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A novel approximate dynamic programming approach for constrained equipment replacement problems: A case study

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ABSTRACT

This paper presents a novel Approximate Dynamic Programming (ADP) approach to solve large-scale nonlinear constrained Equipment Replacement (ER) problems. Since ADP requires accurate estimations of states for future periods, a heuristic estimator based on data clustering was developed for the case study of this paper with small number of sampling periods. This ADP approach uses a Rollout Algorithm to formulate the problem in a Rolling horizon. The model was solved using Genetic Algorithm (GA). This framework was successfully applied for the decision making process of replacement/maintenance of 497 transformers in a power distribution company, which led to a significant reduction in the expected costs. The proposed framework possesses favourable features such as minimizing the effect of uncertainties in the state variables and measurement inaccuracies, which make the model robust and reliable. This work provides a novel general approach that can be employed for other industrial cases and operations research problems.

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1. Introduction

Equipment replacement (ER) is an important decision process that nearly all industries are dealing with. ER optimization is a common topic in management science, and evolves constantly with the progress in operations research techniques. The literature available in this area focuses on decision making regarding maintenance or replacement of equipment over a limited or unlimited horizon, the examination of gradual changes in a technology, as well as the emergence of a new technology. ER problem has always been studied over the last century. In the early twentieth century, Taylor and Hoteling separately considered the cost of depreciation in ER calculations. In recent years, several studies have investigated the replacement of equipment such as transportation fleet [1], conveyor belts [2], medical equipment [3], reactor equipment (considering risk assessment) [4], heavy mining machinery [5], and information technology (IT) equipment [6].

In classical studies of ER, the goal is to find a policy to minimize discounted costs, while the interest rate and equipment costs usually remain constant. Annual costs of equipment operation and ownership are calculated during their lifetime, and an optimal life for the equipment is obtained by minimizing these costs. It can be clearly inferred from the previous works that the answer to a specific problem may considerably vary by changing the assumptions, where the answer usually changes from an optimal to a sub-optimal or even a non-optimal one. Future state variables are predicted from the previous and current ones, and the control parameters. State variables are evaluated based on measurements of a limited number of parameters with certain inaccuracies; therefore, the future state variables can only be predicted approximately. Leung and Tanchoco [7] proposed that some problems can be better handled by making an integrated decision about the equipment replacement. One of the most important factors which should be considered in making decisions, especially in ER, is to determine the prediction horizon, including limited, unlimited, and rolling horizons [8]. Fraser and Posey [9] presented a four-stage framework for analysing ER based on engineering economics, including determination of an alternative approach, prediction of the monetary flow for each approach, calculation of the present value of the monetary flow for each approach, and select of the solution method with the optimal present value. This method addresses ER on limited and unlimited horizons, as well as with and without considering technological changes.

In modern economics literature, two categories of endogenous and exogenous factors are considered for economic fluctuations. In the 1960s, some economic theorists believed that economic fluctuations are similar to echoes, repeating over years with similar intensities and durations. Although this theory has been rejected in modern economics, Boucekkine, Germain, and Licandro [10] believed that this echo model can be modified and used for ER problems. They claimed that it can be shown that this echo is valid for ER. In addition, they proposed that profitability of different equipment with different technologies can be investigated by examination of possible solutions for a problem. However, this needs a huge amount of complicated calculation. Their study showed that the Dynamic Programming approach to ER is largely connected to the economic echoes model.

One of the current problems in Dynamic Programming as well as in optimal control is to find a solution to integral equations to obtain an optimal policy. By applying an appropriate formulation, integral equations were developed for solving ER problems using Dynamic Programming [11], and later employed in several studies. Motamedi, Hadizadeh, and Peyghami [12] tried to find a numerical solution to the integral equations of [11]. They used the Adomian Decomposition Method to solve the equations, and presented a numerical example of ER to present the algorithm solution. Jacobsen [13] employed system dynamic methods for ER decisions. He first identified the subsystems and their components for his case study, and then estimated the future status of these subsystems using the existing data. The decision variable in his study was to repair or replace equipment.

Two important features of a suitable model are its range of applicability for a particular problem, as well as its prevalence. By reviewing the previous studies on ER over the last 70-years, a common point can be clearly found, i.e., practical applications of most of the proposed models are not yet widespread. Therefore, it can be concluded that these models probably have not considered some practical factors with significant effects on decision-making process. According to [14], except a few cases of Stochastic Dynamic Programming, there has been little progress in this area.

Dynamic Programming solves a problem in successive steps, and adopts an optimal policy to satisfy the principle of optimality. An ER model seeks an optimal decision for preserving or replacement of equipment in consecutive time intervals; thus the Dynamic Programming method has been widely used in solving ER problems. Dynamic Programming was introduced in [15] and applied for ER by Bellman [16]. Dynamic Programming is in fact a general solution approach. Unlike linear or quadratic programming in which the structures of input data and analysis are quite clear, in solving a particular problem, the solution method should be adapted to the problem. Using the general structures proposed in Dynamic Programming, a unique solution method is established considering the main principle of Dynamic Programming - the principle of optimality [17]. On the other hand, unlike quadratic programing which can solve problems with many variables, the basic model of Dynamic Programming is only suitable for small-size problems. Increasing the number of variables usually increases the volume of computations, or in other word causes “curse of dimensionality”. Many problems can be modelled and solved using Dynamic Programming. Depending on whether available information and variables are definite or random, various methods can be constructed to solve the problem. In the classical literature of Dynamic Programming, one can find well-known problems such as stagecoach in the shortest path, warehouse, distribution of effort, budgeting, Knapsack, and ER. The basic model of Dynamic Programming is discrete, and mathematical principles and discrete control are employed to solve the model. Three main components of each Dynamic Programming model are state variable(s), decision or control variable(s) and objective function.

A trade-off always should be performed between the simplicity and possibility of a model analysis with the level of model details reflecting the real world conditions. To make a balance between them, two approaches can be applied: either the model can be simplified as much as possible, which is called the limited model, or it can be more complicated, which requires suboptimal methods to solve it. Bertsekas [18] discussed that the efficiency of suboptimal methods is not less than limited models. In this study, sub-optimal methods were used. With the progress in operations research, different Dynamic Programming methods have been developed to build models very similar to real problems. A recent topic in Dynamic Programming has been the use of the ADP approach to solve large-scale and near-real-world problems. Some algorithms used in artificial intelligence such as queuing and game theory problems, and the optimal control are examples of modern problems closely related to Approximate Dynamic Programming [18].

In this study, the ADP approach has been used to formulate and solve an ER problem for the first time. First, the novel ADP framework to solve successive problems is explained, then the ER case study is described. Finally, it is discussed that the framework presented is able to provide optimal decisions for the ER problem studied.

2. Materials and methods

Like other Dynamic Programming models, an ADP model tries to minimize (or maximize) an objective function considering constraints during a decision-making horizon. State variables for a limited or an unlimited horizon should be predicted based on control (decision) variables. The objective function is calculated from the state variables predicted, and then optimized using the optimal decision variables. Sub-optimal methods like heuristic and meta-heuristic algorithms are usually employed for optimization in ADP.

A difference between definitive and approximate models is in their model state spaces, where an approximate model requires to predict the future state variables of a system using available data for the current state variables, which involves some uncertainties. Thus, before solving a model, the state variables should be estimated based on the control variables of the problem. Appropriate definitive data should be determined for the state space used in a model considering the relationship between data. This is usually done using trial and error, econometrics, data mining, heuristic, and dynamic neural programming methods to minimize the approximation error [18]. The conceptual ADP model adopted from [18] is shown in Fig. 1.

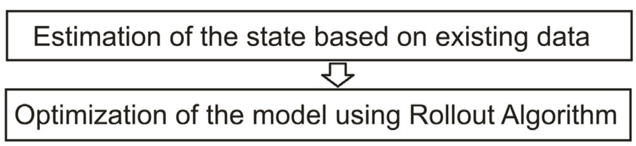

Fig. 1 Conceptual ADP model used for ER problem

2.1 State estimation

The first phase of ADP is the proper prediction of the system state. The applicability of multivariable methods (mostly used in econometrics, such as Auto Regressive Moving Average (ARMA) and Generalized Auto Regressive Conditional Heteroscedasticity (GARCH)) was examined, but appropriate results were not obtained due to low sampling periods of the case study. In similar studies, when the number of samples is high, but the number of measured periods is low, it has been suggested to use data mining techniques for state estimation [19]. Box and Meyer [20] stated that when the number of observations is much less than the number of samples, only a limited portion of data have the major effect on the prediction of samples, and called this situation as "Effect Sparsity". In this study, clustering of data has been used similar to the method proposed by [24]. This means that only those data that can provide the best approximation for future states should be selected. Here, the aim is to apply an algorithm that minimizes the total error which originates from the estimation of existing data ($\tilde{X}_z$) and actual values (X_z). For this purpose, a heuristic algorithm with two estimators was developed, which acts as an intelligent filter to select data. This algorithm will be described in section 3.3 in details.

2.2 Rollout algorithm

The model used for ADP in this study has been constructed based on "Rollout Algorithm", which is a sub-optimal control method for both definitive and stochastic systems. At each stage of decision-making process, the system is converted to a definite state by following specific standard steps, and then the Dynamic Programming or its equivalent optimal control problem is solved on a finite horizon from the current period (also called as rolling horizon). Subsequently, the first element of the decision parameters obtained is taken as the decision element of the current period and the rest are left out. In Rollout Algorithm, the objective function have been considered to be zero after the decision horizon [18].

In the second phase, based on the estimations obtained in the first phase, the problem is formulated and solved using Dynamic Programming. The system state was predicted considering the previous states and the applied decision variables. Although the decision space in this model is extended to several subsequent periods, the goal is to make a decision only in the current period. For the next periods, the process of modelling and problem solving is done again using more accurate inputs for the model.

2.3 Approximate dynamic programming model

The conceptual model is represented as the mathematical model shown in Fig. 2. This a novel general approach that can be used in a variety of problems in Production Engineering and operations research when a regular decision-making process is required. The model is also applicable when the sampling period of measured data is low.

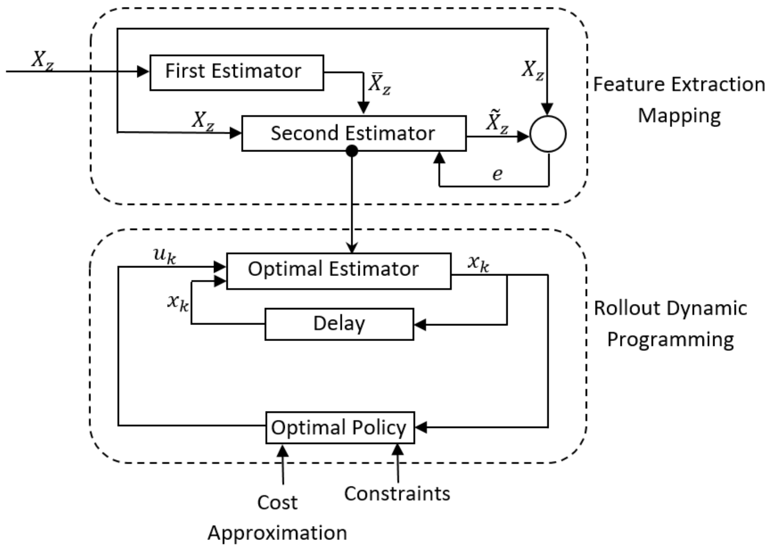

Fig. 2 Schematic representation of the conceptual ADP model

3. Equipment replacement case study

Khorasan-Razavi Electricity Distribution Company (KEDC) has the largest area of activity among other electricity distribution companies in Iran. KEDC distributes electricity at moderate (20 kV) and low (400 V) voltage levels in Khorasan Razavi province. Torbat-e Heydarieh Electricity office with over 103,000 consumers in 2018 is the third largest office of KEDC. Transformers are expensive equipment widely used in electricity distribution networks, and usually provide general or private power supply to one or several consumers. The case study here includes all 497 general pole-mount transformers in Torbat-e Heydarieh, where repairing and replacement of these transformers impose a significant cost to the Company. These transformers have different capacities and ages. No structured method was previously used to make a schedule for maintenance or replacement of the transformers. This study used the transformers data available from 2013 to 2018 to build the ADP model, and identify transformers which possibly require replacement in 2019. For this purpose, it was necessary to choose suitable cost and objective functions, which were determined using a fuzzy method based on the available information and KEDC experts' opinion.

3.1 Formation of the database and normalization

In this study, the Health Index (HI) of transformers is considered as the state variable. Three factors that affect the transformer HI are temperature difference, oil condition, and maximum load. To calculate the temperature difference, a thermo-vision camera was used to monitor the temperature at the outer surface of transformers. The difference of the hottest point of the transformer and the ambient temperature was recorded as the temperature difference. Insulator (oil) breakdown voltage tests were conducted for operating transformer to evaluate the oil condition. In this test, the temperature at which the oil loses its insulating properties was recorded. Finally, the maximum load is defined as the transformer load at peak times divided by its nominal capacity. These data were available for the years from 2013 to 2018. The data has been normalized using the max-min normalization method [21]. In this normalization method, if a factor is desirable to be higher, it is called a positive factor, and normalized as follows,

$$x_s = \frac{x - x_{min}}{x_{max} - x_{min}} \tag{1}$$

however, if it is desirable to be lower, then is called as a negative factor, and normalized as follows,

$$x_s = \frac{x_{max} - x}{x_{max} - x_{min}} \tag{2}$$

where x_s is the normalized value of x, and x_{max} and x_{min} are the maximum and minimum values of the data in the period, respectively. In this study, the maximum load and temperature difference are negative factors, while the oil condition is positive.

3.2 Calculation of the health index as a state variable

HI for the transformer is defined as follows [22],

$$HI = \frac{\sum_{j=1}^{F} w_j \times HIF_j}{\sum_{j=1}^{F} w_j} \tag{3}$$

where F is the number of the factors ($F = 3$ in this study), w_j is the weighting factor, and HIF_j is the HI for each factor. The weighting factors were obtained using a Fuzzy AHP method with the help of the KEDC experts. The decision-making team were asked to prioritize the three HI factors considering their effects on the transformer health. If a transformer is replaced by a new one, the HI of the new transformer is considered to be one until the end of the decision horizon.

3.3 Creating the state estimator model

This section explains how to organise the estimator for predicting the factors affecting the state of the model (HI of the transformers). The actual data (X_z) are available for six periods. As shown in Fig. 2, the data of each period in the first estimation step ($\bar{X}_z$) are estimated using the linear regression model, and then used in the second estimator. In the second estimator, the data of similar transformers are averaged, and the final approximation data ($\tilde{X}_z$) are calculated and compared with the actual data (X_z). This procedure was carried out as follows:

A. The period k in which the data are available is determined and repeated for $k = 1\ to\ K$ (K is number of previous periods with available data);
B. For each transformer t, transformers are sorted from $1\ to\ T$ (except for the period k) in a table based on the similarity of the data matrices (T is total number of transformers);

C. In each step, the following tasks are repeated for $q = 1\ to\ Q$ (q is the number of similar transformers and Q was determined 50):
   C.1. q similar transformers are selected (e.g., if $q = 2$, the transformer itself and the most similar transformer to that are selected). The average matrix is obtained by averaging over similar arrays. This 3D matrix has a dimension of $(K-1) * 3 * T$ (it consists of $K-1$ periods in the first dimension, 3 factors in the second dimension, and T transformers in the third dimension).
   C.2. Using the average matrix of the previous step, the data for the period k are estimated by a linear regression model;
   C.3. Using a multivariable linear regression model for each factor, a linear set of equations are formed based on the other estimated factors in section C.2, and the data for the transformer t are estimated in the period k.
D. The absolute differences of the estimated and actual values are calculated for all transformer factors in all periods, and their summation is considered as the model error. This task is done for different values of q similar transformers, and the error is calculated for each q. For each factor, the q with the minimum error is selected as the optimal number for the model estimators. In other words, using q similar transformers is recommended for the best state estimation.

3.4 Calculating the price of depreciated transformer

In replacing transformers, the price of depreciated transformers should be considered. There are 10 different types of transformers with different capacities. To simplify the calculations, the price of a new 25 kVA transformer is considered one unit, and the price of other transformers is normalized to that. The value of a transformer used for l years is defined as follows,

$$VT(l) = C_0 * e^{-l/\lambda} \tag{4}$$

where C_0 is the price of a new transformer, λ is the depreciation constant, and l is the age of the transformer. λ was determined using fuzzy logic. Fuzzy logic uses fuzzy numbers instead of fixed and definite ones. This study employed the fuzzy logic method introduced by [23] and well described by [24]. The output of fuzzy calculations is a table which indicates the value of a depreciated transformer at different ages compared to a new transformer. For this purpose, a team of KEDC experts were asked to determine a minimum and a maximum price for transformers according to the age of transformers. These data were translated to fuzzy numbers, and tabulated in a data table. Finally, Eq. 4 was fitted to this data table.

3.5 Approximate dynamic programming formulation

After finding the optimal estimator model, the objective function and the constraints should be rewritten in the form of the Rollout algorithm. This is done as follows (the total number of transformers is T, and each transformer and period are represented with indices of t and k, respectively).

A. Using the estimator model proposed in the previous section and the data collected for the transformers, the factors for each transformer are predicted for the next period.
B. Using the estimated factors and Eq. 3, HI values in each decision period (HI_k^t) are obtained for all transformers in a form of a column matrix (HI_k). By assembling this column matrices, a 2D matrix is formed, which shows the general state of the system when no control is applied to the system (no transformer is replaced during the decision horizon).
C. u_k^t is a zero-one control variable which indicates keeping or replacement of the transformer t in the period k; $u_k^t = 0$ if a transformer is preserved, and $u_k^t = 1$ if replacement is required. The state variable in the next period x_{k+1}^t is obtained using the transformer HI as follows,

$$x_{k+1}^t = x_0^t * u_k^t + p_n^t(x_{k-n+1}^t, x_{k-n+2}^t, \dots, x_k^t) * (1 - u_k^t) \tag{5}$$

Eq. 5 shows that if it is decided to replace the transformer t, the situation is the same as the initial condition of the transformer installation (this transformer will not be replaced and its

HI remains constant until the end of the decision horizon). On the other hand, if it is decided to keep a transformer, the state value is estimated using the previous n data states as indicated in the steps A and B; $p_n^t(x_{k-n+1}^t, x_{k-n+2}^t, \dots, x_k^t)$ presents this conditional value calculated by the state estimator.

D. Based on the transformer state vector for each period, the costs of operation, maintenance and repair of each transformer, and a survey for transformers, the value of the transformer t is determined as,

$$CK_k^t = 0.15 * C_0^t * e^{0.065*\left(1-(HI_k^t)^2\right)*l_k^t} \tag{6}$$

where $C0^t$is the price of a new transformer similar to the transformer t, and l_k^t is the age of the transformer t in the period k.

E. CR_k^t is the costs of purchasing new transformers and the replacement operation in the period k. The price of a new transformer is priori known, and the cost of the replacement operation is estimated to be 20 % of the price of a new transformer. Moreover, the depreciated transformer cost is subtracted from the replacement cost. This cost for each transformer t is calculated as,

$$CR_k^t = 1.2 * C_0^t - VT(l) \tag{7}$$

where $VT(l)$ is calculated using Eq. 4 as a function of the transformer age.

F. The expected cost in each period is denoted as g_k and is calculated as follows,

$$g_k = \sum_{t=1}^{T} CR_k^t \times u_k^t + \sum_{t=1}^{T} CK_k^t \times (1 - u_k^t) \tag{8}$$

A constraint of the budget in each period b_k is,

$$\sum_{t=1}^{T} CR_k^t \times u_k^t \leq b_k \tag{9}$$

where b_k was determined to be 50 in this study for all decision periods.

G. Finally, the following equation should be solved,

$$J_k = \min(g_k + \alpha g_{k+1} + \alpha^2 g_{k+2} + \dots + \alpha^n g_{k+N}) \tag{10}$$

where α is the discount rate which is between zero and one (in this study considered to be 1) and N is the number of decision horizon periods (here $N = 4$). At each decision-making step, only transformers with a minimum age of $N + 1$ are examined for possible replacement as follows,

$$\sum_{k=1}^{N} u_k^t \leq 1 \tag{11}$$

Eqs. 5, 8, 9, and 10 are general equations, which can be used for any kind of ER problem. Only Eq. 6 and Eq. 7 are specific to this study, and the should be defined for other types of ER problems. The Eq. 11 may exist or be omitted related to ER conditions.

3.6 Solving the objective problem using genetic algorithm (GA)

A GA method has been employed to solve the problem, which consists of the following steps:

A. Generating control matrices consisting of “random zero and one” elements (497∗5 in this study) as initial population (here 8 samples):
   A.1. Constructing the matrix with random zero and one arrays;
   A.2. Making each population feasible by applying random changes in matrix elements to include the constraints;
   A.3. Local optimization of the population using the random greedy algorithm;

B. Generate the secondary population using GA according to the following steps:
   B.1. Initial composition and formation of several new matrices that are not necessarily feasible and optimal;
   B.2. Making new matrices feasible;
   B.3. Local optimization of the population;
   B.4. Selecting better samples according to the best sample in the previous step;
   B.5. Repeating step B.1 to B.4 until a desired answer is obtained.

4. Results

Fuzzy AHP method was used to find the weights of the three factors. First, the experts were asked to fill in a standard questionnaire to determine the priority of the factors, and then fuzzy numbers and the inconsistency rate were determined for each questionnaire. The weights for temperature difference, oil condition, and maximum load were obtained as 0.53, 0.27, and 0.19, respectively.

The data matrix for each transformer was defined as a matrix $A = [a_{ij}]$ ($i = 1$ to 6 shows the period of the data, and $j = 1$ to 3 presents the three HI factors). For two transformers, the data difference index is defined as the sum of squared differences of elements (the difference index of any transformer relative to itself is zero). For a transformer A, the data difference indices relative to all other transformers were calculated, and transformers were sorted from the minimum difference index to the maximum index.

Next, for each factor, the number of similar transformers required to obtain the best prediction for that factor was found. These numbers for three variables of maximum load, temperature difference, and oil condition were obtained as 11, 32 and 12, respectively as in shown in Fig. 3. This means that for the best prediction of the maximum load, the data of 11 similar transformers (including itself) are required. Using the proposed estimator, the values of these three factors for the coming years (from 2019 to 2023 in this study) were predicted. Having the weights of the factors and the estimated values for the HI factors, the state of the system (transformers HI) was estimated assuming no transformer replacement is done during the decision period.

A fuzzy logic was employed to evaluate the value of in-use transformers. The experts were asked to determine a minimum and a maximum value for transformers over different operating years. These values were converted to fuzzy numbers. Next, fuzzy numbers were converted to crisp values. Table 1 summarizes the results of the survey carried out.

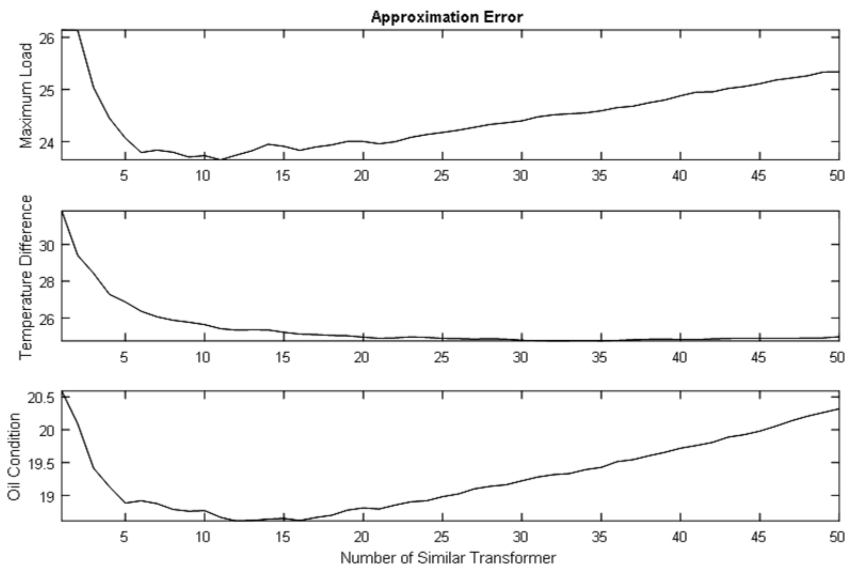

Fig. 3 Approximation error for each health index factor

Table 1 Summary of the survey on evaluation of transformer price over its operation lifetime

Transformer age	Minimum			Maximum			Value (per unit)
	Lower (%)	Geo_Mean (%)	Upper (%)	Lower (%)	Geo_Mean (%)	Upper (%)	
After 10 years	60	67	80	70	84	90	0.751
After 20 years	30	40	50	50	58	70	0.5
After 30 years	20	23	40	30	42	50	0.342
After 40 years	10	10	10	20	28	40	0.19

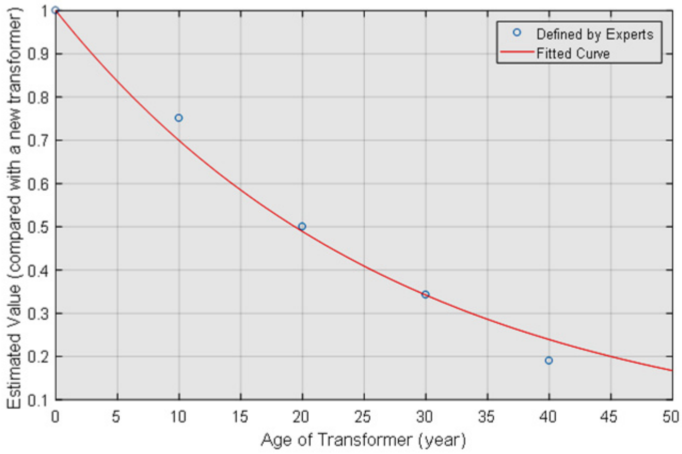

Fig. 4 Fitting a decreasing exponential function to the value of an in-use transformer

These values are normalized to the price of a new transformer, and plotted as a function of the transformer age in Fig. 4. An exponential function according to Eq. 4 was fitted to these data, and the depreciation factor λ was found to be 27.9663.

The decision matrix of this study has dimensions of $497 * 5$ entries (the number of transformers and periods are 497 and 5, respectively). The elements of this matrix are zero or one, which represent preserving (0) or replacement (1) decision for a transformer in a period. For example, if it is decided to replace the transformer 25 in the period 3, the element (25, 3) is one; otherwise, it would be zero. A constraint of the problem is that a transformer can be replaced only one time in the decision horizon (5 periods). Regarding this constraint, the decision matrix can have $6^{497} \cong 5.51 * 10^{386}$ different states, which indicates that the problem is NP-hard.

The primary decision matrix is a zero matrix, which means that the transformer states do not change, and the replacement cost is zero. According to the problem data, the total cost of a period without any replacement is 3696 units. The equivalent state matrix was first estimated according to the control matrix in each period, and then the total cost was calculated. The objective of the problem is to minimize this total cost. A GA algorithm with 8 population samples and 12 iterations of genetics was implemented; the total cost was observed to decrease from 3696 to 3134 units. Table 2 lists the output decisions and the estimated costs obtained by the proposed method. As mentioned earlier, only the decision for the current period (2019) is considered and the rest are neglected. For the next years, this method can be repeated using the updated data.

Table 2 Selected transformers for replacement in different periods of decision-making

Year	Proposed transformers to be replaced	Replacement cost	Maintenance cost	Period cost
2019	9, 101, 102, 130, 160, 209, 211, 212, 225, 273, 310, 316, 319, 366, 444, and 493	49.89	586.96	636.85
2020	10, 21, 70, 150, 156, 166, 169, 181, 213, 277, 311, 326, 347, 409, 459, and 497	49.35	576.90	626.25
2021	68, 105, 122, 152, 159, 175, 224, 240, 242, 244, 261, 279, 293, 304, 307, 318, 337, and 486	49.56	569.59	619.16
2022	2, 28, 36, 106, 108, 123, 125, 131, 182, 184, 195, 233, 276, 305, 400, 419, and 420	50.00	576.53	626.53
2023	23, 64, 90, 94, 154, 176, 200, 232, 259, 268, 303, 325, 327, 341, 354, 454, and 491	40.70	584.45	625.15
Total cost in decision horizon		239.50	2894.44	3133.94

5. Discussion

Previous studies generally solve ER problems by simplifying the model and assuming many parameters to be constant. Many of these methods consider the problem situation to be unchanged or to have slight changes. In real situations, however, existing data of a problem are limited and non-deterministic, and factors such as human and instrumental errors cause uncertainty in system observations. Thus, models must move towards considering more and more uncertainties. Fan et al. [25] classified three distinct approaches for equipment replacement problems: minimum equivalent annual cost, experience/rule based, and dynamic programming approaches. They discussed that the main drawbacks of the first two approaches are the assumption of stationary in the first one and the dependence on experience and judgment of experts. They maintained that uncertainties should be mixed with Dynamic Programming, and then a stochastic dynamic programming approach is used for Equipment Replacement problems which can be classified in the Markovian decision process. Their approach is not suitable when the state space includes several variables, or there are constraints or a huge amount of nonlinearity due to computational constraints. Available models for ER problems become unsuitable when the problem size increases. The new approach proposed in this study is able to handle large constrained problems with several state and decision variables and nonlinear functions.

By applying a few modifications to the ADP model, it can be employed for other production management problems such as location, allocation, inventory, assembly line. For example, in constrained queuing theory problems, this approach can be used for the current period planning; the objective function is defined as the sum of penalties functions and optimized for the next planning periods. The first period results are considered for decision making, and the process is repeated for the next periods.

The interdependence of the state variables was examined in this study, but no relationship between these factors was found. This makes the model used more reliable, because many other hidden factors, which were difficult to measure or not considered, have already affected the data of these three state variables. Because of the clustering of transformers data and considering the Effect Sparsity, these three chosen factors appropriately represent other hidden factors not included in this study.

An important aspect in Rolling Horizon is the number of decision-making periods. Considering a large number increases the data forecast error. On the other hand, if a small number is taken into account, the replacement costs are minimized in the last stages of the forecast process, since the costs of replacement are higher than those of maintenance, and the costs even may not approach the budget constraints. In this study, five periods (including current period) were considered.

An outcome of this study is the prediction of the transformers conditions in coming years, which provides very useful information for the KEDC experts. This prediction can be considered as a Decision Support System. For example, for transformers which have been in use for less than 20 years and were on the list of possible substitution in the next years, a special maintenance program was set up to fix their deficiencies before it becomes necessary to replace them. This supports the use of the Rollout algorithm, which only takes the output of the first period.

6. Conclusion

In order to model ER problems with uncertainty, Only a few limited approaches such as Stochastic Dynamic Programming are available in literature, which cannot be employed for real case problems due to "curse of dimensionality". Previous models cannot deal with constraints and nonlinearities when the volume of a problem expands. In the current study, a new ADP approach was developed for large-scale nonlinear constrained problems. The proposed approach can be applied for a wide range of production management and operations research problems.

Accurate and well-timed decisions on maintenance or replacement of equipment reduces the costs significantly. The mentioned ADP model was used to solve a real transformer replacement problem. The objective was to make an optimal decision considering constraints, and consequently achieve the highest reduction in the expected costs. The parameters measured for transformers were used to define the state space; then by proposing a heuristic estimator, these parameters were predicted in the decision horizon. The objective function was defined as the maintenance and replacement costs, and optimized using GA. These costs were determined according to the available data and conducting surveys. General formulation for the ER problem of this study can be used for other situations such as assembly line, workshop machineries, fleet management.

The model presented has unique features. First, it employs realistic functions which were obtained based on the opinion of some experts in electricity industry. The second feature is its robustness in dealing with data associated with uncertainties and errors caused by human, instrumental, and environmental factors. The state estimation algorithm minimizes the effect of possible inaccuracies in data, and can be applied for a wide range of problems with a few number of sampled periods and large number of samples. The third feature is the consideration of several active factors that affect the problem and some other latent factors, as well as possible interdependency of these factors. The fourth feature is the ability of the model to deal with nonlinear and even zero-one constraints and functions, and meanwhile preserve a high solution speed (which is a trait of the ADP method).

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