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NUMERICAL MODEL FOR DYNAMIC ANALYSIS OF TOOL AND WORKPIECE IN TURNING

Zghal, B. & Haddar, M.
Mechanical Engineering Department, National School of Engineers of Sfax –Tunisia.
Mechanics Modelling and Production Research Unit (U2MP).
National School of Engineers of Sfax (ENIS); BP. W. 3038 – Sfax – Tunisia
E-Mail: bisszghal@yahoo.fr

Abstract:
In this paper is presented the dynamic response of the system tool-workpiece during turning operation by taking into account the regenerative effect. A new model based on the finite element methods was developed to establish the dynamic equation of the system. The tool was modelled by a beam while the dynamic effect of the non deformable elastically suspended workpiece was introduced by considering its displacements. The dynamic response of the entire system confirms the existence of the coupling between the tool and the workpiece. The simulations show the variations of the cutting forces around the static equilibrium and the effect of the vibratory behaviour on the profile of the workpiece during the raising operation.

Key Words: Turning, Regenerative effect, Finite element, Dynamic response

1. INTRODUCTION

Several researchers tried to explain the origin of vibrations and their appearance according to the parameters intervening in the machining operation [1, 2]. These vibrations often called chatter vibrations [3] were the subject of several experimental studies and digital simulations. Tobias has accorded a significant interest to chattering phenomenon in cutting operation by developing the earliest experimental analyses in this field [4].

Fortunately, representing correctly the dynamic behaviour of cutting operation allows avoiding chatter vibration and permitting the good choice of the cutting parameters in order to improve the surface machining quality.

Classical models [5, 6] used in turning adopt the modelization by an elastic tool and a rigid workpiece, or the opposite. These models do not express correctly the dynamic behaviour of the system. Indeed, the dynamic of the operation of cut depends on the geometrical and mechanical characteristics of the tool and the workpiece at the same time, as it depends on the cutting parameters. Unfortunately, in the literature authors started to deal with the problem of cut by finite element only lately and generally with an aim of developing software of simulation of cutting operation and to determine the temperature distribution [7, 8, 9].

In this work, we are interested to investigate the dynamic behaviour of the system tool-workpiece in turning operation. The equation of motion is established by taking account effect of the regeneration. Its resolution by using the Newmark and Newton Raphson method allows the determination of the dynamic response in each point of the tool and for the gravity centre of the workpiece.

2. DYNAMIC EQUATION

In order to determine the dynamic equation of the coupled system we modelled the tool by a beam which is solicited to traction compression and to deflection. For the workpiece we introduce the displacement of its rigid body. Nevertheless, we should consider the boundary conditions applied to the workpiece. So the flexibility of the chuck and the workpiece must be introduced in the form of a rigidity of translation (K_x, K_y, K_z) along the three axes, and a rigidity of rotation according to rotary motion θ_x and θ_y.

Fig. 1 represents the dynamical model for raising operation and tool's discretization.

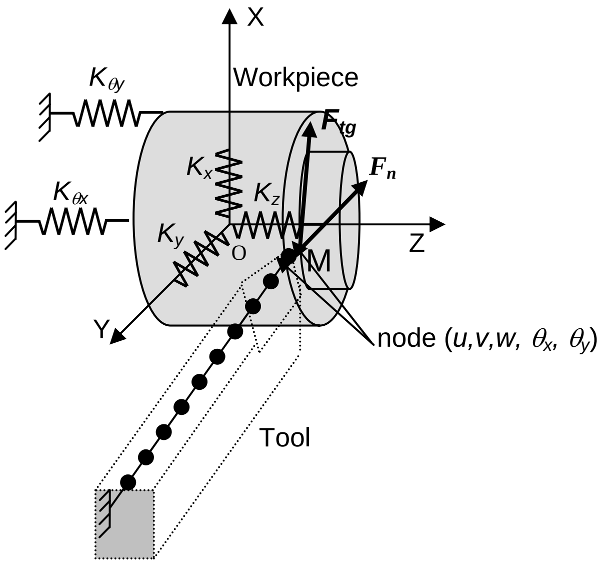

Figure 1: Dynamical model for turning.

The tool is modelled by a beam [10] discretized by ten elements with two nodes and five degrees of freedom by node. The tool is supposed to be clamped to the tool holder. The cutting forces are supposed to be applied to the first node which represents the point of contact between the tool and the workpiece (tool edge).

In fact, the dynamic equation depends on the instantaneous thickness of cut. This last is a scalar quantity which is positive in the case where the tool was inserted in the material and it will be a continuous cutting phase. If the tool was far away from material we adopt that the thickness of cut is null and it will be a discontinuous cutting phase. So, two cases of study are possible:

- In the first case the equation of motion to be solved is as follows:

$$\begin{bmatrix} [M_t] & 0 \\ 0 & [M_p^r] \end{bmatrix} \{\ddot{X}\} + \begin{bmatrix} [C_t] & 0 \\ 0 & [C_p^r] \end{bmatrix} \{\dot{X}\} + \begin{bmatrix} [K_t] & 0 \\ 0 & [K_p^r] \end{bmatrix} \{X\} = \begin{Bmatrix} F_t \\ F_p \end{Bmatrix} \qquad (1)$$

This is the dynamic equation of the forced mode at a raising operation. The program of resolution is based on the combination of Newmark's method and Newton Raphson's one.

Where

$$[M_p^r] = \begin{bmatrix} m_p & & & & \\ & m_p & & 0 & \\ & & m_p & & \\ & 0 & & J_p & \\ & & & & J_p \end{bmatrix}, \; [C_p^r] = \begin{bmatrix} 0 & & & & \\ & 0 & 0 & & \\ & & 0 & & \\ & 0 & & 0 & \omega J_p \\ & & & -\omega J_p & 0 \end{bmatrix}, \; [K_p^r] = \begin{bmatrix} K_x & & & & \\ & K_y & & 0 & \\ & & K_z & & \\ & 0 & & K_{\theta x} & \\ & & & & K_{\theta y} \end{bmatrix} \qquad (2)$$

$[M_p^r]$, $[C_p^r]$, $[K_p^r]$ are respectively the mass, damping and rigidity matrix of the non deformable workpiece.

$[M_t]$, $[C_t]$ and $[K_t]$ are respectively is the mass, damping and rigidity matrix of elastic tool, Kx, Ky and Kz represent the rigidity of translation, $K_{\theta x}$ and $K_{\theta y}$ represents the rigidity of rotation, $x = (x_1 \; \cdots \; x_n)^t$ is the displacement vector of the complete system formed by the tool and the workpiece, $\dot{x} = (\dot{x}_1 \; \cdots \; \dot{x}_n)^t$ is the velocity vector, $\ddot{x} = (\ddot{x}_1 \; \cdots \; \ddot{x}_n)^t$ is the acceleration vector, $F = (F_t \; F_p)^t$ is the vector of force applied to the system and which depends on the nature of the operation of cut and the solution at the moment 't' and 't-τ', F_t is the vector of force applied to the tool, F_p is the vector of force applied to the workpiece, J_p is the inertia moment of the workpiece, m_p is mass of the workpiece, ω is the Spindle speed and τ is the period of one revolution of the workpiece.

Anti-symmetric terms presented in the damping matrix of workpiece represent the effect of its rotary motion.

- In the second case there is not contact between the workpiece and the tool and the equation to be solved becomes

$$\begin{bmatrix} [M_t] & 0 \\ 0 & [M_p^r] \end{bmatrix} \{\ddot{X}\} + \begin{bmatrix} [C_t] & 0 \\ 0 & [C_p^r] \end{bmatrix} \{\dot{X}\} + \begin{bmatrix} [K_t] & 0 \\ 0 & [K_p^r] \end{bmatrix} \{X\} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{3}$$

This equation expresses the free oscillations of the system into absence of machining. The solution of this equation was established by using the method of Newmark.

2.1 Modelling of the forces applied to the workpiece

The workpiece is subjected to the action of the tangential cutting force and thrust one (normal component). In addition to the cutting forces we take account of the effect of their moments along X and Y axes.

The projection of the thrust force F_n and tangential force F_{tg} on the basis of study is as follows:

$$\vec{F}_n = -F_n \vec{Y}, \; \vec{F}_{tg} = F_{tg} \vec{X} \tag{4}$$

Where $\vec{X}$ and $\vec{Y}$ are unit vectors along tangential and normal directions.

The moments of cutting forces are written:

$$\vec{M}(F_n)_{/0} = \vec{F}_n \times \overrightarrow{MO}, \; \vec{M}(F_{tg})_{/0} = \vec{F}_{tg} \times \overrightarrow{MO} \tag{5}$$

Where $\vec{M}(F_n)_{/0}$ is the moment of the thrust force around the gravity centre of the workpiece, $\vec{M}(F_{tg})_{/0}$ is the moment of the tangential force around the gravity centre of the workpiece, $\overrightarrow{MO}$ is the vector between the point 'M', which represents the contact between the workpiece and the tool, and the point 'O' which is the gravity centre of the workpiece,

$$\vec{M}(F_n)_{/0} = \begin{pmatrix} 0 \\ -F_n \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -D_p/2 \\ -L_p/2 \end{pmatrix} = \begin{pmatrix} F_n L_p/2 \\ 0 \\ 0 \end{pmatrix}, \; \vec{M}(F_{tg})_{/0} = \begin{pmatrix} F_{tg} \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -D_p/2 \\ -L_p/2 \end{pmatrix} = \begin{pmatrix} 0 \\ F_{tg} L_p/2 \\ 0 \end{pmatrix} \tag{6}$$

Where D_p is diameter of the workpiece and L_p is its length.

2.2 Model of cut

The selected model of cut is a nonlinear model extracted from the bibliography [7].

$$\begin{cases} F_{tg} = 0.5\ 10^{3}\ K_t w \left(0.5\ 10^{4}\ h\right)^{0.49} & \text{: is the tangential force} \\ F_N = 0.5\ 10^{3}\ K_N w \left(0.5\ 10^{4}\ h\right)^{0.83} & \text{: is the thrust force} \end{cases} \tag{7}$$

Where K_t and K_v are terms which express the intensity of cutting pressure, *w* represents the cutting width (width of chip) and h is the instantaneous thickness of cut.

2.3 Modelling of the complete system

By gathering the various degrees of freedom of the system the vector of displacement is written:

$$X^T = (u_{t1}\ v_{t1}\ \ ...\ \ \theta_{yt11}\ \ X_p\ Y_p\ Z_p\ \theta_x\ \ \theta_y) \tag{8}$$

Indeed, the vector of displacement includes the nodal variables of the elastic tool $(u_{t1}\ v_{t1}\ \ ...\ \ \theta_{yt11})$ and variables relating to the movement of the rigid body of the workpiece $(X_p\ Y_p\ Z_p\ \theta_x\ \ \theta_y)$.

In this study we are interested to the case of raising operation. The vector of force applied to the system depends on the components of cutting model.

$$F^T = (-F_n\ \ -F_{tg}\ 0\ ...\ \ 0\ \ F_{tg}\ \ -F_n\ \ 0\ \ \frac{L_p F_N}{2}\ \ \frac{L_p F_{tg}}{2}\ ) \tag{9}$$

The instantaneous thickness of cut 'h' is expressed according to the displacement of the tool edge. Indeed, this model permits the investigation of the regeneration effect on the level of thickness cut [11] and it will be written as:

$$h = (h_0 + u_{t1}(t - \tau) - u_{t1}(t))\ \ g(h) \tag{10}$$

Where u_{t1} is the displacement of the tool edge in the direction of thrust force, h_0 is the nominal thickness of the chip and *g(h)* is a function defined by

$$g(h) = \begin{cases} 1 & if\ h \succ 0 \\ 0 & if\ h\ \le\ 0 \end{cases} \tag{11}$$

The equation of instantaneous thickness of cut traduces the existing coupling between the tool and the workpiece. In fact, the cutting force applied to the workpiece depends on the displacement of the tool edge. So, the dynamic behaviour of the workpiece will be mainly influenced by the tool vibration.

3. NUMERICAL RESULTS

The results of the digital simulation are obtained by developing a program able to solve the dynamic equation obtained.

For the body of the tool a square section was used while the workpiece is supposed to be cylindrical. Table 1 represents the geometrical and mechanical characteristics of the tool and the workpiece and the cutting conditions adopted during a raising operation.

Table I: Cutting parameters.

Cutting conditions	Workpiece characteristics	Tool characteristics
Depth of cut :2mm Advance : 0.25mm/tr Spindle speed :1000tr/mn K_x : 10^9 N/m; K_y : 10^9 N/m; K_z : 10^{11} N/m; $K_{\theta x} = K_{\theta y}$: 10^4 N.m/rd	Length : 40mm Diameter : 50mm Inertia moment:1.92 10^{-4}Kg m^2 Young modulus: 21 10^4 MPa Masse/volume : 7800Kg/m^3	Length : 100mm Section : 10x10 mm^2 Young modulus: 21 10^4 MPa Masse/volume : 7800Kg/m^3

Fig. 2 represents the temporal displacement of the tool edge corresponding to the first degree of freedom of the first node of the tool "*ut1*".

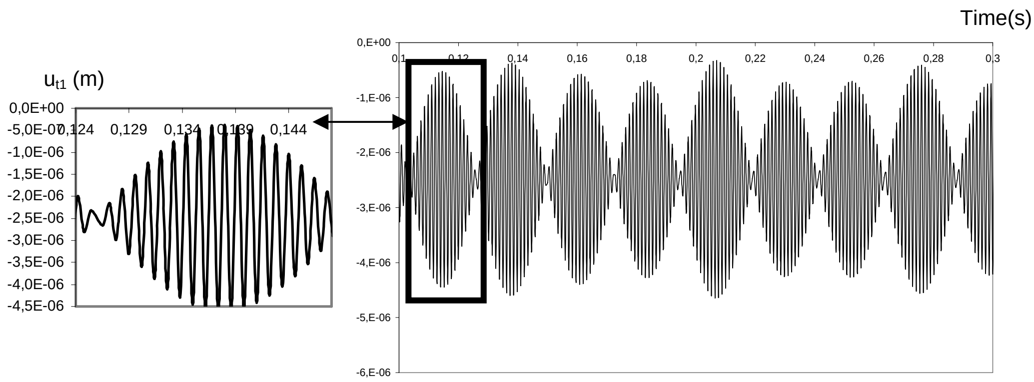

Figure 2: Displacement of the tool edge.

Fig. 2 shows that the tool vibrates around a static value about "- 2.5 10^{-6}m" which corresponds to the displacement due to the effect of the static component of the thrust cutting force. The period of small vibrations corresponds to the period of the first eigen frequency of the tool which is about 838 Hz.

Fig. 3 represents the displacement of the gravity centre of the workpiece corresponding to the 67^{th} degree of freedom.

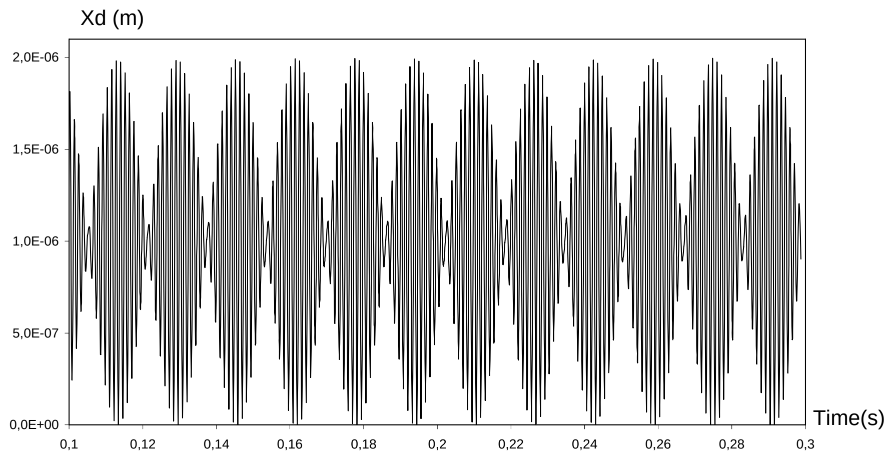

Figure 3: Displacement of the gravity centre of the workpiece.

Fig. 3 shows that the workpiece vibrates around a static value about "10^{-6}m" which corresponds to the displacement due to the effect of the static component of the normal cutting force. This displacement is less important than the tool one since the rigidity of the system of maintains of the workpiece is more important. This figure shows well the coupling between the tool and the workpiece. Indeed, the period of the small vibrations is equal to that of the tool.

Fig. 4 represents temporal and radial displacement of the gravity centre of the workpiece during the third revolution. This value is given by:

$$r = \sqrt{X_p^2 + Y_p^2} \tag{12}$$

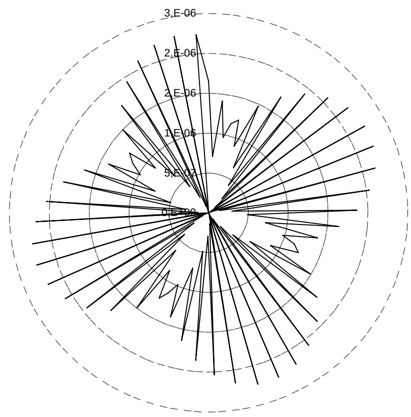

Figure 4: Radial displacement of the gravity centre of the workpiece.

This figure shows that the gravity centre of the workpiece vibrates during machining without exceeding the acceptable limit. Indeed, this displacement is less than 2.24 μm during the third revolution.

Fig. 5 represents the variation of the cutting thickness during the first five revolutions.

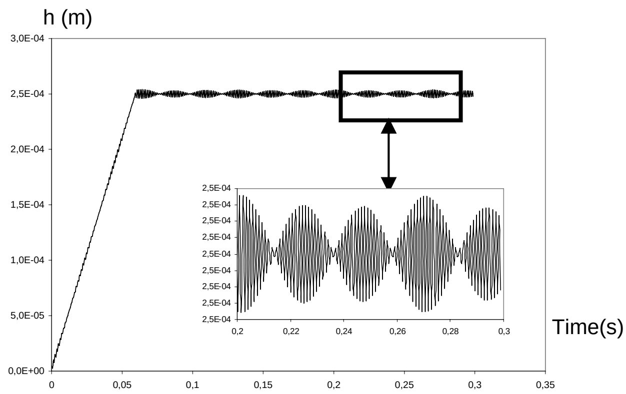

Figure 5: Variation of the cutting thickness.

This figure shows the variation of the instantaneous cutting thickness during the first five revolutions. According to this curve, the fluctuations are around the static value thickness of cut. The maximum of variation does not exceed 0.254 mm.

In this curve we notice that the cutting thickness is not interrupted during the cut. This observation implies that the cut is continuous during machining and that the contact between the tool and material is always assured. This result is also observed for the cutting forces.

Figs 6.a and 6.b represent, respectively, the variation of the tangential and thrust cutting forces during the first five revolutions.

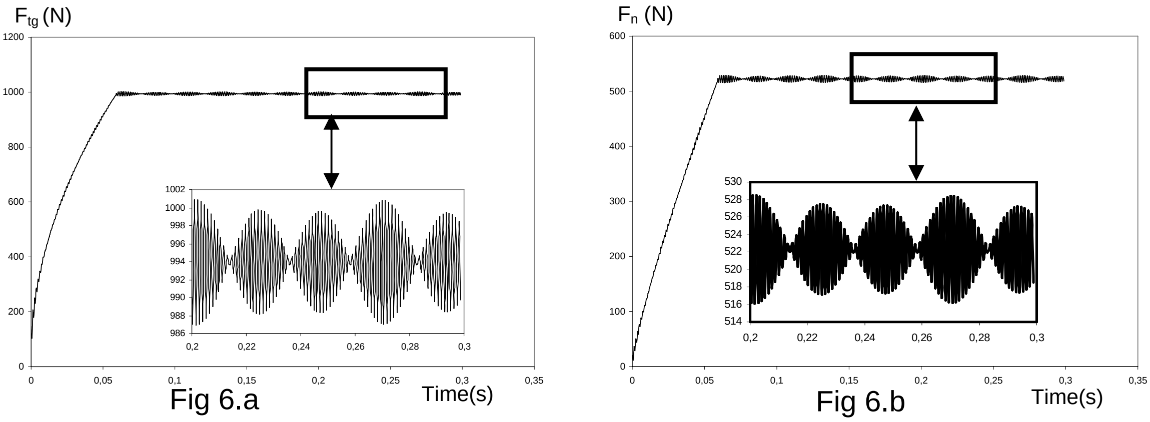

Figure 6: Variation of cutting forces.

The variation of the cutting forces according to time show the existence of two areas:

- The first area expresses the beginning of the contact between the tool and material. During this period the cutting force increases according to time since the cutting thickness increases from zero to the advance's value. Indeed, a little fluctuation exists around the static value of the cutting force.
- The second area begins just after the first one i.e. after the period of one revolution. This area represents the effective variation of the cutting force according to time. Indeed, in the case of the tangential cutting force the variation is about 980 N which corresponds to the static value. In the case of the normal cutting force the variation is around 540 N.

The rate and the value of the cutting force show that it is a stable case and there is not chatter vibration. Indeed, the cutting force does not diverge and remains under limits acceptable since it does not exceed 5% of the static value.

This behaviour is directly transmitted on the surface's profile of the workpiece during machining.

Fig. 7 presents the profile of surface during the third revolution. It is interesting to note that the central part of the workpiece is not schematized, for a radius lower than 24.2 mm, and this is with an aim of making the profile visible.

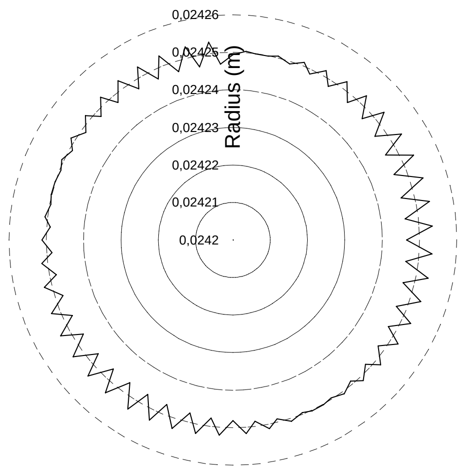

Figure 7: Surface profile.

According to fig. 7 fluctuation of the profile of the generated surface does not exceed some micrometers. These oscillations are regarded as being the influence of the vibration of the tool edge on the workpiece. Indeed, they represent the print of the tool edge during machining. However, these results do not consider the effect of the angles of cut and the nose radius of the tool.

4. CONCLUSION

In the present study a new nonlinear model was developed to investigate the dynamic behaviour of the coupled system tool-workpiece in turning operation. The variation of different temporal variables shows that the tool and the workpiece are in interaction during machining. Indeed, the same period of vibration is observed for the dynamic response of the tool and the workpiece. Moreover, the simulation of the cutting force shows the existence of little fluctuation around static value of the cutting force. Due to these oscillations also visible at dynamic response of the tool and workpiece, there are undulations on surface profile during machining.

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