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# Designing a warehouse internal layout using a parabolic aisles based method

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**A B S T R A C T**

Refined layout is a basis of warehousing efficiency. Straight aisle is a typical feature of current warehouse internal layouts. The purpose of this paper is to explore the possibility of using curve aisles for warehouse layout. By Choosing typical non-traditional layouts and transforming their inclined cross-aisle trajectory into parabola, two parabolic aisle layouts, parabolic Flying-V and parabolic Fishbone, are constructed. For unit-load warehouses, based on the morphological characteristic analysis and the parabolic types selection, the picking distance model and the cross-aisle length formula are presented. Interval Numerical Simulation Method (INSM) and Genetic Algorithms (GA) are adopted to solve the model respectively in order to verify the results. This research breaks through the realistic situation of straight aisle leading warehouse layout, and enriches the relevant layout theory. The calculation results of 100 warehouses with different sizes show that the picking distance of parabolic Flying-V could be reduced by 0.22-0.62 % compared with the straight layout, and the theoretical possible improvement space has been compressed by 2.42-12.26 %. Its length of cross-aisle is shortened by −0.03-3.10 %. The picking distance of parabolic Fishbone could be only reduced by 0.02-0.04 %. The theoretical possible improvement space has been compressed by 1.27-1.83 %. But its length of cross-aisle will increase by 4.63-19.50 % significantly. We believe that the layout of non-rectangular complex special-shaped warehouses based on curve trajectory aisles would become an important research topic. In addition, after some necessary modifications to the objectives and constraints, the proposed method in this paper may also be used for the arrangement of machines and devices in a workshop in principle.

**A R T I C L E   I N F O**

*Keywords:*
Layout design;
Warehouse internal layout;
Parabolic aisle layout;
Layout efficiency;
Simulation;
Optimization;
Interval numerical simulation method (INSM);
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## 1. Introduction

Warehouse is an important social logistics infrastructure. The level of warehouse system planning and design directly affects the overall logistics efficiency of a country. We know that the travel distance of a picking tour is an imperative factor in improving warehouse operation efficiency [1]. The efficiency of warehouse processes could be improved by reducing travel time and cost in replenishment and order picking [2]. The layout of internal aisle in warehouses is the basis of storage space planning and warehouse operation scheduling management, which directly affects the area utilization rate, the access efficiency, the difficulty of layout adjustment, the overall operation energy consumption and the total cost of warehousing. With the rapid development of warehouse operation and management technology, warehouse enlargement and automation are the future development trends. The benefit of lean warehouse layout research will be more fully reflected in the construction of more and more intelligent super-large warehouses.

Traditional warehouse layouts lack detailed quantitative research on the efficiency mechanism of aisle layout itself. In practice, it is usually based on the qualitative analysis to choose a basic layout mode, and then to match the specification parameters such as aisle width, function area size and shelf size according to relevant design specifications. For the common rectangular warehouses, most of them are based on experience and intuition, using the layout of parallel shelves and orthogonal straight track aisles (as shown in Fig. 1) [3].

In 2009, Gue and Meller break away from the traditional conventions, and innovatively put forward two new warehouse internal layouts with inclined cross-aisles: Flying-V and Fishbone (as shown in Fig. 2) [4]. Numerical simulation results show that, for the unit-load warehouse, these two new non-traditional layouts based on the cross-aisle angle modelling and optimization can reduce the average total picking distance by about 10 % and 20 % respectively compared with the traditional layouts based on empiricism.

Literature review shows that both the traditional layouts based on parallel orthogonal aisle mode selection and the non-traditional layouts based on aisle angle modelling and optimization have a problem or shortcoming. The aisle trajectory is basically a straight line or piecewise straight line by default. The layout design revolves around the direction and distribution of the cross and picking aisles of the straight-line trajectory. There is no research on warehouse aisle layout based on curved line trajectory has been found in the literature. Choosing straight aisle has become a default research paradigm of warehouse internal layout design.

Aiming at the problem or shortcoming, this research widens the view of aisle trajectory selection and proposes the conception of exploring the curve path layout method. Focusing on the curve aisle layout problem, we selected two typical non-traditional layouts, Flying-V and Fishbone, to carry out the curve transformation of the parabolic cross-aisle trajectory. The straight trajectory of the inclined cross-aisles in the original layout is changed into a parabola, but the straight characteristics of the horizontal or vertical cross-aisle remain unchanged (as shown in Fig. 3).

The parabolic aisle layout expands the trajectory shape of the cross-aisle, and could obtain a layout scheme with higher picking efficiency (Because it has enlarged the feasible region of the optimization model.) without significantly affecting the utilization ratio of warehouse area (Because it does not change the number of the cross-aisles.). In order to balance the contradiction between picking efficiency and utilization ratio of warehouse area better, we introduce the cross-aisle length index in our model.

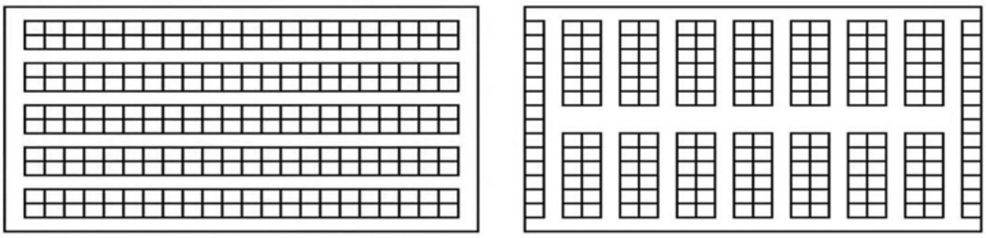


**Fig. 1** Two typical kinds of traditional warehouse internal layouts

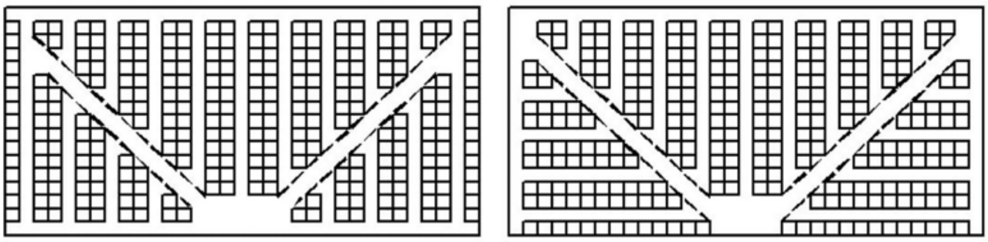


**Fig. 2** Flying-V and Fishbone internal layouts of warehouse

This paper continues with the following content. The next section is a literature review. The third section describes the modelling process of the parabolic layout. The fourth section provides the model of the corresponding straight layout. The fifth section gives out the methodology of the model solving. The sixth section presents our numerical results with discussions. The last section concludes this paper.

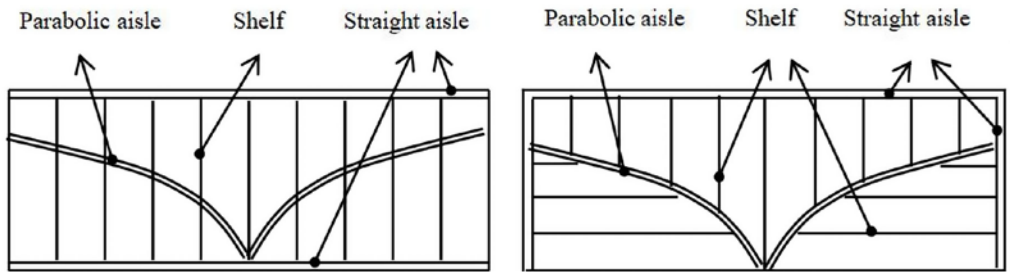



**Fig. 3** Parabolic Flying-V and parabolic Fishbone internal layouts of warehouse

## 2. Literature review

The new non-traditional layout of Gue and Meller attracted wide attention from academia, and became a research hotspot soon. Pohl *et al*. studied the optimization of Fishbone layout under the dual-command [5]. The results show that the background of dual-command significantly reduces the advantage of Fishbone layout compared with traditional layout. Pohl *et al*. investigated the impact of different warehousing strategies on Flying-V and Fishbone layout under single and dual-command [6]. The results show that the optimal warehouse design parameters under stochastic strategy also perform well under turnover rate strategy. Gue *et al*. presented the aisle layout optimization problem in multiple P&D (Pickup and Deposit) points and unit-load warehouses [7]. Flying-V and Inverted-V warehouse models are established respectively. The numerical results show that the advantage of Flying-V aisle design is not as significant as that of the single P&D point. Cardona *et al*. made an in-depth analysis of the cross-aisle angle in Fishbone layout [8]. According to the fixed and uncertain length and width of warehouse, the determination of optimal aisle angle and its robust stability are discussed, which provides a theoretical basis for the flexible decision-making of actual warehouse layout design. Ö.Öztürkoğlu *et al*. based on the idea of increasing the number of cross-aisles in traditional warehouse layout and optimizing the angles of its cross and picking aisles, presented three non-traditional warehouse layouts, Chevron, Leaf and Butterfly [9]. Angular modelling and optimization analysis show that these three non-traditional warehouse layouts can reduce the total picking distance by 19.53 %, 21.72 % and 22.52 % respectively compared with traditional warehouse layout. Some practical applications of non-traditional warehouse layout are also discussed. Clark and Meller introduced the vertical travel parameter to modify the picking time model, and investigated the impact of vertical movement on Flying-V and Fishbone layout on multi-storey shelves [10]. The results show that the vertical operation of shelves has a great impact on Flying-V layout, but not on Fishbone layout. Jiang *et al*. improved Fishbone layout by considering the characteristics of drive-in racking, and obtained a Leaf-like layout [11]. Liu *et al*. based on Fishbone layout, according to the general storage principle, studied the optimization of storage location allocation by using of the genetic Algorithms and MATLAB tool [12]. Ö.Öztürkoğlu *et al*. considered the relationship between the location of a single pallet and the aisle, constructed a constructive multiple P&D points unit-load warehouse aisle design network-based model [13]. Particle swarm optimization is used to optimize the aisle design scheme. Cardona *et al*. proposed a very practical three-dimensional detailed design method of Fishbone layout [14]. This method considers the influence of aisle width, forklift speed, rental fee and maintenance cost, etc. Marco Bortolini *et al*. proposed a layout of inserting one or more straight non-orthogonal cross-aisles into rectangular warehouses [15]. The picking distance and warehouse area loss of this kind of layout under unit-load are also analysed. The average picking time and warehouse parameters under random warehousing strategy and individual operation instructions are obtained. The proposed layout is essentially a straight aisle Flying-V and its expansion. When the optimal layout is adopted, the picking distance can be reduced by 7-17 %. Liu *et al*. studied the problem of solving the optimal Fishbone layout by means of the genetic Algorithms [16]. Akhilesh Mesa discussed the non-traditional layout method of multiple P&D points and unit-load warehouses with multiple cross aisles [17]. Unlike other designs in the past, the aisles are arranged in diamonds. In essence, the layout proposed in this paper can still be regarded as a combination of two Fishbone layouts. Mowrey *et al*. applied the idea of non-orthogonal inclined aisle to the internal layout of retail stores [18]. In order to make full use of the limited space in the store and show more goods to customers, this research expands the practical application scope of non-traditional layout. Zhang *et al*. proposed the Twin Leaf method of warehouse aisle layout in view of the deficiencies of Leaf layout [19]. Three basic characteristics of the Twin Leaf layout are analysed. Compared with Leaf layout, Twin Leaf layout can reduce the picking distance by 1.02 % on average, and the theoretical possible improvement space can be compressed by 45.45 %.

To sum up, in the past ten years, non-traditional warehouse layout methods have been deeply studied by scholars all over the world and applied by some American enterprises. These researches can be roughly classified into two major categories: the first one is the innovation of basic layout methods. Five non-traditional layout methods, Flying-V, Fishbone, Chevron, Leaf and Butterfly are presented. The second one is the efficiency analysis of basic layouts in specific warehousing operating environment. For example, the impact of different practical operating environments, such as single-command, dual-command, multiple P&D points warehouses, specific warehousing strategy requirements, location assignment optimization, etc.

In fact, the budding idea of non-orthogonal design of warehouse aisles could be traced back to the 1960s. Moder and Thornton analysed the influence of pallet placement angle and aisle width on the utilization ratio of warehouse area through mathematical modelling [20]. Francis studied the optimal layout of rectangular warehouses based on the assumption of straight path and the consideration of picking and construction cost [21, 22]. Berry put forward a proposal to arrange pallets around a diagonal aisle into different roadways according to the characteristics of inventory units [23]. White studied the Euclidean efficiency estimation of radial aisles in non-rectangular warehouses [24]. Under the assumption of continuous space, Euclidean efficiency of four and six radial aisles is estimated. Bassan *et al*. based on Francis's research, considered the aisle structure parameters, and analysed the impact of the internal layout of warehouse on the overall operating cost of warehouse [25]. The total cost function is constructed to optimize the warehouse layout. In the early stage, these scattered discussions on the design ideas and methods of non-orthogonal warehouses layout were relatively shallow. Their basic limitation is that they are only preliminary theoretical discussions, no practical design specifications, lack of practical application case testing, and thus they failed to arouse attention. Since then, relevant research has entered a relatively quiet period of about 30 years.

## 3. Research models

Based on the analysis of morphological characteristics, the selection of parabolic type and the efficiency modelling of layout, the research is carried out in turn. For the convenience of analysis and comparison, the idea of continuous space modelling is adopted to build the efficiency model of the parabolic layout in unit-load warehouses [9]. The cross-aisle length is introduced as a supplementary evaluation index to the optimization of the layout efficiency.

### 3.1. Assumptions and symbols

There are many factors affecting the layout of warehouse aisles. For the convenience of discussion, the following premise assumptions and parameter symbols are specially made.

*Assumptions*：

1) The warehouse is rectangular.
2) Neglect the influence of warehouse height on the layout of warehouse aisle.
3) Only one storage or picking operation is carried out for goods at a certain location in the warehouse, and one-way moving distance is used.

4) One-time completion the operation of goods in the whole warehouse.
5) Consider the situation of a single P&D point, and every warehousing operation passes through the P&D point.
6) The widths of the cross and picking aisles in warehouse are neglected in our optimization models. For the convenience of comparison, we adopted the same assumption described in Ö.Öztürkoğlu *et al.* [9]. Although this continuous model with zero width aisles is not completely consistent with the reality, it is close enough for our research purposes, especially for super-large warehouses.
7) The volume of goods and the size of pallets or shelves in the warehouse are all neglected when modelling and analysing.

*Symbols*：

P&D: Pickup and Deposit point, the entrance of goods;

$O, x, y$: O is the origin of the coordinate system. The origin is located at the P&D point. The length direction of the warehouse is set to $x$ axis and the width direction is set to $y$ axis.

$w$: Half of the length of warehouse horizontal direction;

$h$: Total vertical width of warehouse;

$N(x, y)$: The location coordinates of any storage point in the warehouse: $-w \leq x \leq w$, $0 \leq \mathrm{y} \leq \mathrm{h}$;

$\alpha$: The cross-aisle angle of the right half of the warehouse under the straight Flying-V layout;

$\beta$: The cross-aisle angle of the right half of the warehouse under the straight Fishbone layout;

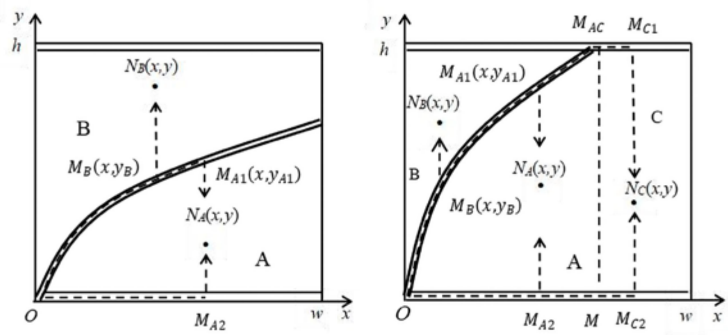
$\alpha^*$: The optimal angle of the cross-aisle when the picking distance of the straight Flying-V layout is the smallest;

$\beta^*$: The optimal angle of the cross-aisle when the picking distance of the straight Fishbone layout is the smallest.

### 3.2. Morphological characteristics of the parabolic layout

These two parabolic layouts are both bilateral symmetrical, so we only need to take the discussion on the right half of the warehouse. According to the different possible intersection positions of the parabolic cross-aisle and the warehouse edge, they are divided into two different sub-morphologies (as shown in Fig. 4 and Fig. 5).

In the sub-form 1 of parabolic Flying-V as shown in Fig. 4(a), the right half warehouse is divided into two picking sub-areas, A and B, by the parabolic cross-aisle. In the sub-form 2 of parabolic Flying-V as shown in Fig. 4(b), the right half warehouse is divided into three picking sub-areas, A, B and C, by the parabolic cross-aisle and the line segment. Similarly, in the two sub-forms of parabolic Fishbone shown in Fig. 5(a) and Fig. 5(b), their right half warehouses are both divided into two picking sub-areas, A and B, by the parabolic cross-aisle respectively.




(a) (b)

**Fig. 4** Two sub-forms of parabolic Flying-V layout

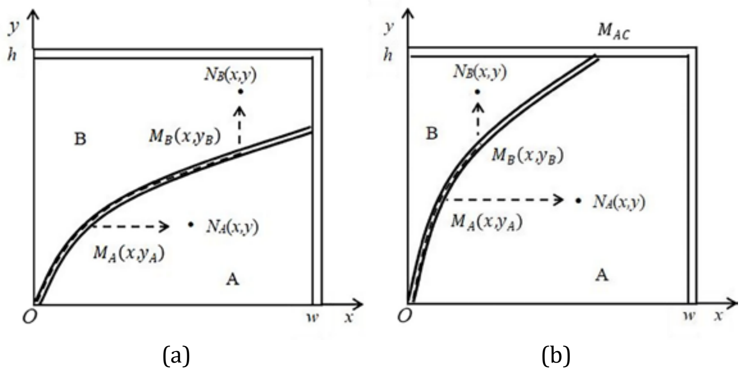



**Fig. 5** Two sub-forms of parabolic Fishbone layout

### 3.3. Efficiency model for the parabolic layout

The parabolic layout efficiency model consists of two parts. The first part is the total picking distance model. The second is the length formula of the cross-aisle. Picking distance is the core index reflecting the efficiency of picking operation. The cross-aisle length is the main factor affecting the utilization rate of warehouse area, which can be used as a supplementary index for layout efficiency evaluation. The parabolic type of the parabolic cross-aisle should be determined first, and the basic picking distance formula of each picking sub-region is derived. Then the average total picking distance model and the length formula of the cross-aisle are obtained by integration.

***Determination of the parabolic type***

Because our discussions are all taken in the right half of the warehouse, the parabolic cross-aisle must pass through the P&D point (the coordinate origin $O$) of the warehouse, so the selected parabola should pass through the first quadrant and the origin. There are two basic types of parabola passing through the origin, they take *x* axis and *y* axis as the axis of symmetry respectively. That are $x = ay^2 + by$ and $y = ax^2 + bx$. The opening direction of the parabola is determined by the positive and negative of parameter $a$. Obviously, the parabola corresponding to $a < 0$ does not meet this basic requirement and should be discarded. Therefore, the only optional parabola is: $x = ay^2 + by, a > 0$ *and* $y = ax^2 + bx, a > 0$. On this basis, combined with the characteristics of two kinds of parabolic layout (especially, the dual equivalence of thin-high warehouse and flat warehouse under Fishbone layout), through the trial calculation of typical thin-high, square and flat warehouse, the parabola of type $x = ay^2 + by, a > 0, b > 0$ is synthetically determined as the basis of modeling. In addition, because the study only involves the first quadrant, there are, $x > 0, y > 0$, see Eq. 1.

$$x(y) = ay^2 + by \ \ (a > 0, b > 0, x > 0, y > 0) \tag{1}$$

Based on the selected basic parabola type, according to the different integral variables, two expressions of the corresponding parabolic arc length (S) formula are given incidentally, as shown in Eq. 2, Eq. 3 and Eq. 4.

$$S = S(x) = S(y) \tag{2}$$

$$S(x) = \frac{1}{4a}\left[\sqrt{4ax + b^2 + 1} \times \sqrt{4ax + b^2} + \ln\left|\sqrt{4ax + b^2 + 1} + \sqrt{4ax + b^2}\right| - \sqrt{b^2 + 1} \times b - \ln\left|\sqrt{b^2 + 1} + b\right|\right] \tag{3}$$

$$S(y) = \frac{1}{4a}\left[\sqrt{(2ay + b)^2 + 1} \times (2ay + b) + \ln\left|\sqrt{(2ay + b)^2 + 1} + (2ay + b)\right| - \sqrt{b^2 + 1} \times b - \ln\left|\sqrt{b^2 + 1} + b\right|\right] \tag{4}$$

***Picking distance model of the parabolic layout***

For convenience of comparison, the optimization model of average total picking distances of the parabolic layouts under unit-load are constructed by using the idea of continuous space modelling [9].

1) Picking distance of parabolic Flying-V. Based on the results of morphological analysis in Fig. 4. and the selected parabolic equation type (1), we can obtain that:

$$y_{A1} = \frac{\sqrt{4ax+b^2}-b}{2a} \tag{5}$$

- Picking distance $D_{PFV}^{A^1}(x,y)$ in A region of sub-form 1.

$$D_{PFV}^{A^1}(x,y) = min\left\{D_{PFV}^{A_1^1}(x,y), D_{PFV}^{A_2^1}(x,y)\right\} \tag{6}$$

$$D_{PFV}^{A_1^1}(x,y) = d_{OM_{A1}} + d_{M_{A1}N_A} \tag{7}$$

$$d_{OM_{A1}} = S(x) \tag{8}$$

$$d_{M_{A1}N_A} = y_{A1} - y \tag{9}$$

$$D_{PFV}^{A_2^1}(x,y) = x + y \tag{10}$$

- The basic picking distance $D_{PFV}^{B^1}(x,y)$ in B region of sub-form 1.

$$D_{PFV}^{B^1}(x,y) = d_{OM_B} + d_{M_BN_B} \tag{11}$$

$$d_{OM_B} = S(x) \tag{12}$$

$$d_{M_BN_B} = y - y_{A1} \tag{13}$$

- Picking distance of region A in sub-form 2 $D_{PFV}^{A^2}(x,y)$.

$$D_{PFV}^{A^2}(x,y) = min\left\{D_{PFV}^{A_1^2}(x,y), D_{PFV}^{A_2^2}(x,y)\right\} \tag{14}$$

$$D_{PFV}^{A_1^2}(x,y) = D_{PFV}^{A_1^1}(x,y)(0 < x < ah^2 + bh) \tag{15}$$

$$D_{PFV}^{A_2^2}(x,y) = x + y \tag{16}$$

- Picking distance of region B in sub-form 2 $D_{PFV}^{B^2}(x,y)$.

$$D_{PFV}^{B^2}(x,y) = D_{PFV}^{B^1}(x,y)(0 < x < ah^2 + bh) \tag{17}$$

- Picking distance of area C in sub-form 2 $D_{PFV}^{C^2}(x,y)$.

$$D_{PFV}^{C^2}(x,y) = min\left\{D_{PFV}^{C_1^2}(x,y), D_{PFV}^{C_2^2}(x,y)\right\} \tag{18}$$

$$D_{PFV}^{C_1^2}(x,y) = d_{OM_{AC}} + d_{M_{AC}M_{C2}} + d_{M_{C2}N_C} \tag{19}$$

$$d_{OM_{AC}} = S(y)\ (y = h\ ) = \frac{1}{4a}\left[\left(\sqrt{(2ah+b)^2+1} \times (2ah+b) + \ln\left|\sqrt{(2ah+b)^2+1} + (2ah+b)\right|\right) - \sqrt{b^2+1} \times b - \ln\left|\sqrt{b^2+1} + b\right|\right] \tag{20}$$

$$d_{M_{AC}M_{C2}} = x - (ah^2 + bh) \tag{21}$$

$$d_{M_{C2}N_C} = h - y \tag{22}$$

$$D_{PFV}^{C_2^2}(x,y) = x + y \tag{23}$$

- Based on the above continuous space modelling assumption, the optimal average total picking distance of parabolic Flying-V is $E_{PFV}^*$ as shown in Eq. 24.

$$E_{PFV}^* = min\left\{min(E_{PFV}^1), min(E_{PFV}^2)\right\} \tag{24}$$

$$E_{PFV}^1 = \frac{1}{wh}\left(\int_0^w \int_0^{y_{A1}(x)} D_{PFV}^{A^1}(x,y)dydx + \int_0^w \int_{y_{A1}(x)}^h D_{PFV}^{B^1}(x,y)dydx\right) \tag{25}$$

$$E_{PFV}^2 = \frac{1}{wh}\left(\int_0^{ah^2+bh} \int_0^{y_{A1}(x)} D_{PFV}^{A^2}(x,y)dydx + \int_0^{ah^2+bh} \int_{y_{A1}(x)}^h D_{PFV}^{B^2}(x,y)dydx + \int_{ah^2+bh}^w \int_0^h D_{PFV}^{C^2}(x,y)dydx\right) \tag{26}$$

2) Picking distance of parabolic Fishbone. From the analysis of morphological features shown in Fig. 5, it is found that the basic picking distance formulas for the two sub-forms of parabolic Fishbone layout are the same. They are recorded as $D^{A}_{PFB}(x,y)$ and $D^{B}_{PFB}(x,y)$.

$$D^{A}_{PFB}(x,y) = d_{OM_A} + d_{M_A N_A} \tag{27}$$

$$d_{OM_A} = S(y) \tag{28}$$

$$d_{M_A N_A} = x - (ay^2 + by) \tag{29}$$

$$D^{B}_{PFB}(x,y) = D^{B1}_{PFV}(x,y) \tag{30}$$

Based on the assumption of continuous modelling, the optimal average total picking distance of parabolic Flying-V layout is $E^{*}_{PFB}$ see Eq. 31.

$$E^{*}_{PFB} = min\,\{min(E^{1}_{PFB})\,, min(E^{2}_{PFB})\} \tag{31}$$

$$E^{1}_{PFB} = \frac{1}{wh}\left(\int_0^w \int_0^{y_{A1}(x)} D^{A}_{PFB}(x,y)dydx + \int_0^w \int_{y_{A1}(x)}^{h} D^{B}_{PFB}(x,y)dydx\right) \tag{32}$$

$$E^{2}_{PFB} = \frac{1}{wh}\left(\int_0^h \int_{ay^2+by}^{w} D^{A}_{PFB}(x,y)dxdy + \int_0^h \int_0^{ay^2+by} D^{B}_{PFB}(x,y)dxdy\right) \tag{33}$$

***Main aisle length formula for parabolic layout***

From Figs. 4 and 5, the optimal parabolic cross-aisle lengths $T_{PFV}$ and $T_{PFB}$ of Flying-V and Fishbone are respectively:

$$T_{PFV} = \begin{cases} S^{*}(y)(y=h), & a^{*}h^2 + b^{*}h < w \\ S^{*}(x)(x=w), & a^{*}h^2 + b^{*}h \geq w \end{cases} \tag{34}$$

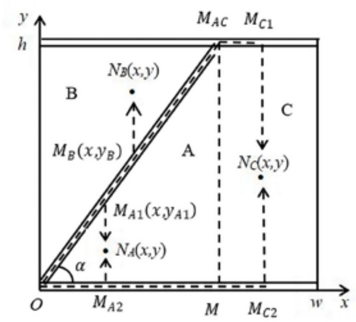
$$T_{PFB} = \begin{cases} S^{*}(y)(y=h), & a^{*}h^2 + b^{*}h < w \\ S^{*}(x)(x=w), & a^{*}h^2 + b^{*}h \geq w \end{cases} \tag{35}$$

## 4. Reference models

To analyse the efficiency of parabolic layout, the corresponding straight layout should be taken as a reference. However, in the historical literature, the layout efficiency analysis of straight Flying-V and straight Fishbone does not consider the cross-aisle length, and is discrete modelling. To unify the comparison benchmark, it is necessary to reconstruct the picking distance model of straight Flying-V and straight Fishbone and the length formula of their cross-aisle.

### 4.1 Picking distance model of straight layout

Similarly, according to the different intersection positions between the cross-aisle and the edge of the warehouse, the straight Flying-V layout also has two different sub-form, as shown in Fig. 6. In sub-form 1, the right half warehouse is divided into two regions, A and B, by the cross-aisle; in sub-form 2, the right warehouse is divided into three sub-regions A, B and C, by the cross-aisle and line segment $MM_{AC}$. Similarly, the straight Fishbone layout also has two different sub-forms, as shown in Fig. 7. The right half warehouse is divided into two picking sub-areas, A and B, by the cross-aisle.



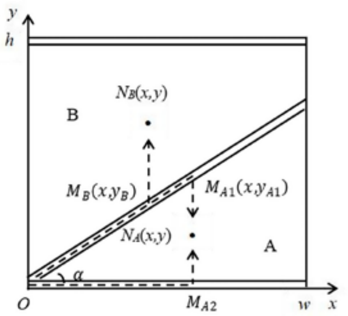
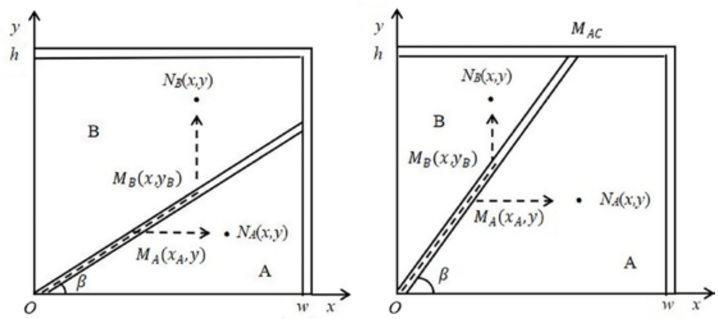




**Fig. 6** Two sub-forms of straight Flying-V layout




(a) (b)

**Fig. 7** Two sub-forms of straight Fishbone layout

***Picking distance for straight Flying-V***

In the following passage, $D_{SFV}^{A^1}(x,y)$ and $D_{SFV}^{B^1}(x,y)$ respectively represent the basic picking distances of A and B sub-regions in sub-form 1; $D_{SFV}^{A^2}(x,y)$, $D_{SFV}^{B^2}(x,y)$ and $D_{SFV}^{C^2}(x,y)$ represent the basic picking distances of A, B and C sub-regions in sub-form 2, respectively.

$$D_{SFV}^{A^1}(x,y) = min\left\{D_{SFV}^{A_1^1}(x,y), D_{SFV}^{A_2^1}(x,y)\right\} \tag{36}$$

$$D_{SFV}^{A_1^1}(x,y) = \frac{x}{\cos\alpha} + x\tan\alpha - y \tag{37}$$

$$D_{SFV}^{A_2^1}(x,y) = x + y \tag{38}$$

$$D_{SFV}^{B^1}(x,y) = \frac{x}{\cos\alpha} + y - x\tan\alpha \tag{39}$$

$$D_{SFV}^{A^2}(x,y) = min\left\{D_{SFV}^{A_1^2}(x,y), D_{SFV}^{A_2^2}(x,y)\right\} \tag{40}$$

$$D_{SFV}^{A_1^2}(x,y) = \frac{x}{\cos\alpha} + x\tan\alpha - y \tag{41}$$

$$D_{SFV}^{A_2^2}(x,y) = x + y \tag{42}$$

$$D_{SFV}^{B^2}(x,y) = \frac{x}{\cos\alpha} + y - x\tan\alpha \tag{43}$$

$$D_{SFV}^{C^2}(x,y) = min\left\{D_{SFV}^{C_1^2}(x,y), D_{SFV}^{C_2^2}(x,y)\right\} \tag{44}$$

$$D_{SFV}^{C_1^2}(x,y) = \sqrt{\left(\frac{h}{\tan\alpha}\right)^2 + h^2} + \left(x - \frac{h}{\tan\alpha}\right) + (h - y) \tag{45}$$

$$D_{SFV}^{C_2^2}(x,y) = x + y \tag{46}$$

Based on the assumption of continuous modelling, the optimal average total picking distance of straight Flying-V is $E_{SFV}^*$ shown in Eq. 47.

$$E_{SFV}^* = min\,\{min(E_{SFV}^1)\,, min(E_{SFV}^2)\} \tag{47}$$

$$E_{SFV}^1 = \frac{1}{wh}\left(\int_0^w \int_0^{x\tan\alpha} D_{SFV}^{A^1}(x,y)dydx + \int_0^w \int_{x\tan\alpha}^h D_{SFV}^{B^1}(x,y)dydx\right) \tag{48}$$

$$E_{SFV}^2 = \frac{1}{wh}\left(\int_0^{\frac{h}{\tan\alpha}} \int_0^{x\tan\alpha} D_{SFV}^{A^2}(x,y)dydx + \int_0^{\frac{h}{\tan\alpha}} \int_{x\tan\alpha}^h D_{SFV}^{B^2}(x,y)dydx + \int_{\frac{h}{\tan\alpha}}^w \int_0^h D_{SFV}^{C^2}(x,y)dydx\right) \tag{49}$$

***Picking distance for straight fishbone***

From Figure 7, we know that the basic picking distance formula of A and B sub-regions in the two sub-forms of straight-line Fishbone layout are the same respectively. So, we can use $D_{SFB}^{A}(x,y)$ and $D_{SFB}^{B}(x,y)$ to express the basic picking distance formulas of A and B picking area respectively.

$$D_{SFB}^{A}(x,y) = \frac{y}{\sin\beta} + x - y\cot\beta \tag{50}$$

$$D_{SFB}^{B}(x,y)=\frac{x}{\cos\beta}+y-x\tan\beta \tag{51}$$

Based on the continuous modelling assumption mentioned above, the optimal average total picking distance of straight-line Fishbone layout $E_{SFB}^{*}$ can be obtained by the double integral of the above basic picking distance formulas.

$$E_{SFB}^{*}=min\left\{min(E_{SFB}^{1}),min(E_{SFB}^{2})\right\} \tag{52}$$

$$E_{SFB}^{1}=\frac{1}{wh}\left(\int_{0}^{w}\int_{0}^{x\tan\beta}D_{SFV}^{A}(x,y)dydx+\int_{0}^{w}\int_{x\tan\beta}^{h}D_{SFV}^{B}(x,y)dydx\right) \tag{53}$$

$$E_{SFB}^{2}=\frac{1}{wh}\left(\int_{0}^{h}\int_{\frac{y}{\tan\beta}}^{w}D_{SFV}^{A}(x,y)dydx+\int_{0}^{h}\int_{0}^{\frac{y}{\tan\beta}}D_{SFV}^{B}(x,y)dydx\right) \tag{54}$$

### 4.2 Main aisle length for straight layout

From Fig. 6 and Fig. 7, the optimal inclined cross-aisle lengths $T_{SFV}$ and $T_{SFB}$ of straight Flying-V and Fishbone are Eq. 55 and Eq. 56 respectively.

$$T_{SFV}=\begin{cases}\frac{h}{\sin\alpha^{*}}, & \alpha^{*}>\arctan(h/w)\\ \frac{w}{\mathrm{con}\,\alpha^{*}}, & \alpha^{*}\leq\arctan(h/w)\end{cases} \tag{55}$$

$$T_{SFB}=\begin{cases}\frac{h}{\sin\beta^{*}}, & \beta^{*}>\arctan(h/w)\\ \frac{w}{\mathrm{con}\,\beta^{*}}, & \beta^{*}\leq\arctan(h/w)\end{cases} \tag{56}$$

## 5. Model solving

Since the double integral of the average total picking distance models established in this paper are all extremely difficult to obtain the analytical expressions, the analytical optimization analysis process which has been successfully used in aisle angle optimization by Öztürkoğlu *et al.* couldn't be carried out [9]. In order to solve this problem and to ensure the reliability of the results, we decided to adopt the Interval Numerical Simulation Method (INSM) and the Genetic Algorithms (GA) respectively to solve this model. INSM which has been successfully applied by Zhang Zhiyong *et al.* [19]. GA is widely used to solve this type of model. Both methods require differential discretization of the model first.

### 5.1 Differential discrete processing

According to the general principle of differential discretization, $w$ and $h$ of the right half warehouse are discretized by $m$ and $n$ meshes respectively. （$x_i, y_j$）is used to represent the coordinates of the central points of each grid block.

$$\left.\begin{aligned}x_i&=\frac{w}{m}\times(i-0.5) \quad i=1,2,\dots,m\\ y_j&=\frac{h}{n}\times(j-0.5) \quad j=1,2,\dots,n\end{aligned}\right\} \tag{57}$$

In order to calculate and interpret the results directly and conveniently, the pallet can be understood as a square with a side length of 1, and the length and width of the warehouse can be set as an integral multiple of the side length of the square pallet. See Eq. 58. It is worth pointing out that this setting will not affect the conclusions of relevant analysis.

$$\frac{w}{m}=\frac{h}{n}=1 \tag{58}$$

### 5.2 Interval numerical simulation method

Based on the above discretization results, INSM could be carried out. It should be noted that the parameters a and b of parabolic trajectory equation have a non-closed theoretical range, and they also have a certain compensation effect on the layout efficiency of warehouse aisles with each other. The so-called mutual compensation effect of parabolic aisle trajectory parameters means that when one parameter value is fixed and unchanged, the layout efficiency will change regularly with the change of another parameter value. Therefore, reasonable determination of bounded numerical simulation interval fixing method for a and b needs to be specially dealt with in conjunction with specific problems. After a lot of trial calculations and analysis, the simulation-based optimization intervals selected for the 100 warehouses in this paper are: $a \in [0,\ 2], b \in [0.1,\ 100]$.

### 5.3 Genetic algorithms

The specific parameters of applying Genetic Algorithms (GA) to solve the model in this paper are as follows:

The individual coded with natural numbers. The individual consisted of $a$ and $b$. The value ranges are： $a \in [0,\ 2], b \in [0.1,\ 100]$, same as the optimization interval selected in INSM above. The size of the population is 200. The fitness function by taking the inverse of the objective function (Eq. 59).

$$F1 = \frac{1}{E^*_{PFV}};\quad F2 = \frac{1}{E^*_{SFV}};\quad F3 = \frac{1}{E^*_{PFB}};\quad F4 = \frac{1}{E^*_{SFB}} \tag{59}$$

According to the adaptive priority determined by the weighted roulette method, individuals with higher fitness values have a higher probability of contributing to the next generation for one or more offspring [26]. Once individuals were selected, they were subjected to a two-point crossover operation. After a lot of trial calculations and analysis, the crossover probability is 0.8 in this paper. The single point mutation method has been adopted. After a lot of trial calculations and analysis, the mutation probability is 0.1. The maximum number of generations is 200 in this paper.

## 6. Results and analysis

### 6.1 Efficiency analysis indicators

As a reference, we also calculated the minimum total picking distance of the 100 sample warehouses using the traditional orthogonal straight aisle layout and the ideal direct flight operation: $EC^*$ and $EZ^*$. They are regarded as the upper and lower bounds of the total picking distance of various non-traditional layout, respectively, as shown in Eq. 60 and Eq. 61.

$$EC^* = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(x_i + y_j\right) \tag{60}$$

$$EZ^* = \sum_{i=1}^{m} \sum_{j=1}^{n} \sqrt{\left({x_i}^2 + {y_j}^2\right)} \tag{61}$$

Other efficiency analysis indicators are:

$$\left.\begin{aligned} EB_{PFV} &= \frac{E^*_{PFV}}{EC^*} \times 100 \\ EB_{SFV} &= \frac{E^*_{SFV}}{EC^*} \times 100 \\ EZB &= \frac{EZ^*}{EC^*} \times 100 \end{aligned}\right\} \tag{62}$$

$$\left.\begin{aligned} TS_{PFV} &= EB_{PFV} - EZB \\ TS_{SFV} &= EB_{SFV} - EZB \end{aligned}\right\} \quad \text{(The theoretical possible improvement space of parabolic Flying-V and straight Flying-V)} \tag{63}$$

$$TGB_{FV} = \frac{(T_{SFV} - T_{PFV})}{T_{SFV}} \times 100 \tag{64}$$

$$\left.\begin{aligned} EB_{PFB} &= \frac{E^*_{PFB}}{EC^*} \times 100 \\ EB_{SFB} &= \frac{E^*_{SFB}}{EC^*} \times 100 \end{aligned}\right\} \tag{65}$$

$$\left.\begin{aligned} TS_{PFB} &= EB_{PFB} - EZB \\ TS_{SFB} &= EB_{SFB} - EZB \end{aligned}\right\} \quad \text{(The theoretical possible improvement space of parabolic Fishbone and straight Fishbone)} \tag{66}$$

$$TGB_{FB} = \frac{(T_{SFB} - T_{PFB})}{T_{SFB}} \times 100 \tag{67}$$

### 6.2 Numerical results

With the help of MATLAB tools, the minimum total picking distance and the length of cross-aisle of 100 warehouses with lengths and widths ranging from 10 to 100 (interval 10) were analysed and solved. The data range covers the size of conventional warehouses, and the results are representative. If we ignore the calculated error, the results of INSM and GA are basically consistent with each other. Among them, the percentage values of picking distance of two kinds of straight-line layout and direct-flying operation of square warehouse are about 85.29, 80.47 and 76.52 respectively, which are basically consistent with the results of the relevant references [4, 7]. This shows that the accuracy of the calculation results in this paper is satisfactory. We draw Figs. 8 to 13 with the data.

### 6.3 Characteristics of parabolic Flying-V

Parabolic Flying-V layout has three characteristics compared with straight Flying-V layout in rectangular warehouses with unit-load.

- Semi-square warehouses ($w/h = 1$). The total picking distance can be reduced by about 0.35-0.37 %, and the theoretical possible improvement space has been compressed by about 3.38-3.57 %. The picking efficiency is improved slightly. The length of the cross-aisle could be shortened by 2.01-2.62 %. The area utilization rate is slightly improved.
- Semi-flat warehouses ($w/h > 1$). The total picking distance can be reduced by about 0.38-0.62 %, the theoretical possible improvement space has been compressed by 3.66-12.26 %. The picking efficiency is improved obviously, and the change rule is roughly gradually greater, the flatter the warehouse, the bigger the value. The length of the cross-aisle could be shortened by 1.92-0.03 %, and the change rule is roughly gradually smaller, the flatter the warehouse, the smaller the value. That is to say, the more significant the picking efficiency improved, the less the warehouse area utilization rate increased.
- Semi-thin and tall warehouses ($w/h < 1$). Only when ($2/3 \leq w/h < 1$), the total picking distance could be reduced by about 0.22-0.31 %, the theoretical possible improvement space has been compressed by 2.42-3.12 %, and the picking efficiency is improved slightly. The length of cross-aisle could be shortened by 2.38-3.10 %, and the area utilization rate is also improved slightly. For other warehouses with $w/h < 2/3$ (ignoring $7/10$ outliers), straight Flying-V layouts are better.

### 6.4 Characteristics of parabolic fishbone

Parabolic Fishbone layout also has three characteristics compared with straight-line Fishbone layout in rectangular warehouses with unit-load.

- Semi-square warehouses ($w/h = 1$). The optimal parabola Fishbone degenerates to the optimal straight Fishbone. That is to say, the straight Fishbone layout is better. It is not difficult to understand that this feature is determined by the symmetrical distribution of the aisles in Fishbone layout.
- Semi-flat warehouses ($w/h > 1$). Only when $w/h \geq 3$(ignoring calculation error), the total picking distance could be reduced by about 0.02-0.04 %, the theoretical possible improvement space has been compressed by about 1.27-1.83 %, and the picking efficiency is improved slightly. However, its length of the cross-aisle will increase by 4.63-19.50 %, which means the reduction of the utilization rate of warehouse area. For other warehouses with $1 < w/h < 3$ (ignoring calculation error), straight Fishbone layouts are better.

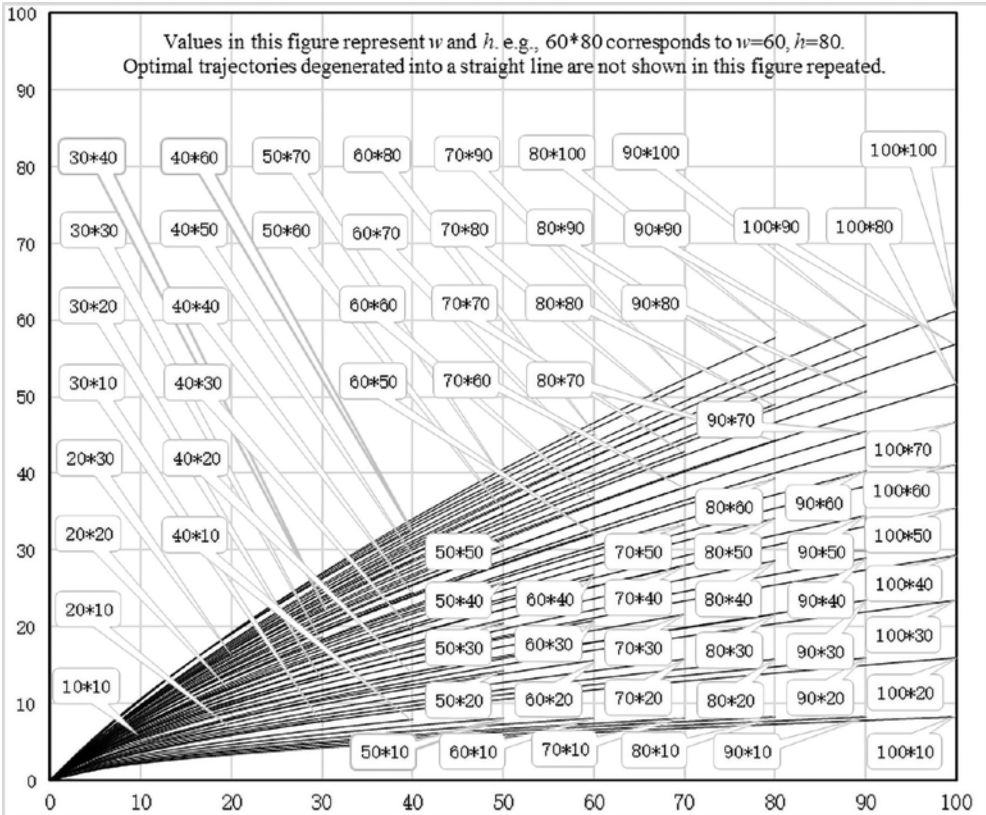



**Fig. 8** Optimal main aisle trajectory map of parabolic Flying-V of 100 warehouses

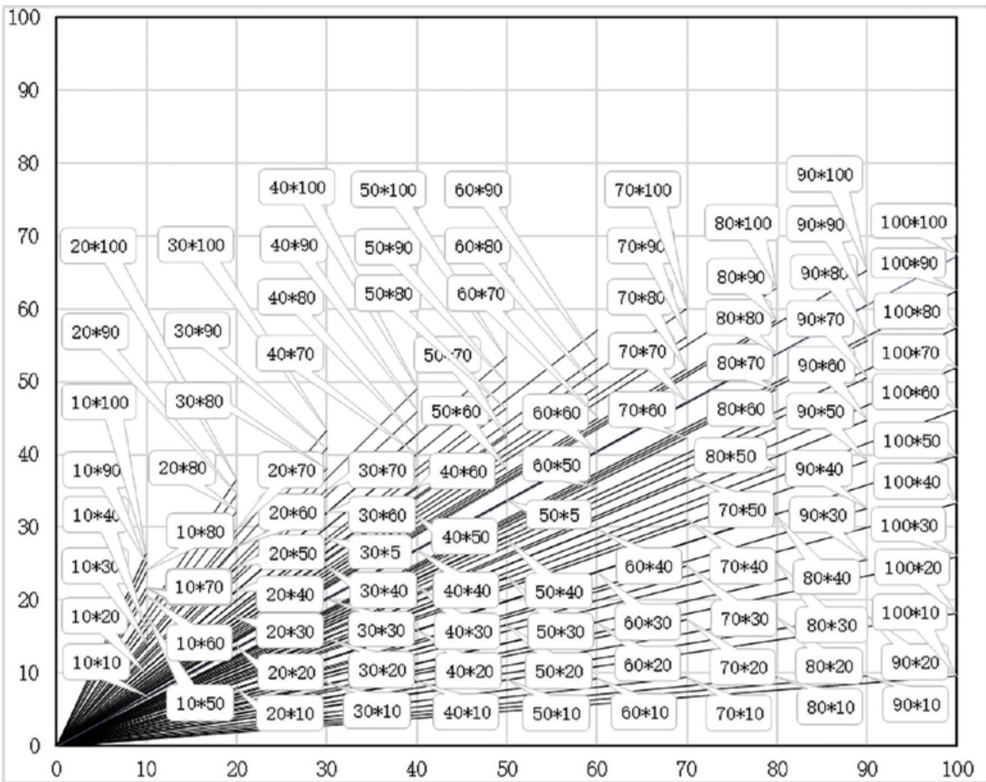



**Fig. 9** Optimal main aisle trajectory map of straight Flying-V of 100 warehouses

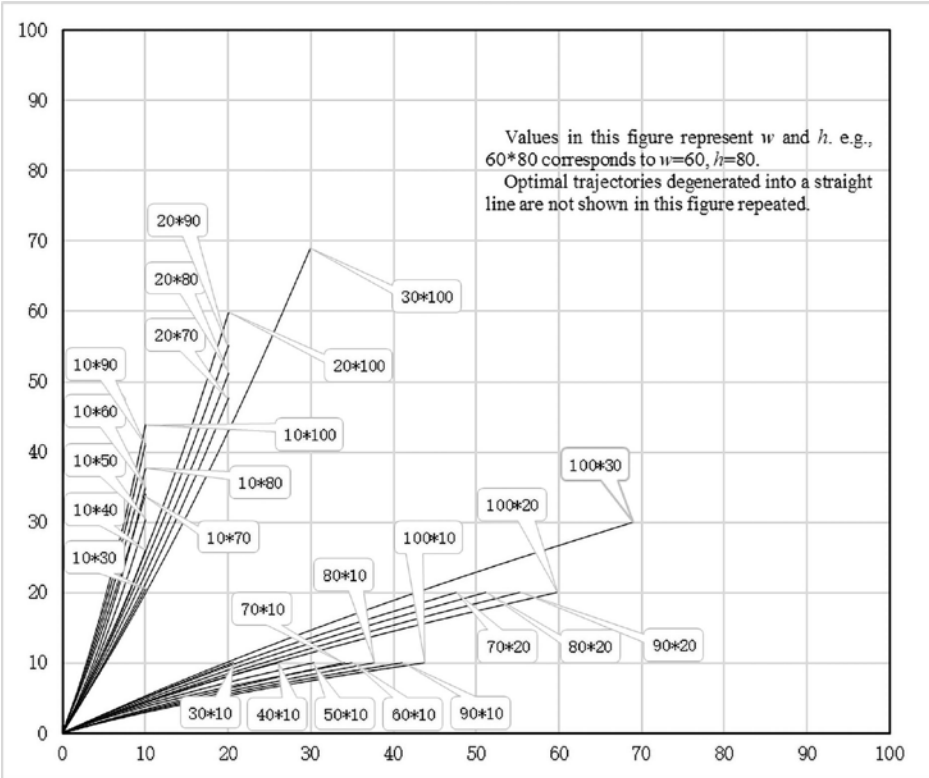



**Fig. 10** Optimal main aisle trajectory map of parabolic Fishbone of 100 warehouses

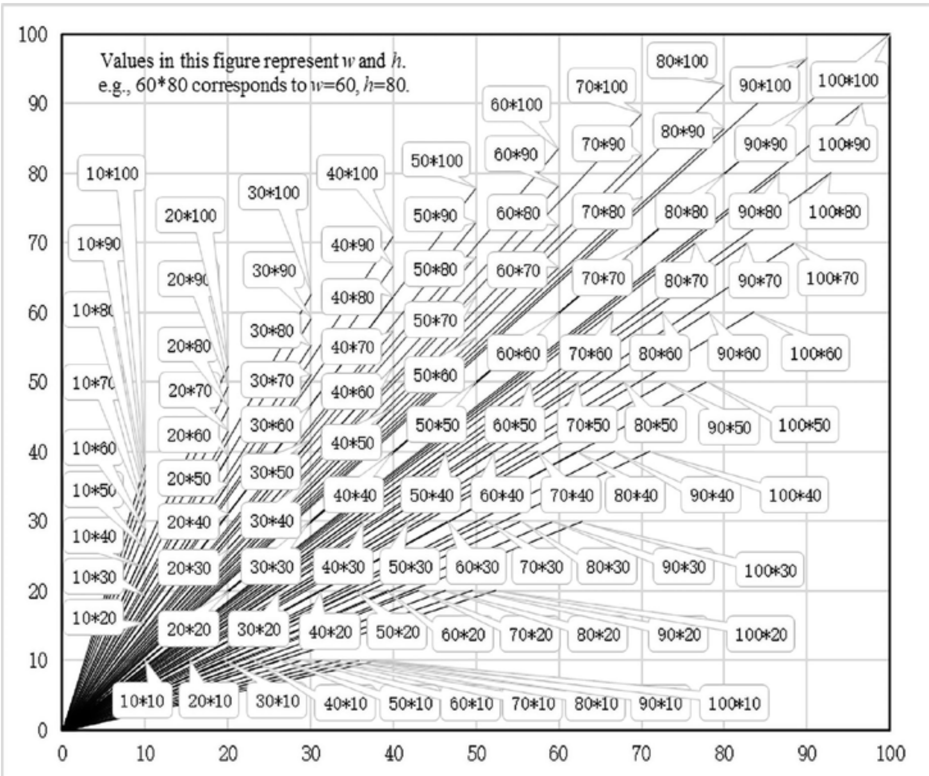
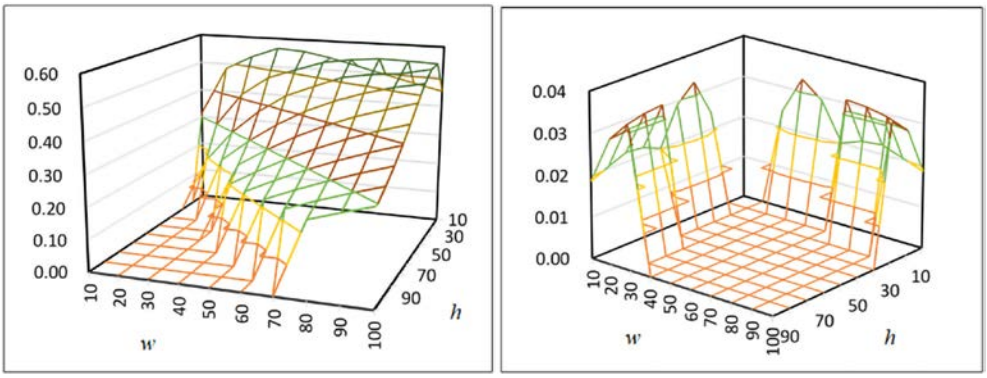
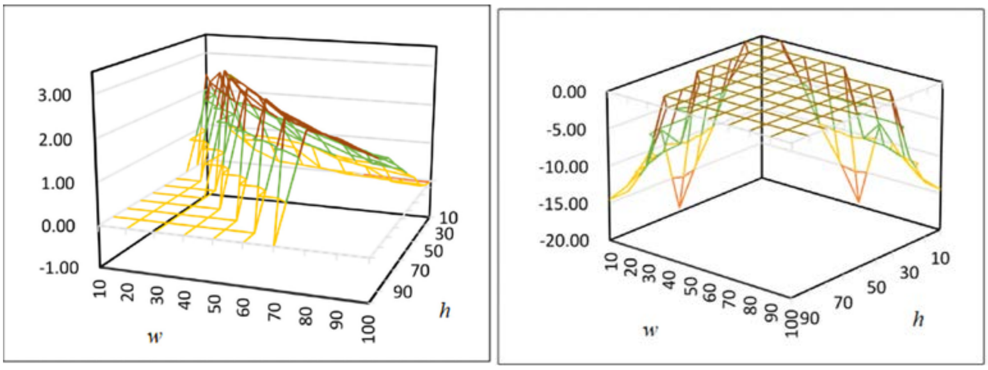



**Fig. 11** Optimal main aisle trajectory map of straight Fishbone of 100 warehouses




(a) $TS_{SFV}$-$TS_{PFV}$ (b) $TS_{SFB}$-$TS_{PFB}$

**Fig. 12** Distribution diagrams of $TS_{SFV}$-$TS_{PFV}$ and $TS_{SFB}$-$TS_{PFB}$ and TSSFB-TSPFB of 100 warehouses




(a) $TGB_{FV}$ (b) $TGB_{FB}$

**Fig. 13** Distribution diagrams of $TGB_{FV}$ and $TGB_{FB}$ of 100 warehouses

- Semi-thin and tall warehouses ($w/h < 1$). They have the same characteristics as the semi-flat warehouse. For Fishbone layout, thin and tall warehouses and flat warehouses with inverted aspect ratio are dual layouts. Let the optimal layout of the flat warehouse rotated 90 degrees counter clockwise with the P&D point as the centre first, and then turn 180 degrees around the longitudinal axis, the optimal layout of thin and tall warehouse with inverted width-height ratio can be obtained. The optimal parabolic trajectory equation of another warehouse can be obtained by interchanging x and y in the optimal parabolic trajectory equation of one warehouse.

## 7. Conclusion

Parabolic layout has its own characteristics and advantages. Although the non-closed theoretical range of parabolic trajectory equation parameters $a$ and $b$ and the mutual compensation effect in layout efficiency bring some technical difficulties to the related analysis, the parabolic transformation effect of two typical straight non-traditional layouts preliminarily verifies the scientificity and feasibility of the idea. This research enriches the relevant theory of warehouse aisle layout, and provides certain theoretical and technical basis for further tapping the efficiency potential of warehouse layout, breaking through the realistic situation of straight aisle leading warehouse layout.

Through the analysis of layout characteristics, the selection of parabolic type and the introduction of cross-aisle length index, the efficiency optimization models of two parabolic layouts under unit-load are constructed. The calculation results of 100 warehouses with different sizes show that the picking distance of parabolic Flying-V could be reduced by 0.22%-0.62% compared with the straight layout, and the theoretical possible improvement space has been compressed by 2.42-12.26 %. Its length of cross-aisle is shortened by $-0.03$-3.10 %. The picking distance of parabolic Fishbone could be only reduced by 0.02-0.04 %. The theoretical possible improvement space has been compressed by 1.27-1.83 %. But its length of cross-aisle will increase by 4.63-19.50 % significantly.

The results contribute to the solution of manufacturing logistics problems, and the formation of green production mode. The objective of logistics solutions is to influence and manage material flows in the right way which is very important for manufacturing systems. The solution of manufacturing logistics problems in specific market area requires the utilization of the means of algorithmization, heuristics, mathematical statistics, modelling and computer simulation [27]. Green production mode is an advanced manufacturing mode. From the perspective of enterprises, reducing the operating cost of green manufacturing mode through scientific and technological innovation is a very good decision-making scheme for enterprises [28].

Future research directions include: based on the parabolic layout, combined with the actual needs of warehousing operation and management, putting forward more scientific and reasonable target of multi-attribute comprehensive measurement of efficiency; investigating the effects of task interleaving (dual-command), multiple P&D points, and different warehousing strategies; and further studying the warehouse design guidelines to enrich and perfect the layout principle of curve trajectory aisle [29]. Based on the analysis of efficiency mechanism, the practical design rules of the corresponding layout will be studied, which can provide reference for the formulation of non-linear warehouse layout design specifications. Also, many practical warehousing problems, such as the order assignment, the order batching and the picker routing of large wave picking warehouses which have been modelled by Ardjmand *et al*. [30], can be integrated with the non-linear warehouse layout. With the continuous development of social economic technology, intelligent control units have become the trend of manufacturing enterprises [31], we believe that the layout of non-rectangular complex special-shaped warehouses based on curve trajectory aisle could become an important research topic.

In addition, the proposed method in this paper may also be used in principle for the arrangement of machines and devices in a workshop. However, compared with the internal layout of a warehouse, the layout in a workshop is much more complex. For example, there are many big differences in sizes, shapes, site occupations, handling tools and process relations of different machines and devices. The specific processing technology relationship between machines and devices and materials and personnel must also be considered. Therefore, although the optimization methods can be used to finely study the workshop layout problem, the objectives and constraints of the corresponding mathematical model will be much more complex. However, we do believe that with the popularization of modern large-scale intelligent factories, the fine workshop layout using mathematical models and simulation optimization technology will become an important research direction.

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