

A two-stage construction heuristic approach for vehicle routing problem with split deliveries and pickups: Case studies and performance comparison

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ABSTRACT

The vehicle routing problem with split deliveries and pickups is a hot research topic in recent years, where a customer can be served multiple times with split deliveries and pickups. The objective is to minimize the travel distance, use the fewest number of vehicles and increase the load rate, which will further reduce the carbon emissions that damage the environment. In this paper, we use a two-stage construction heuristic approach to solve this problem. First, partitioning algorithms based on the multi-restart-iterative sweep algorithm are adopted to partition the customer domain into sub-domains according to the vehicle capacity, and to determine the split points and the corresponding values. Second, a modified Clarke-Wright savings algorithm is used to check the possibility of each point in each route based on the load of each point and the vehicle load limitation. The three case studies with 12 instances per each from the reconstructed Solomon benchmark datasets were conducted to evaluate the effectiveness and feasibility of the proposed approaches-Unsplit, Both-Split and Enhanced-Both-Split. The comparison among these approaches reveals that the splits reduce the total travel cost and vehicles used, and increase the average loading rate considerably, especially when customers have larger demand values. Our computation results prove that the vehicle routing problem with split deliveries and pickups is highly beneficial for transportation and logistics enterprises.

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1. Introduction

1.1 General

The vehicle routing problem (VRP) plays an important role in modern logistics. The vehicle route optimization has a considerable impact on reducing transport distance, the number of transport vehicles and carbon emissions that damage the environment [1-3]. The VRP with simultaneous delivery and pickup (VRPSDP) [4] can reduce the energy consumption that results from the empty return trips of vehicles under the single vehicle delivery or pickup system. It can reduce not only the transport costs that benefit enterprises, but also increase the protection of environments. Therefore, the VRPSDP and its variants have become a popular research topic in logistics. With the relaxation of the constraint imposed on the classic VRPSDP in which each customer can be visited once and only once [5], the delivery and pickup demands of customers can be split (i.e., each customer is allowed to be visited multiple times), and greater vehicle reduction and path

savings can be achieved [6, 7]. Therefore, the VRPs concerning split demands have received increasing attention from researchers. The corresponding problem with only the split delivery is called the split delivery VRP (SDVRP); the corresponding problem splitting both delivery and pickup demands is called the split VRP with deliveries and pickups (SVRPDP). The SVRPDP has three models based on the sequence of deliveries and pickups [8]. In the first model, all pickups are done on the backhaul way after all deliveries, which is similar to the VRP with backhauling. In the second model, deliveries and pickups can occur in any sequence on a vehicle route, which is similar to the VRP with mixed deliveries and pickups. In the third model called the VRP with split deliveries and pickups (VRPSPDP), both the delivery and the pickup occur simultaneously (similar to the VRPSPDP). This study focuses on this third model and its solution, also on the performance comparisons between the VRPSPDP and the VRPSPDP in terms of transportation distances, vehicles used and the load rates which are the important factors affecting transportation costs and environmental costs [9, 10].

In the SDVRP, which was introduced by Dror and Trudeau [5] in 1989, a customer point can be visited multi-times by vehicles via the split deliveries. Therefore, numerous researchers have focused on solutions to the SDVRP problem because this model reduces the distance travelled and the number of vehicles used by splitting customer demands [11-13]. A lengthy article would be needed to present the survey of all researchers in the area; nonetheless, typical theoretical analysis and experimental results can be glimpsed from the previous articles [14-23].

In the VRPSPDP, which was introduced by Mitra [24] in 2005, a vehicle carrying goods within its load-carrying capacity, departs from the depot, delivers the loads to customers en route, and picks up their returnable items; finally, the vehicle returns to the depot with these items within its load-carrying capacity. The VRPSPDP allows an unlimited number of deliveries and pickings up, which means that each customer can be serviced multiple times by the same vehicle or by different vehicles. The goal of [24] is to determine a group of vehicle paths that minimizes the total travel distance under the premise that the load of each path does not exceed the vehicle's load-carrying capacity. To address this problem, a mixed-integer linear programming formulation was proposed [24], and a route construction heuristic was developed based on the cheapest insertion criterion with the fewest vehicles. Subsequently, better solutions have been proposed [25] that employ a parallel clustering technique and a new route construction heuristic. Based on [24, 25], Wang [26, 27] designed two heuristics (viz., the farthest node split load algorithm and competitive decision algorithm) for the VRPSPDP model without the vehicle number limitation and two heuristics (viz., the farthest node full split algorithm and nearest node full split algorithm) for the VRPSPDP model with the vehicle number limitation. Yin *et al.* [28] proposed a VRPSPDP model with two special preconditions: a maximum travel distance constraint and a restriction that each customer's demand should be split only once. Wang *et al.* [29] developed a two-stage heuristic method that integrated the initial heuristic algorithm and a hybrid heuristic algorithm to solve the VRPSPDP. Polat [30] proposed a parallel approach based on the variable neighbourhood search to solve the VRP with divisible deliveries and pickups. Qiu *et al.* [31] developed a tabu search algorithm based on specially designed batch combinations and item creations for the VRPSPDP.

So far, the focus has been mainly on SDVRP solutions, and there have been only few articles on VRPSPDP. Consequently, there is considerable room for improvements in terms of the comprehensive optimization effect, methods and the time spent in solving the VRPSPDP. With these in mind, we propose a two-stage approach based on the principle "clustering first, routing later." The proposed approach will solve the VRPSPDP in as little as possible computation time to reduce the travel distance, the number of vehicles used and increase the loading rate. In the first stage, partitioning algorithms based on the multi-restart-iterative sweep algorithms (MRISA) are adopted to cluster customers into sub-domains and to determine the split points and the corresponding values. In the second stage, a modified Clarke-Wright (C-W) savings algorithm is applied to optimize the travel distance of each route in each cluster. The three case studies with 12 instances per each from the reconstructed Solomon benchmark datasets were adopted to evaluate the effectiveness and feasibility of the proposed approaches, which are used for un-splitting customer demands, splitting both customer deliveries and pickups and fine-tuning splitting both customer deliveries and pickups. The computational results present that the VRPSPDP model is more effective in the

transportation and logistics enterprises than the VRPSDP because of the reduced the total travel distances, the reduced number of vehicles used, and the increased the loading rate.

2. Problem statement

The VRPSDP mentioned in this paper refers to n customers and m vehicles with the same model in the problem domain. Each vehicle leaves the depot $i = 0$ carrying only the delivery goods that are within its load-carrying capacity $\sum_{i=1}^n d_i \leq Q$. Each vehicle delivers d_i and picks up (e.g., recycles) p_i simultaneously along the route for the customers $i \mid i = (1, 2, \dots, n)$. Finally, the vehicle returns to the depot with only the picked-up goods that are within its load-carrying limit $\sum_{i=1}^n p_i \leq Q$. At each customer location, there may be a mixture of goods (deliveries and pickups), and the load on the vehicle may increase or decrease because of simultaneous loading and unloading. Therefore, the load feasibility needs to be checked for each point along a route. The distance between the customers i and j is c_{ij} ; there is no $c_{ii} = 0$ loop, and the path is undirected $c_{ij} = c_{ji}$. The quantity of deliveries between the customers i and j is $0 \leq d_{ij} \leq Q$, and the quantity of pickups between the customers i and j is $0 \leq p_{ij} \leq Q$. Each customer may have both delivery and pickup demands, either of which may exceed the vehicle's load-carrying capacity. Each customer's deliveries and pickups can be split; that is, each customer can be visited by multiple vehicles or by the same vehicle multiple times. If the vehicle k travels from the customer i to j , then x_{ijk} is equal to 1; otherwise $x_{ijk} = 0$. If the customer i is served by the vehicle k , then y_{ik} is equal to 1; otherwise, $y_{ik} = 0$. The minimum number of vehicles used is $\lceil \max(\sum_{i=1}^n d_i, \sum_{i=1}^n p_i) / Q \rceil$, where x denotes the smallest integer that is equal to or greater than x [24, 25]. For the sake of simplicity, we assume that there is no time window limit and no limit on the maximum driving time and distance. The goal is to minimize the total driving distance using few vehicles and keeping a high load rate to meet the demands of all the customers.

The formulation [32] is given as:

$$\min \sum_{i=0}^n \sum_{j=0}^n \sum_{k=1}^m c_{ij} x_{ijk} \tag{1}$$

s. t.

$$\sum_{k=1}^m d_{0j} y_{jk} = d_j, j = 1, 2, \dots, n; \tag{2}$$

$$\sum_{k=1}^m p_{j0} y_{jk} = p_j, j = 1, 2, \dots, n; \tag{3}$$

$$\sum_{i=1}^n d_{0i} y_{ik} \leq Q, k = 1, 2, \dots, m; \tag{4}$$

$$\sum_{i=1}^n p_{i0} y_{ik} \leq Q, k = 1, 2, \dots, m; \tag{5}$$

$$\sum_{i=0}^{\theta} p_i y_{ik} + \sum_{i=\theta+1}^n d_i y_{ik} \leq Q, i = 0, 1, \dots, \theta, \theta + 1, \dots, n; k = 1, 2, \dots, m; \tag{6}$$

$$\sum_{i=0}^n d_{i0} = 0; \tag{7}$$

$$\sum_{i=0}^n p_{0i} = 0; \tag{8}$$

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^n (x_{ijk} - x_{j(j+1)k}) = 0, k = 1, 2, \dots, m; \quad (9)$$

$$\sum_{j=0}^n x_{0jk} = 1, \sum_{j=0}^n x_{j0k} = 1, k = 1, 2, \dots, m; \quad (10)$$

Eq. 1 is the objective function to minimize the total travel distance. Eq. 2 and 3 are the customer demand constraints to ensure that the delivery/pickup demands of the customer j are satisfied by multiple visits. Eq. 4 and 5 are the vehicle-loading constraints to ensure that the delivery/pickup quantities of a vehicle in one tour do not exceed the vehicle's load-carrying capacity. Eq. 6 is the real-time vehicle-loading constraint [32], which is used to check the fluctuating load on every point of each route; this constraint ensures that the gross load of both the delivery and pickup at any node will not exceed the vehicle capacity in one tour. The sum of the picked-up quantity at the customer node θ (including node θ) and the delivery quantity after the customer node θ (starting from node $\theta + 1$) along the route of the vehicle k cannot exceed the vehicle capacity. Eq. 7 and 8 are the depot load-type constraints to ensure that no delivery loads are directed to the depot and no pickup loads come from the depot. Eq. 9 is the vehicle access conservation constraint to ensure that a vehicle that arrives at the customer location j also leaves this location. Eq. 10 states the depot access restriction that each vehicle enters/exits the depot only once per tour.

3. A two-stage approach

A two-stage approach based on "clustering first and routing later" is proposed to solve the VRP-SPDP. In the first stage, partitioning algorithms based on the MRISA are employed to partition customers into some clusters and to ascertain the split points and the corresponding values. In the second stage, a modified C-W savings algorithm that satisfies the customer particular double requirements is adopted to optimize the travel distance.

3.1 Initialization of the coordinate system

A sweep algorithm proposed by Gillett and Miller in 1974 [33] groups the nearest points into a cluster under certain preconditions. It is performed in a polar coordinate system, so that the rectangular coordinate system needs to be transformed into a polar coordinate system, all points have to be sorted in ascending or/and descending order [34], and a set of variables *optimal* should be defined to store the best distance and the corresponding customer points (i.e., the points and their deliveries and pickups) and the split points (i.e., the split points and the corresponding values) in each cluster.

3.2 Sweep algorithm for the VRPSDP

The sweep algorithm for the VRPSDP (SA-VRPSDP), which satisfies the delivery/pickup double demands, is the modification of the basic sweep algorithm. In the VRPSDP, a vehicle visits a point only once; therefore, the customer demands cannot be split. Thus, SA-VRPSDP is also called SA-Unsplit. In addition, before the clustering, the delivery or pickup demand of each point needs to be checked to ensure that this demand is less than the vehicle's load-carrying capacity Q . If $d_i, p_i \geq Q$, the quantity Q would be individually delivered or picked-up. The remaining part would participate in SA-VRPSDP. The SA-Unsplit procedure is described as follows.

Step 1: Specify one point as the first starting point.

Step 2: SA-Unsplit (Partitioning of the customer area).

a) Sweep points gradually into an initial cluster in one direction (clockwise or anticlockwise). Three possible cases arise at point i .

Case 1: When $((MCV_d^i < Q)$ and $(MCV_p^i < Q))$, continue to sweep points gradually into this cluster.

Case 2: When $((MCV_d^i = Q)$ or $(MCV_p^i = Q))$, the point i becomes the last point lp of this cluster, and record lp and this cluster. There are following three situations:

- If $((MCV_d^i = Q)$ and $(MCV_p^i = Q))$, then denote as $((MCV_d^{lp} = Q)$ and $(MCV_p^{lp} = MCV_p^i))$.
- If $((MCV_d^i = Q)$ and $(MCV_p^i < Q))$, then denote as $((MCV_d^{lp} = Q)$ and $(MCV_p^{lp} = MCV_p^i))$.
- If $((MCV_d^i < Q)$ and $(MCV_p^i = Q))$, then denote as $((MCV_d^{lp} = MCV_d^i)$ and $(MCV_p^{lp} = Q))$.

Case 3: When $((MCV_d^i > Q)$ or $(MCV_p^i > Q))$, the point preceding i becomes the lp of this cluster, and record lp and this cluster. The following three situations are possible:

- If $((MCV_d^i > Q)$ and $(MCV_p^i > Q))$, then denote as $((MCV_d^{lp} = MCV_d^i - d_i)$ and $(MCV_p^{lp} = MCV_p^i - p_i))$.
- If $((MCV_d^i > Q)$ and $(MCV_p^i < Q))$, then denote as $((MCV_d^{lp} = MCV_d^i - d_i)$ and $(MCV_p^{lp} = MCV_p^i - p_i))$.
- If $((MCV_d^i < Q)$ and $(MCV_p^i > Q))$, then denote as $((MCV_d^{lp} = MCV_d^i - d_i)$ and $(MCV_p^{lp} = MCV_p^i - p_i))$.

- b) Taking the next point of lp as the first starting point of the next cluster, repeat Step 2 a) until the endpoint is swept.
- c) Calculate the total travel distance ttd of every cluster based on the spatial distribution of the points.
- d) Compare the ttd with the value in the *optimal*, and store the smaller value and the corresponding points (the customer points and the split points) into the *optimal* variables.

Step 3: End the SA-Unsplit procedure.

3.3 Sweep algorithm for the VRPSPDP

In the VRPSPDP, the constraint imposed on the VRPSPDP that a customer point can be visited only once for both delivery and pickup, is relaxed. Therefore, a point can be visited more than once, and the customer demands of both delivery and pickup can be split. The sweep algorithm for VRPSPDP (SA-VRPSPDP) is then employed to determine the split points and the split values in each cluster of the VRPSPDP. It is also called SA-Both-Split. The SA-Both-Split procedure of is described in the following.

Step 1: Specify one point as the first starting point.

Step 2: SA-Both-Split (Partitioning of the customer area).

- a) Sweep points gradually into an initial cluster in one direction (clockwise or anticlockwise) until $[\max(\sum_{i=1}^i d_i, \sum_{i=1}^i p_i) \geq Q]$ reaches point i , the point i becomes the last point lp of this cluster. The following four cases can arise:

Case 1: When $((MCV_d^i = Q)$ or $(MCV_p^i = Q))$, record lp and this cluster. There are three situations.

- If $((MCV_d^i = Q)$ and $(MCV_p^i = Q))$, then denote as $((MCV_d^{lp} = Q)$ and $(MCV_p^{lp} = Q))$.
- If $((MCV_d^i = Q)$ and $(MCV_p^i < Q))$, then denote as $((MCV_d^{lp} = Q)$ and $(MCV_p^{lp} = MCV_p^i))$.
- If $((MCV_p^i = Q)$ and $(MCV_d^i < Q))$, then denote as $((MCV_d^{lp} = MCV_d^i)$ and $(MCV_p^{lp} = Q))$.

Case 2: When $((MCV_d^i > Q)$ and $(MCV_p^i < Q))$, split lp into $lp1$ and $lp2$, and end the grouping at the point $lp1$. Then, there are $MCV_d^{lp1} = Q$, $d_{lp2} = MCV_d^{lp} - Q$, $MCV_p^{lp1} =$

MCV_p^i and $p_{lp2} = 0$. Record $lp, d_{lp1}, d_{lp2}, p_{lp1}, p_{lp2}$ and this cluster; $lp2$ becomes the starting point of the next clustering.

Case 3: When $((MCV_d^i < Q)$ and $(MCV_p^i > Q))$, split lp into $lp1$ and $lp2$, and end the grouping at the point $lp1$. Then, there are $MCV_d^{lp1} = MCV_d^i, d_{lp2} = 0, MCV_p^{lp1} = Q$ and $p_{lp2} = MCV_p^{lp} - Q$. Record $lp, d_{lp1}, d_{lp2}, p_{lp1}, p_{lp2}$ and this cluster; $lp2$ becomes the starting point of the next clustering.

Case 4: When $((MCV_d^i > Q)$ and $(MCV_p^i > Q))$, split lp into $lp1$ and $lp2$, and end the grouping at the point $lp1$. Then, $MCV_d^{lp1} = Q, d_{lp2} = MCV_d^i - Q, MCV_p^{lp1} = Q,$ and $MCV_p^{lp2} = MCV_p^i - Q$. Record $lp, d_{lp1}, d_{lp2}, p_{lp1}, p_{lp2}$ and this cluster; $lp2$ becomes the starting point of the next clustering.

- b) Taking $lp2$ (with the demand d_{lp2} and p_{lp2}) as the starting point of the next cluster, repeat Step 2 until the endpoint is swept.
- c) Calculate the total travel distance ttd of every cluster based on the spatial distribution of points.
- d) Compare the ttd with the value in *optimal* variables, and store the smaller values and the corresponding points (the customer points and the split points) in the *optimal* variables.

Step 3: End the SA-Both-Split procedure.

3.4 Multi-restart-iterative mode

When SA-Unsplit and SA-Both-Split are performed in the multi-restart-iterative (MRI) mode, the operations are called MRISA-Unsplit and MRISA-Both-Split, respectively.

- MRISA-Unsplit: Execute the SA-Unsplit procedure from each customer point (0- n) successively in the clockwise and anticlockwise directions to partition the customer area.
- MRISA-Both-Split: Execute the SA-Both-Split procedure from each customer point (0- n) successively in the clockwise and anticlockwise directions to partition the customer area.

The smallest ttd in the *optimal* is extracted, and the corresponding cluster becomes the most appropriate customer sub-domains.

3.5 Route optimization in a cluster

After the operations described above, there is one route in each cluster, and each customer point has both delivery and pickup demands simultaneously. Therefore, the routing problem becomes a VRPSDP. A modified C-W algorithm is adopted as follows [32]:

- Step 1: Form an initial solution set $L = \{L_i\}, \forall i \in \{1, 2, 3, \dots, n\}$, where L_i states one point i .
- Step 2: Compute the distance-saving degree of point-pair (i, j) $\Delta c_{ij} = c_{0i} + c_{0j} - \Delta c_{ij}$. Sort $\Delta c_{ij} (i, j = 1, 2, \dots, n)$ in descending order Dc_{ij} .
- Step 3: Initialize the new route set $L'_0 = \emptyset$ and the load quantity set $R'_0 = 0$ for both the delivery demands d_i and pickup demands p_i .
- Step 4: Scan the descending order Dc_{ij} from the top to the bottom and stop at the bottom.
- Step 5: Look for the merge possibility of point-pairs according to the following discriminant conditions during scanning.
 - a) Go to the next point-pair in the descending order Dc_{ij} if both points i and j are not on the route, that is, $L_i \not\subseteq L'_0$ and $L_j \not\subseteq L'_0$.
 - b) Go to the next point-pair in the descending order Dc_{ij} if both points i and j are on the route, that is, $L_i \subseteq L'_0$ and $L_j \subseteq L'_0$.
 - c) Merge one point i or j , which is not in L'_0 , that is, $(L_i \cup L'_0)$ or $(L_j \cup L'_0)$, and calculate the route distance if one point j or i is on the route, that is, $L_j \subseteq L'_0$ or $L_i \subseteq L'_0$. This can give rise to four scenarios:
 - Merging point j behind point i if i is the end of the route.
 - Merging point j before point i if i is the beginning of the route.

- Merging point i behind point j if j is the end of the route.
 - Merging point i before point j if j is the beginning of the route.
- d) Check the load feasibility for every point of each route because the gross load of the vehicle fluctuates with increasing or decreasing loads along the route. If the load in the current route meets the following conditions, go to the next point-pair; otherwise, delete all operations in Step 5 and return to Step 4. This leads to the following scenarios:
- The load quantity at the starting point is equal to $R'_{s0} = \sum_{i=1}^n d_i y_{ik} \leq Q$.
 - The load quantity at the end point is equal to $R'_{e0} = \sum_{i=1}^n p_i y_{ik} \leq Q$.
 - The load quantity at the current point i is equal to $R'_{0,i} = R'_{0,i} - d_i + p_i + R'_{(i+1),0} \leq Q$, that is $R'_{0,i} = R'_{s0} - \sum_{i=1}^{i-1} d_i y_{ik} - d_i + \sum_{i=1}^{i-1} p_i y_{ik} + p_i \leq Q$.

4. Case studies

Computational experiments were performed to verify the feasibility and effectiveness of the proposed algorithms in reducing travel distances and the number of vehicles used, and increasing the loading rates. The executed results of the VRPSPDP and the VRPSDP were compared to determine which heuristic performs better than others. The 25-, 50-, and 100-customer datasets were chosen from the Solomon datasets of VRP Web [35]. However, the Solomon datasets cannot be used directly in the VRPSPDP model because they do not include the pickup demand data. The delivery and pickup demands in a new constructed dataset come from two different original datasets. For example, for the new constructed dataset CR101, the delivery is from C101, and the pickup is from R101. The experiments were implemented using C in a 64-bit Windows 7 machine with an Intel (R) Core processor of 2.50 GHz speed and 8 GB memory as in [32].

4.1 Execution on the constructed Dataset 1

Our initial experiment was performed on the constructed Dataset 1. The results of MRISA-Both-Split + Modified C-W (hereinafter B-Split) for the VRPSPDP and MRISA-Unsplit + Modified C-W (hereinafter Unsplit) for the VRPSDP are shown in Table 1. Here, the distance reduction (Δ Dist. %) and the route reduction (Δ Rt. %) are respectively given as follows:

$$\Delta_{\text{B-Split to Unsplit}} \text{Dist. \%} = |(\text{Dist. of B-Split} - \text{Dist. of Unsplit})| / (\text{Dist. of Unsplit}) \times 100 \% \quad (11)$$

$$\Delta_{\text{B-Split to Unsplit}} \text{Rt. \%} = |(\text{Rt. of B-Split} - \text{Rt. of Unsplit})| / (\text{Rt. of Unsplit}) \times 100 \% \quad (12)$$

The results do not show any benefits for B-Split as compared with Unsplit. We note the following points:

- The average Δ Dist % of the B-Split is 1.95 %, and Δ Dist % is between 0.0 % and 3.76 %.
- The average Δ Rt. % of the B-Split is 0.0 %.

Table 1 Results for the constructed Dataset 1 ($Q = 200$ kg)

Instance	Demands (D/P) kg	B-Split				Unsplit	
		Distance (m)	Route	Δ Dist. %	Δ Rt. %	Distance (m)	Route
CR101-25	460/332	236	3	3.28	0	244	3
CR201-25	460/332	266	3	2.21	0	272	3
R_C101-25	332/460	395	3	2.95	0	407	3
RCR101-25	540/332	311	3	2.81	0	320	3
CR101-50	860/721	488	5	0.61	0	491	5
CR201-50	860/721	633	5	2.91	0	652	5
R_C101-50	721/860	677	5	0.00	0	677	5
RCR101-50	970/721	580	5	0.68	0	584	5
CR101-100	1810/1458	976	10	3.27	0	1009	10
CR201-100	1810/1458	1034	10	0.48	0	1039	10
R_C101-100	1458/1810	1032	10	0.39	0	1036	10
RCR101-100	1724/1458	1406	9	3.76	0	1461	9
Average	—	—	—	1.95	0	—	—

Note: Demand (D/P) denotes Deliveries/Pickups.

These results might be the consequence of the average demand values being too small in the original constructed dataset with average values being 8.96 %, 8.53 %, and 8.50 % of the vehicle capacity for the deliveries and 7.28 %, 7.56 %, and 7.73 % of the vehicle capacity for the pickups in the 25-, 50-, and 100-customer cases, respectively. All these values are below 9 % of the vehicle capacity.

4.2 Execution on the constructed Dataset 2

To verify the benefits of splitting, we constructed Dataset 2 from the Dataset 1. We retained the data locations but changed the delivery and pickup values by multiplying all the values by 4 and adding $0.1 \times Q$ to all the points; here, Q denotes the capacity of the vehicle. Thus, Dataset 2 had delivery and pickup values between 12 % and 110 % of the vehicle capacity with average values of 45.84 %, 44.11 %, and 44.01 % of the vehicle capacity for the deliveries, and 39.12 %, 40.23 %, and 40.92 % of the vehicle capacity for the pickups in the 25-, 50-, and 100- customer cases, respectively.

The results of applying B-Split and Unsplit to the Dataset 2 are shown in Table 2. Here, the load rate increase (Δ Lr. %) is given as follows:

$$\Delta_{\text{B-Split to Unsplit}} \text{Lr. \%} = |(\text{Lr. of B-Split} - \text{Lr. of Unsplit})| / (\text{Lr. of Unsplit}) \times 100\% \quad (13)$$

The results show that B-Split has obvious advantages over Unsplit:

- The Δ Dist.% of the B-Split lies between 3.71 % and 16.66 %, with an average of 11.64 %. This distance reduction directly decreases the transportation cost.
- The Δ Rt. % of the B-Split lies between 12.50 % and 25.40 %, with an average of 20.71 %. Thus, the vehicle start up fee, manpower costs and the transportation cost will decrease with the reduction of the number of vehicles required.
- The Δ Lr. % of the B-Split lies between 14.46 % and 33.80 %, with an average of 26.72 %. Therefore, the number of vehicles used will decrease with increasing load rate, and the transportation cost will be also reduced.
- The resulting routes are shaped like a petal around the depot. The beginning and end points of most routes are split.

Table 2 Results obtained for Dataset 2 ($Q = 200$ kg)

Instance	Demand (D/P), kg	B-Split						Unsplit		
		Dist., m	Rt.	Lr.	Δ Dist. %	Δ Rt. %	Δ Lr. %	Dist., m	Rt.	Lr.
CR101-25	2340/1828	643	12	0.98	10.57	14.29	16.67	719	14	0.84
CR201-25	2340/1828	782	12	0.98	4.98	14.29	16.67	823	14	0.84
R_C101-25	1828/2340	831	12	0.98	8.98	25.00	34.25	913	16	0.73
R_C101-25	2660/1828	1246	14	0.95	3.71	12.50	14.46	1294	16	0.83
CR101-50	4440/3884	1356	23	0.97	14.93	20.69	25.97	1594	29	0.77
CR201-50	4440/3884	1592	23	0.97	10.66	20.69	25.97	1782	29	0.77
R_C101-50	3884/4440	1685	24	0.93	8.32	20.00	25.68	1838	30	0.74
R_C101-50	4880/3884	2354	25	0.98	16.61	21.88	28.95	2823	32	0.76
CR101-100	9240/7832	3437	48	0.96	16.66	25.00	33.33	4124	64	0.72
CR201-100	9240/7832	3648	48	0.96	15.73	25.00	33.33	4329	64	0.72
R_C101-100	7832/9240	3280	48	0.96	12.88	23.81	31.51	3765	63	0.73
R_C101-100	8896/7832	4021	47	0.95	15.60	25.40	33.80	4764	63	0.71
Average	—	—	—	—	11.64	20.71	26.72	—	—	—

Note: Demand (D/P) denotes Deliveries/Pickups; Dist. denotes Distance; Rt. denotes Route; Lr. denotes Load rate.

4.3 Execution on the constructed Dataset 3

We constructed Dataset 3 from Dataset 1 to better show the benefits of the splitting. We retained the locations from Dataset 1 but changed the delivery and pickup values by adding $0.75 \times Q$ and $0.2 \times Q$ to the delivery and pickup at each odd point and adding $0.2 \times Q$ and $0.75 \times Q$ to the delivery and pickup at each even point [32]. Thus, Dataset 3 had demand values for either delivery or pickup between 20.5 % and 100 % of the vehicle capacity. In the 25-, 50-, and 100-customer cases, the average values were 57.56 %, 56.03 %, and 56.00 % of the vehicle capacity for the deliveries and 53.68 %, 55.06 %, and 55.23 % of the vehicle capacity for the pickups, respectively.

Further, in Cases 2 and 3 of the B-Split, the split of the last point lp depends only on $\max(\sum_{i=1}^i d_i, \sum_{i=1}^i p_i)$, regardless of how much $\min(\sum_{i=1}^i d_i, \sum_{i=1}^i p_i)$ differs from Q . This may affect the vehicle load rate, resulting in an increase in the number of vehicles used. Therefore, a coefficient of fine-tuning is introduced to control the “difference,” and to form an Enhanced Both-Split (hereinafter E-B-Split). If $(\sum_{i=1}^i X_i < coef \times Q)$, where X indicates $\min(\sum_{i=1}^i d_i, \sum_{i=1}^i p_i)$, then the clustering does not terminate on X at the point i and the sweeping continues; If $(\sum_{i=1}^i X_i \geq coef \times Q)$, then the clustering terminates on X at the point i and the sweeping stops. This implies that d (delivery) and p (pickup) could have separate partition termination points. In E-Both-Split, Cases 2 is given as follows:

*Case 2: When $((MCV_d^i > Q)$ and $(MCV_p^i < Q))$ at point i ,

- If $((MCV_d^i > Q)$ and $(MCV_p^i = ef \times Q))$, and point i is the last point lp for both deliveries and pickups, then split lp into $lp1$ and $lp2$, end the current cluster on both the deliveries and pickups, and start the next clustering at $lp2$. Therefore, $MCV_d^{lp1} = Q$; $d_{lp2} = MCV_d^{lp} - Q$; $d_{lp1} = d_{lp} - d_{lp2}$; $MCV_p^{lp1} = MCV_p^i = ef \times Q$ and $p_{lp2} = 0$.
- If $((MCV_d^i > Q)$ and $(MCV_p^i < ef \times Q))$, and point i is the last point lp on the deliveries, split the delivery into $lp1$ and $lp2$, and end the current cluster on delivery. There are $MCV_d^{lp1} = Q$; $d_{lp2} = MCV_d^{lp} - Q$ and $d_{lp1} = d_{lp} - d_{lp2}$. Go to the next i and continue to sweep. Three situations can arise on $(MCV_p^i + p_{i+1})$:
 - If $((MCV_p^i + p_{i+1}) > ef \times Q)$, and point $i + 1$ is the lp on the pickup, split the point, and end the cluster on the pickup. Then, $MCV_p^{lp1} = Q$; $p_{lp2} = (MCV_p^i + p_{i+1}) - Q = MCV_p^{lp} - Q$ and $p_{lp1} = p_{lp} - p_{lp2}$.
 - If $((MCV_p^i + p_{i+1}) = ef \times Q)$, then point $i + 1$ is the lp on the pickup, and the current cluster is ended on the pickup. Thus, $MCV_p^{lp1} = MCV_p^{lp} = Q$ and $p_{lp2} = 0$.
 - If $((MCV_p^i + p_{i+1}) < ef \times Q)$, then continue to sweep.

*Case 3 has the same operations as *Case 2, but only d and p are swapped.

Table 3 shows the results of applying the MRISA-E-Both-Split + Modified C-W (hereinafter E-B-Split), B-Split, and Unsplit to Dataset 3. Table 4 presents the comparisons among the results of E-B-Split, B-Split, and Unsplit.

The results prove that the E-B-Split has significant advantages over the other two algorithms (especially as compared with Unsplit). The main advantages are as follows:

- The Δ Dist. % of the E-B-Split as compared with the Unsplit lies between 30.65 % and 38.35 %, with an average of 33.92%. The Δ Dist. % of the E-B-Split as comparison with the B-Split lies between 12.53 % and 24.23 %, with an average of 17.38 %. The Δ Dist. % of the B-Split relative to the Unsplit lies between 11.66 % and 26.85 %, with an average of 19.89 %. This distance reduction shown as Fig. 1 would directly decrease the transportation cost.
- The average load rate obtained by the E-B-Split, B-Split, and Unsplit are 0.96, 0.89, and 0.57, respectively. The Δ Lr. % of the E-B-Split as compared with the Unsplit lies between 65.52 % and 73.21 %, with an average of 68.83 %. The Δ Lr. % of the E-B-Split in comparison with the B-Split lies between 0.0 % and 13.79 %, with an average of 8.65 %. The Δ Lr. % of the B-Split relative to Unsplit lies between 47.46 % and 66.07 %, with an average of 55.53 %. This load rate increase shown as Fig. 2 would directly decrease the number of vehicles used.
- The number of routes in Unsplit is equal to the number of points, that is, one route corresponds to one point. The number of routes in the E-B-Split is reduced by approximately 40.75 % on the average as compared with the number of routes in Unsplit. The number of routes in B-Split is reduced by approximately 36 % on average as compared with the number of routes in Unsplit shown as Fig. 3. Therefore, by applying the E-B-Split, the number of vehicles and manpower costs can be reduced, which will substantially decrease the transportation cost.

- The resulting routes are shaped like a petal around the depot, and a point splitting often occurs at the beginning or the end of sub-domains in the sweeping procedure.
- When the E-B-Split is applied, there might be two splitting points between two adjacent routes: one for deliveries and another for pickups.

Table 3 Results obtained for Dataset 3 ($Q = 200$ kg)

Instance	Demand (D/P), (kg)	E-B-Split			B-Split			Unsplit		
		Dist. (m)	Rt.	Lr.	Dist. (m)	Rt.	Lr.	Dist. (m)	Rt.	Lr.
CR101-25	2890/2652	734	0.96	15	891	0.90	16	1132	0.58	25
CR201-25	2890/2652	880	0.96	15	1091	0.90	16	1275	0.58	25
R_C101-25	2762/2780	859	0.93	15	982	0.93	15	1246	0.56	25
RRCR101-25	2970/2652	1217	0.99	15	1485	0.87	17	1887	0.59	25
CR101-50	5610/5471	1555	0.97	29	1807	0.91	31	2411	0.56	50
CR201-50	5610/5471	1778	0.97	29	2250	0.88	32	2679	0.56	50
R_C101-50	5471/5610	1757	0.94	30	2319	0.83	34	2625	0.56	50
RRCR101-50	5720/5471	2503	0.95	30	2970	0.87	33	4060	0.57	50
CR101-100	11310/10958	3724	0.98	58	4455	0.90	63	5771	0.57	100
CR201-100	11310/10958	3942	0.98	58	4773	0.88	64	5943	0.57	100
R_C101-100	10958/11310	3460	0.96	59	4156	0.87	65	4989	0.57	100
RRCR101-100	11224/10958	4372	0.94	60	5182	0.88	64	6618	0.56	100
Average	—	—	0.96	34.42	—	0.89	38	—	0.57	58

Note: Demand (D/P) denotes Deliveries/Pickups; Dist. denotes Distance; Rt. denotes Route; Lr. denotes Load rate.

Table 4 Comparisons of E-B-Split, B-Split and Unsplit

Instance	E-B-Split to Unsplit (%)			E-B-Split to B-Split (%)			B-Split to Unsplit (%)		
	Δ Dist.	Δ Lr.	Δ Rt.	Δ Dist.	Δ Lr.	Δ Rt.	Δ Dist.	Δ Lr.	Δ Rt.
CR101-25	35.16	65.52	40	17.62	6.67	6.25	21.29	55.17	36
CR201-25	30.98	65.52	40	19.34	6.67	6.25	14.43	55.17	36
R_C101-25	31.06	66.07	40	12.53	0	0.00	21.19	66.07	40
RRCR101-25	35.51	67.80	40	18.05	13.79	11.76	21.30	47.46	32
CR101-50	35.50	73.21	42	13.95	6.59	6.45	25.05	62.50	38
CR201-50	33.63	73.21	42	20.98	10.23	9.38	16.01	57.14	36
R_C101-50	33.07	67.86	40	24.23	13.25	11.76	11.66	48.21	32
RRCR101-50	38.35	66.67	40	15.72	9.2	9.09	26.85	52.63	34
CR101-100	35.47	71.93	42	16.41	8.89	7.94	22.80	57.89	37
CR201-100	33.67	71.93	42	17.41	11.36	9.38	19.69	54.39	36
R_C101-100	30.65	68.42	41	16.75	10.34	9.23	16.70	52.63	35
RRCR101-100	33.94	67.86	40	15.63	6.82	6.25	21.70	57.14	36
Average	33.92	68.83	40.75	17.38	8.65	7.81	19.89	55.53	36

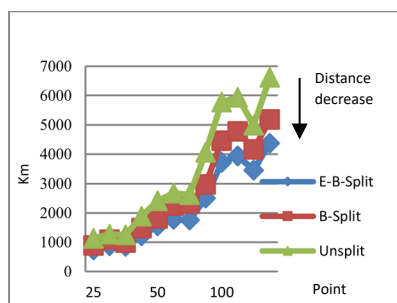


Fig. 1 Distance reduction of E-B-Split

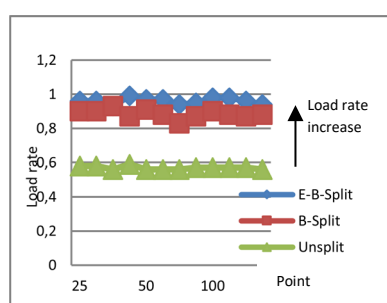


Fig. 2 Load rate increase of E-B-Split

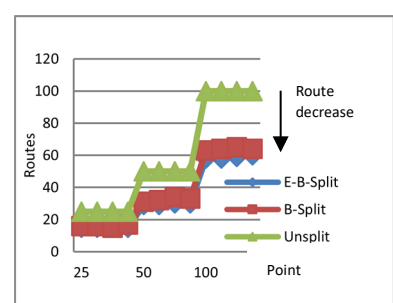


Fig. 3 Routes reduction of E-B-Split

5. Conclusion

In this study, we developed a mathematical formulation to model the VRPSPDP problem. A two-stage construction heuristic approach was used under the strategy of “clustering first and routing later.” In the first stage, the partitioning algorithms (B-Split and E-B-Split) based on MRISA and the coefficient of fine-tuning were employed to divide the customer domain into sub-domains. The minimum number of vehicles used was decided by the number of sub-domains. The split points and the corresponding values in each sub-domain were determined after the partitioning

algorithms. In the second stage, a modified C-W savings algorithm was adopted to check the load feasibility for every point of each route and to optimize the route distance in each sub-domain. Three cases studies with 12 instances per each from the reconstructed three Solomon datasets were employed to verify the feasibility and effectiveness of the proposed approaches (Unsplit, Both-Split and Enhanced-Both-Split). Our computation results proves that the vehicle routing problem with split deliveries and pickups is highly beneficial for transportation and logistics enterprises. The results of the comparison among these approaches reveal the following:

- The proposed algorithms of E-B-Split and B-Split for the VRPSPDP considerably reduce the total travel distance and the number of vehicles used, and increase the average loading rate.
- Particularly, the E-B-Split applied to the constructed Dataset 3 shows significant advantages over Unsplit. E-B-Split reduced the travel distance by 33.92 % on average, and increased the loading rate by 68.83 % on average. In addition, the number of vehicles used also decreased by 40.75 % on average. When compared with the results [9-11] of SDVRP, where the optimal route length and the optimal number of vehicles may be as little as half of the corresponding VRP, the results of the E-B-Split performed on Dataset 3 showed trends similar to those of SDVRP.
- The E-B-Split applied to the constructed Dataset 3 has some advantages over the B-Split. The travel distance was reduced by 17.38 % on average; the loading rate was increased by 8.65 % on average, and the number of vehicles used was decreased by approximately 10.50 % on average. These results prove that the E-B-Split can get more optimization results than the B-Split.
- The B-Split applied to the Dataset 3 can provide greater benefits than when it is applied to the Dataset 2 because Dataset 3 has larger customer demand values (about 55 % on average) than Dataset 2 (about 40 % on average).
- Routes are shaped similar to petals around the depot, and point splitting often occurs at the start or end of the sub-domains in the sweep.

The approaches used in this study could achieve the better computational results with a little time. But more work would be done in future to improve the accuracy and the extrapolation of the partitioning algorithms. The intelligent algorithms would be used to focus on obtaining more optimized solution in a bearable time to fit more complexity of problems.

In this study, time-depended problems (such as, customer service time windows, travel speeds, travel start time, etc.) were not considered. These are the very important factors of effecting the transportation fuel consumption and the carbon emission [36-38]. Therefore, multi-objective vehicle routing problem will be our next research target as well.

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Appendix A

Whole-word-descriptions of all the abbreviations used in the paper

VRP	Vehicle Routing Problem
SDVRP	Split Delivery Vehicle Routing Problem
VRPSDP	Vehicle Routing Problem with Simultaneously Delivery and Pickups
VRPSPDP	Vehicle Routing Problem with (Simultaneously) Split Delivery and Pickups
SA-VRP	Sweep Algorithm for VRP
SA-SDVRP	Sweep Algorithm for SDVRP
SA-VRPSDP	Sweep Algorithm for VRPSDP
SA-VRPSPDP	Sweep Algorithm for VRPSPDP
MRI	Multi Restart Iterative
MRISA	MRI +SA Multi Restart Iterative Sweep Algorithm
MRISA-Unsplit	Unsplit based on MRI +SA
MRISA-B-Split	B-Split based on MRI +SA
MRISA-E-B-Split	E-B-Split based on MRI +SA
Unsplit	Non split
B-Split	Split for Both Deliveries and Pickups
E-B-Split	Enhanced Both Split (Fine-Turning Split for Both Deliveries and Pickups)
C-W savings	Clarke-Wright savings