tag{5}
$$

Eq. 6 is equilibrium equation for nodes of various levels of preventive maintenance.

$$2\left(asl_{lmp}\right)\left[\left(\left(1-\beta_{mp}\right)\lambda IS_{mp}\right)+\left(\frac{1}{\theta s_{mp}n_{mp}}\right)\right]\pi s_{mp}-\sum_{i=1}^{f-1}ali_{limp}\lambda li_{limp}\pi l_{lmp}=0 \tag{6}$$

$$\forall m\in M,\quad \forall p\in P,\quad l\in L$$

Eq. 7 is equilibrium equation for inspection node (false alarm).

$$2\left[\left(a_{mp}\lambda IS_{mp}\right)+\left(\frac{1}{\theta s_{mp}n_{mp}}\right)\right]\pi s_{mp}-\lambda ins_{mp}\pi ins_{mp}=0\qquad \forall m\in M,\ \forall p\in P \tag{7}$$

Eq. 8 calculates the sampling interval.

$$h_{mp}=\frac{1}{\lambda IS_{mp}}\qquad \forall m\in M,\qquad \forall\, p\in P \tag{8}$$

Eq. 9 calculates the preventive maintenance interval.

$$\tau_{mp}=\left(\frac{1}{\left(1-\beta_{mp}\right)\left(\lambda IS_{mp}\right)}\right)+\left(\frac{1}{\theta_{mp}n_{mp}}\right)\qquad \forall m\in M,\ \forall p\in P \tag{9}$$

Now, Eq. 10, which is necessary to obtain the percentage of process time remaining in each of the modes.

$$\sum_{i=1}^{f-1}\pi i_{imp}+\sum_{l=1}^{L}\pi l_{lmp}+\pi f_{mp}+\pi ins_{mp}+\pi s_{mp}=1\qquad \forall m\in M,\qquad \forall\, p\in P \tag{10}$$

Eq. 11 calculates the probability of type I error.

$$\alpha_{mp}=2\phi(-k_{mp})\qquad \forall m\in M,\qquad \forall\, p\in P \tag{11}$$

$\phi(x)$ is the cumulative distribution function of normal distribution. Eq. 12 calculates the probability of type II error.

$$\beta_{mp}=\phi(k_{mp}-\delta_{mp}\sqrt{n_{mp}})-\phi(-k_{mp}-\delta_{mp}\sqrt{n_{mp}})\qquad \forall m\in M,\qquad \forall\, p\in P \tag{12}$$

It should be noted that the duration of stay in different modes of the machine (process) up to entering the sampling mode, $f-1$ type hyper exponential random variable, is considered. Because of failure mode, there is no input for sampling mode. The duration until the exit from the sampling mode is also considered to be a hyper exponential random variable of type 4. Because after the sampling, the process is detected either in-control or out-of- control, then the probability of entering the in control mode is 0.5 and the probability of entering the out-of-control modes is 0.5. Also, with the probability α, the process enters the false alarm state, and with the probability β, it is not specified out-of-control of the process, and enters from the sampling mode into out-of-control modes. In addition, with the probability $1-\alpha$, enters the in-control state and, with the probability $1-\beta$, enters the preventive maintenance state. The duration of stay in each machine mode (process), as well as the duration of stay in a failure mode, and preventive maintenance and inspection for false alarm, is an exponential random variable.

3. Results and discussion: Case studies and sensitivity analysis

In order to validate and evaluate the proposed model, a numerical example is presented here, and then a sensitivity analysis is performed to examine the effect of model parameters on optimal solutions.

Consider a manufacturing system that includes a machine that produces a product by the machine. The process of controlling the machine is performed by the $\bar{X}$ control chart. The process of this machine involves various machine (process) modes and failure mode, which, if the process is in a state of failure, corrective maintenance transform the process into a in control state. All modes of the machine (process) and failure mode, preventive maintenance mode, and sampling mode and false alarm mode are considered in the form of a continuous time Markov chain. In the jobshop production system, for each machine and product, a format of the continuous time Markov chain should be considered and solved. In the numerical example presented, a machine is considered to produce a product. . The machine has four modes: the in control mode (mode 1), the out of control modes (mode 2 and 3) and the mode of failure (mode 4). We know that the process state, in the absence of maintenance, only goes to worse condition. A preventive maintenance level is considered. The costs, the arrival rates and the probability of transfers and other assumed parameters, for this example, are presented in Tables 1-4.

To solve the nonlinear model presented in section 2, taking into account the above parameters, a program is written in GAMS software (version 24.9.1). The optimal solution obtained using the BARON solver is $Z^* = 94.108$, $h_{11}^* = 134$, $\tau_{11}^* = 2014$, $n_{11}^* = 5$, $k_{11}^* = 2.3$. It should be noted that the unit of time in this example is in minutes.

Sensitivity analysis was performed to observe the effect of model parameters on an optimal solution. The parameters we are interested in examining their impact are the cost of preventive maintenance, the cost of corrective maintenance and the magnitude of the changes.

The sensitivity analysis parameters are presented in Table 5. The results of the sensitivity analysis are summarized in Table 6.

As we see, the change in magnitude δ_{11} affects the value k_{11}. As δ_{11} increases, k_{11} also increases. And it is logical that larger process changes necessarily require a larger control limit.

Table 1 The exit rates between each mode (node)

λij_{0111}	λij_{0211}	λij_{0311}	λij_{1211}	λ_{ij1311}	λ_{ij2311}	λl_{111}	λf_{11}	λins_{11}
0.00025	0.000166	0.000125	0.0002	0.00011	0.000222	0.0222	0.00555	0.05

Table 2 The probability of transition between each mode (node)

ais_{011}	ais_{111}	ais_{211}	asi_{111}	asi_{211}	asi_{311}	ali_{1011}	ali_{1111}	ali_{1211}
0.0001	0.2999	0.7	0.3	0.3	0.4	0.6	0.3	0.1

Table 3 The costs

ci_{011}	ci_{111}	ci_{211}	ccm_{11}	cl_{111}	cf_{11}	cv_{11}	$cins_{11}$	$cstop_{11}$
0	100	200	2000	200	20	5	50	1000

Table 4 Other parameters

δ_{11}	θs_{11}
1.5	20

Table 5 Changes ccm_{11} and cl_{111} and δ_{11} for sensitivity analysis

Different states	ccm_{11}	cl_{111}	δ_{11}
State1	1000		
State2	4000		
State3		100	
State4		400	
State5			1
State6			3

Table 6 Sensitivity analysis results

Different states	Z^*	h_{11}^*	τ_{11}^*	n_{11}^*	k_{11}^*
State1	71.220	143	2023	3	1.85
State2	139.748	119	1999	5	2.25
State3	92.610	126	2006	3	1.67
State4	96.983	150	2030	4	2.3
State5	108.999	130	1930	3	1.98
State6	82.409	156	1680	3	2.9

As shown in Table 6, the cost of preventive maintenance affects both the sampling interval and the preventive maintenance interval and the sample size. As the cost of preventive maintenance is reduced, the sampling interval and the preventive maintenance interval and the sample size increases to ensure the performance of the production system.

When the cost of corrective maintenance increases, the sampling interval and the preventive maintenance interval decreases, but the sample size increases.

From Table 6, we can see that the relationship between the cost changes of model parameters and the optimal cost derived from the integrated model is not linear. By changing the cost parameters, the model variables that are related to repair and statistical quality control are changed so that the total cost of the integrated model is minimized. Therefore, this analysis shows that a potential cost reduction is done by applying an integrated model for determining repair and quality control policies. In today's competitive environments, cost reduction plays an important role in the performance of the production system.

Changing non-cost parameters of the model also affected the optimal cost of the integrated model. This is also due to changes in the model variables, which simultaneously changed the variables related to repairs and quality control.

The analysis results show that the change in input parameters affects both the preventive maintenance policy and the statistical process control policy, and simultaneously optimizes repair and quality control policies by minimizing the total cost of both policies. Moreover, these results indicate the dependence between these two policies.

4. Conclusion

This paper presents an integrated model for optimizing statistical process control policies (sampling interval, sample size and control limit) and preventive maintenance (the preventive maintenance interval). The information obtained from the quality control charts was used to decide on the preventive maintenance interval. The proposed model was modeled in the form of a continuous time Markov chain, and the model was optimized with the cost-per-unit time scale.

A numerical example is done to clarify the problem, and the sensitivity analysis shows the dependence between preventive maintenance and statistical process control.

The contribution of this paper was to develop an integrated model to optimize preventive maintenance policy and statistical process control policy, which was modeled in the form of a continuous time Markov chain considering the length of time for preventive maintenance and corrective maintenance. The goal was to reduce costs per unit time.

In this model, the duration of preventive maintenance and duration of corrective maintenance are not zero. Considering the length of time for corrective and preventive maintenance, this model is consistent with the reality of the production system. In addition, this assumption makes this model applicable to industrial environments, because in most cases, the duration of corrective and preventive maintenance is not negligible. This issue has not been considered in the literature on the integrated consideration of preventive maintenance and quality control of the process in the form of a Markov chain. This research gap was considered in this article. According to the results and findings of this research, it is possible in future researches to introduce production planning policies in this model. Considering the simultaneous optimization of production planning, preventive maintenance and statistical quality control is an interesting topic for future research.

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