

## A multi-product pricing and inventory model with production rate proportional to power demand rate

Keshavarzfar, R.<sup>a</sup>, Makui, A.<sup>b</sup>, Tavakkoli-Moghaddam, R.<sup>c,d,\*</sup>

<sup>a</sup>School of Industrial Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran

<sup>b</sup>School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

<sup>c</sup>School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran

<sup>d</sup>Arts et Métiers ParisTech, LCFC, Metz, France

### ABSTRACT

This paper deals with an economic lot size model when demand follows a power law and changes with time, owing to the fact that this kind of pattern is suitable for so many real situations. Production rate is presumed to be proportional to demand rate. Also since that demand is price sensitive in reality, we suppose that demand decreases linearly with price. With regard to these points, in this article a joint pricing and inventory model is presented where demand depends on time with a power pattern and price linearly, production rate changes pro rata with demand rate and multiple items are considered. The principal consideration of the study is to satisfy the demand and optimize the profit for all items in the system, simultaneously. Setup, holding, backlogging and production costs are involved in the inventory system. The aim is to maximize total profit function and achieve optimum values of scheduling period, reorder point and price. Employing mathematical modelling and optimization methods, the existence of the optimal solutions is proved, and then a simple heuristic algorithm is presented to maximize total inventory profit and determine the best values of variables. A numerical analysis is carried out to illustrate the applications of the proposed models.

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#### \*Corresponding author:

tavakoli@ut.ac.ir  
(Tavakkoli-Moghaddam, R.)

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# Model oblikovanja cen in zalog z več izdelki s stopnjo proizvodnje, sorazmerno z eksponentno stopnjo povpraševanja

Keshavarzfar, R.<sup>a</sup>, Makui, A.<sup>b</sup>, Tavakkoli-Moghaddam, R.<sup>c,d,\*</sup>

<sup>a</sup>School of Industrial Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran

<sup>b</sup>School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

<sup>c</sup>School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran

<sup>d</sup>Arts et Métiers ParisTech, LCFC, Metz, France

## POVZETEK

Prispevek obravnava ekonomski model velikosti serije, pri katerem se povpraševanje eksponentno spreminja s časom, kar je značilen vzorec za mnogo resničnih situacij. Domneva se, da je stopnja proizvodnje sorazmerna s stopnjo povpraševanja. Ker je povpraševanje odvisno od cene, domnevamo, da se povpraševanje linearno zmanjšuje s ceno. V zvezi s temi točkami je v prispevku predstavljen skupni model cen in zalog za več artiklov, pri katerem je povpraševanje eksponentno odvisno od časa, cena linearno, stopnja proizvodnje pa je sorazmerna s stopnjo povpraševanja. Glavna naloga študije je zadovoljiti povpraševanje in optimizirati dobiček za vse artikle v sistemu hkrati. Stroški postavitve, zadrževanja, zaostankov in stroškov proizvodnje so vključeni v sistem zalog. Cilj je maksimirati funkcijo skupnega dobička in doseči optimalne vrednosti obdobja načrtovanja, točke naročila in cene. Z uporabo metod matematičnega modeliranja in optimizacije se dokaže obstoj optimalnih rešitev, nato pa je predstavljen preprost hevristični algoritem, ki maksimira skupni dobiček in določi najboljše vrednosti spremenljivk. Za ponazoritev uporabe predlaganih modelov je izvedena številčna analiza.

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## PODATKI O ČLANKU

*Ključne besede:*

Model cen;

Model zalog;

Gospodarna količina proizvodnje (EPQ);

Pomanjkanje ponaročil;

Eksponentno povpraševanje

*\*Kontaktna oseba:*

[tavakoli@ut.ac.ir](mailto:tavakoli@ut.ac.ir)

(Tavakkoli-Moghaddam, R.)

*Zgodovina članka:*

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