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IMPORTANCE OF OVERLAPPING DECOMPOSITION FOR A WEB TENSION CONTROL SYSTEM

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Abstract:
The increasing requirement on the control performance for systems handling web material led to investigation for sophisticated control strategies. Actually many industrial web-transport systems use decentralized PI-type controllers. But when decentralized control is applied to a web tension control system, the interactions among control stations are the major problem for the controller design.

One of the authors has proposed a control methodology based on overlapping decomposition of the controlled object showing that it is possible to have good control performance.

But since there exist many sources of disturbance for this kind of system e.g. roller non circularity, roller change, web sliding, change of web-elasticity etc., a robust control technique (e.g. H-infinity control) has been chosen and tested for the system model. Then an interesting comparison has been carried out in this paper, comparing the results of the overlapping decomposition technique with the conventional decentralized controller based on disjoint decomposition.

Experimental data about the sensors behavior and noise are considered for evaluating the performances of the controllers.

Key Words: Winding systems, Decentralized control, H_∞ control, Overlapping subsystems

1. INTRODUCTION

Since a web tension system consists of many subsystems connected with certain dynamics and centralized control is not feasible because it has too many drive rolls, decentralized control is essential and many industrial web-transport systems have used decentralized PI-type controller [1,2]. But when decentralized control is applied to a web tension control system, the interactions among control stations is the major problem for the controller design and the situation can become serious when designing the decentralized control system via a disjoint decomposition. Subsequently, one of the authors [1] has proposed a control methodology based on overlapping decomposition of the controlled object showing, for a model, that it is possible to have good control performance.

In order to design a better control system or to identify the plant parameters experimentally, correct modeling is necessary. Moreover there exist many sources of disturbance e.g. roller no circularity, roller change, web sliding, change of web-elasticity; due to the strong coupling between web velocity and web tension, these disturbances, introduced by elastic web, are transmitted to the web tension that can result in a web break or fold. So a target of the control strategy concerns the robustness during the process; for this reason more efficient control strategies such as high-gain adaptive control, H-infinity control, fuzzy control, etc. have recently been presented [3-5].

In this paper the Voigt model experimentally validated in [11] is used for testing controller performances.

2. CONTROL DESIGN: STATE FORM OF THE VOIGT MODEL

The classical Voigt model of a web-tension control system with four control section [11], may be expressed in a state form (1) considering the state vector $x=[F_{uw}\ v_1\ v_2\ v_5\ v_3\ F_{dr}\ v_4]^T$ and considering the matrices *A,B,C,D* defined in (2), (3), (4), (5). The geometrical parameters used are defined and estimated in [11] and v_1, v_2, v_3, v_4 are the speed referred to the 4 motors of the system (corresponding respectively to unwinder, lead, draw-roll and winder section) and respectively termed $(v_{un}, v_l, v_{dr}, v_w)$, v_5 is another state variable (indicated in Fig. 4 of part 1) and F_{uw} and F_{dr} are the tension force of unwinder section and draw-roll section.

The model introduced in [11] part 1, has 4 input (electrical tension of the 4 servomotors) and 4 outputs $(F_{uw}\ v_2\ F_{dr}\ v_4)$ and 7 state variables as a minimal representation.

$$\begin{aligned} E\cdot\dot{x} &= B\cdot x + C\cdot u \\ y &= D\cdot x \end{aligned} \tag{1}$$

$$E=\begin{bmatrix} 1 & -\dfrac{A\cdot\eta}{L_{un}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & A\cdot\eta & 1 & A\cdot\eta & 0 & -A\cdot\eta \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\dfrac{A\cdot\eta}{L_{dr}} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$B=\begin{bmatrix} 1 & \dfrac{A\cdot\eta}{L_{uw}\cdot T_v} & 0 & 0 & 0 & 0 & 0 \\ -\dfrac{r_{uw}^2}{J_{uw}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\dfrac{r_l^2}{J_l} & 0 & 0 & \dfrac{r_l^2}{J_l} & 0 & 0 & 0 \\ 0 & 0 & \dfrac{-A\cdot\eta}{T_v} & 0 & \dfrac{-A\cdot\eta}{T_v} & 0 & \dfrac{A\cdot\eta}{T_v} \\ 0 & 0 & 0 & \dfrac{r_{dr}^2}{J_{dr}\cdot L_l} & 0 & \dfrac{r_{dr}^2}{J_{dr}} & 0 \\ 0 & 0 & 0 & 0 & \dfrac{A\cdot\eta}{L_{dr}\cdot T_v} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\dfrac{r_w^2}{J_w} & 0 \end{bmatrix} \tag{3}$$

$$C=\begin{bmatrix} 0 & 0 & 0 & 0 \\ -\dfrac{r_{uw}}{J_{uw}} & 0 & 0 & 0 \\ 0 & \dfrac{r_l}{J_l} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{r_{dr}}{J_{dr}} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{r_w}{J_w} \end{bmatrix} \tag{4}$$

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

3. CONTROL DESIGN: CONTROL PERFORMANCE USING DISJOINT DECOMPOSITION

The system under study, like the industrial systems, is composed by many rolls, and it has to be decomposed in many subsystems for web tension control and web speed control (decentralized control). It is not appropriate to design a robust controller for the entire system for the complexity of the dynamics; in this respect a decomposition of the model shown in part 1 is necessary.

One of the authors showed [1] that it is possible to improve the performance of the controller for a web tension control system if the decomposition is made with an overlapping methodology in stead of a disjoint decomposition. A conventional disjoint decomposition of the system in four subsystems is shown in Figure 1. For each subsystem it is not difficult to tune a PI controller; but when the four controllers are applied to the complete system the control performance is not so good when a noise comparable with the disturbances foreseen is applied to the system model.

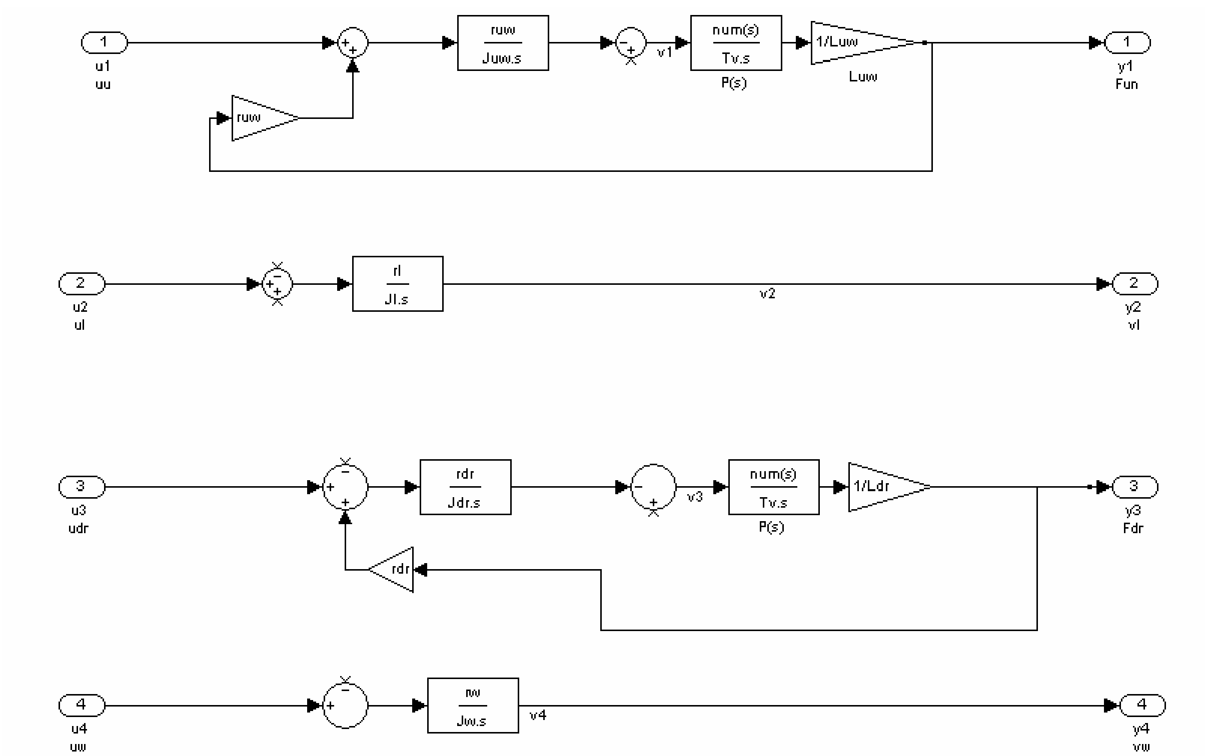

Figure 1: Disjoint decomposed system model.

4. CONTROL DESIGN: THE USE OF ROBUST H∞ CONTROL

The necessity of using a robust control strategy for winding systems has recently been introduced in the literature, [2, 4, 6]; robustness of the controller provides a safe control of the web throughout the whole industrial process.

For the validated model of this work, each subsystem of the disjoint decomposed system has been controlled using *H∞* controller in a weighted mixed sensitivity problem;

calling the transfer function of each subsystem the augmented plant, *H∞* controller is the controller of the augmented plant shown in Figure 2 [7,8].

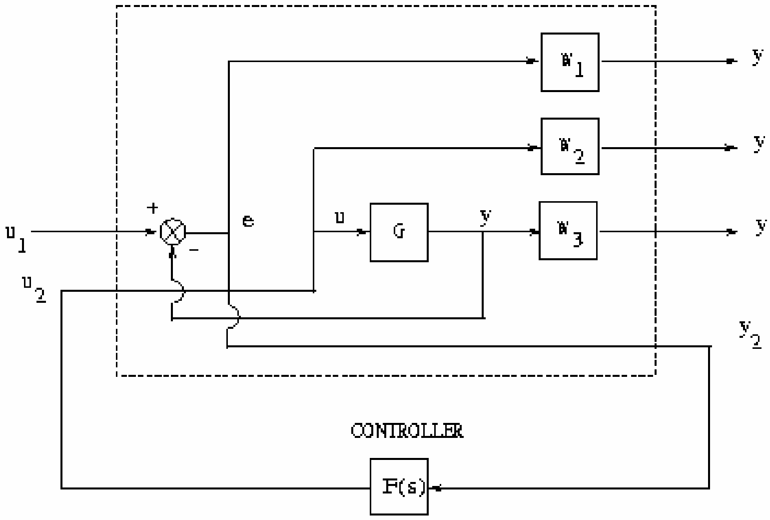

Figure 2: *H∞* controller augmented plant.

The *H∞* controller scheme is shown in Figure 3, where *K1, K2, K3, K4* are the disjoint subsystems robust controllers [7,8].

The values of the weights matrices W_1, W_2, and W_3, designed for each subsystem are shown in Table I and the calculated *K1, K2, K3, K4* robust controllers equation are reported in Table II.

Table I: Weight matrices for the four subsystems.

	$W_1(s)$	$W_2(s)$	$W_3(s)$
Subsystem 1	(s+10)/(10s+1)	(s+18)/(18s+1)	s/10
Subsystem 2	$(s+4)/s^2$	(0.00001s+0.0001)/(0.5s+10)	0.1/1000
Subsystem 3	$(s+100)/(s^3+3s^2)$	$(s^3+5s^2+s)/(s^3+s^2+18s+1)$	1/5
Subsystem 4	$(s+4)/s^2$	(0.00001s+0.0001)/(0.5s+10)	0.1/1000

Table II: H controller transfer functions for disjoint decomposition.

K1	-17.93 s^3 - 2180 s^2 - 338.9 s - 12.11 -- s^4 + 1.241e004 s^3 + 3177 s^2 + 262.9 s + 6.939
K2	5.08e004 s^4 + 1.318e006 s^3 + 6.483e006 s^2 + 9.216e006 s + 4e006 -- s^5 + 523.8 s^4 + 1.008e004 s^3 + 7.984e-009 s^2 + 2.013e-007 s
K3	2.811 s^7 - 33.32 s^6 - 220.4 s^5 - 835.3 s^4 - 2695 s^3 - 2746 s^2 - 396.7 s - 14.04 --- s^8 + 292.4 s^7 + 1203 s^6 + 6228 s^5 + 1.648e004 s^4 + 2452 s^3 + 86.52 s^2 + 1.903e-010 s + 6.858e-012
K4	5.062e004 s^4 + 1.314e006 s^3 + 6.475e006 s^2 + 9.212e006 s + 4e006 --- s^5 + 669.6 s^4 + 1.299e004 s^3 - 1.669e-008 s^2 - 3.118e-007 s

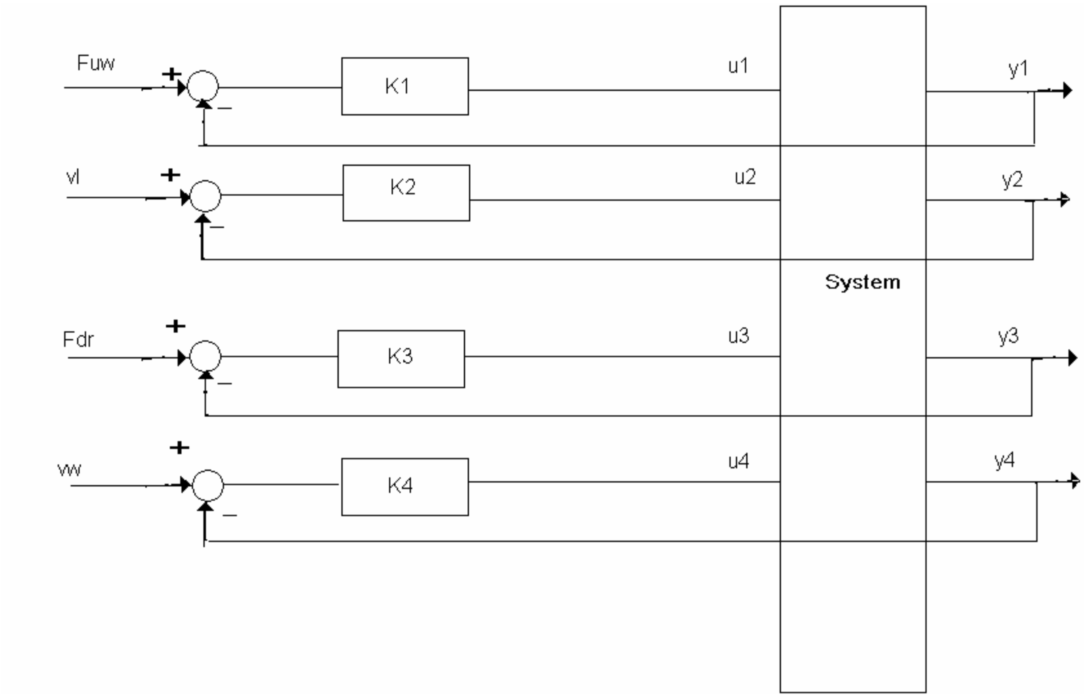

Figure 3: Controller scheme for the disjoint decomposed.

In particular the transfer function G referred to subsystems 2 and 4 (Fig. 1) have one pole on the *jw* axis; in this case a loop transformation technique [9] was necessary for designing the controllers. In fact for subsystems 2 and 4 [9] the H_∞ controller problem is not standard; but, following the approach expressed in [9], the controller design become possible.

The results of each designed controller considering an individual subsystem is very good for each subsystem in presence of noise too; in Figure 4 the behaviour of the subsystem 2 with H_∞ controller is shown with unitary step setpoint and noise added with noise power 10^{-6}.

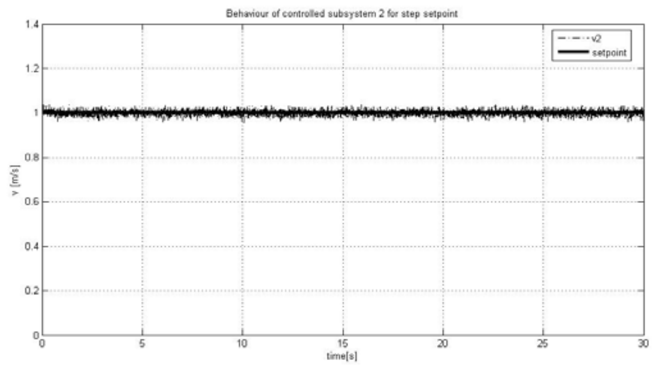

Figure 4: $H\infty$ controller step tracking performance for subsystem 2 with noise added.

The behaviour of each subsystem with $H\infty$ controller is better than using a PI controller when a noise comparable with the experimental one is applied to the sensor feedback; but when the controllers are applied to the entire model, because of the disjoint decomposition, the results are not so good. In Figures 5-8 the behaviour of the four system outputs with the same noise added for subsystem 2 considering the $H\infty$ controllers obtained with disjoint decomposition is shown. The setpoint chosen try to consider a constant value of tension when a speed change is required.

The robust control designed with a disjoint decomposition is not able to control properly the output variables, especially considering the behaviour of F_{un} (Fig. 5) and F_{dr} (Fig. 7).

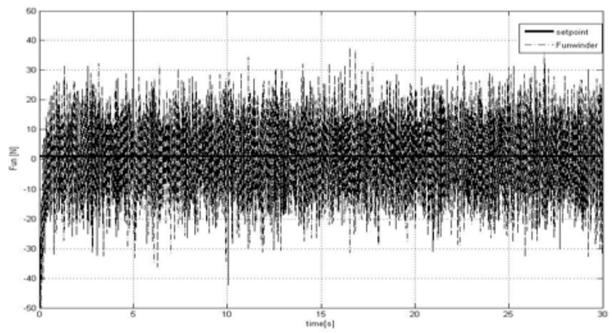

Figure 5: $H\infty$ controller performance with disjoint decomposition for output F_{un}.

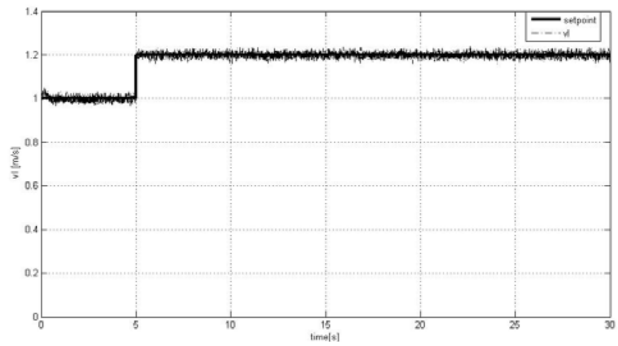

Figure 6: $H\infty$ controller performance with disjoint decomposition for output v_l.

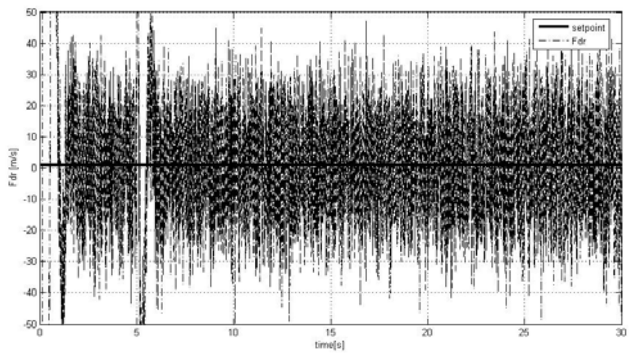

Figure 7: $H\infty$ controller performance with disjoint decomposition for output F_{dr}.

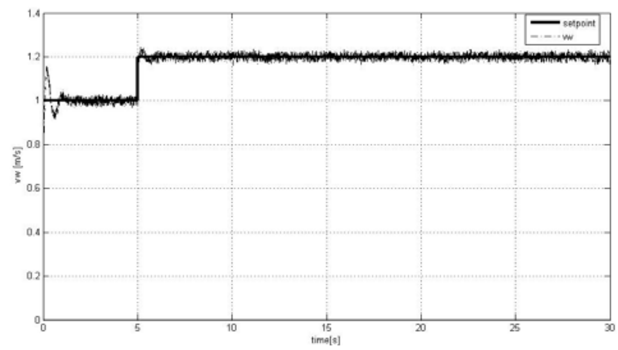

Figure 8: $H\infty$ controller performance with disjoint decomposition for output v_w.

5. CONTROL DESIGN: THE IMPORTANCE OF OVERLAPPING DECOMPOSITION

The model of the considered system has been divided in 4 subsystems, so as suggested in [1] (Fig. 9). The overlapping decomposed system may be expressed by new inputs (u_{11}, u_{22}, u_{33}, u_{44}) by the transformation in (6), in matricial form in (7):

$$
\begin{aligned}
u_{11} &= -\frac{r_{uw}}{J_{uw}} \cdot u_1 + \frac{r_l}{J_l} \cdot u_2 \\
u_{22} &= u_2 \\
u_{33} &= -\frac{r_{dr}}{J_{dr}} \cdot u_3 + \frac{r_w}{J_{wl}} \cdot u_4 \\
u_{44} &= u_4
\end{aligned}
\tag{6}
$$

$$
\begin{bmatrix} u_{11} \\ u_{22} \\ u_{33} \\ u_{44} \end{bmatrix} = N^{-1} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}
\tag{7}
$$

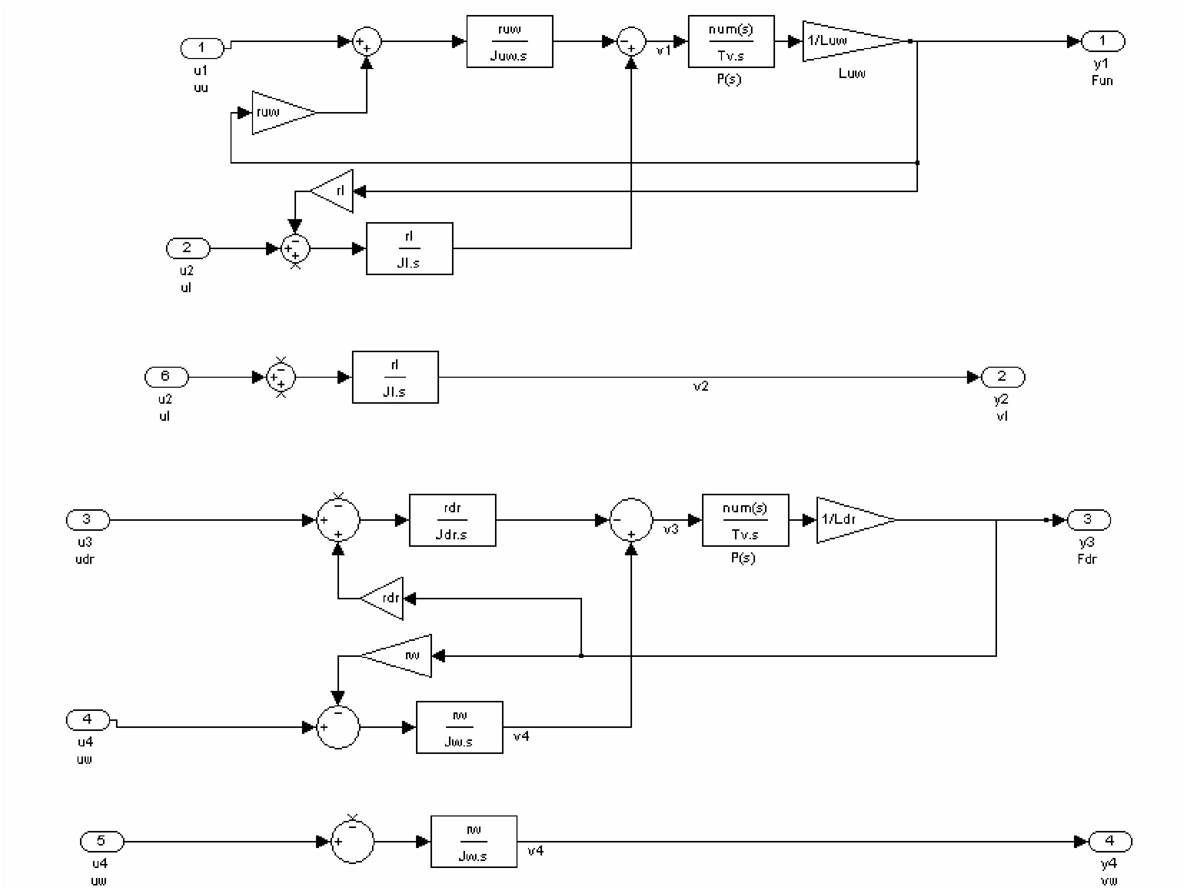

Figure 9: Overlapping decomposed system.

The new subsystem equations contain new inputs with overlapped variables. For each new subsystem is possible to design its own controller (called respectively *K1, K2, K3, K4*). The new controller obtained with overlapping are listed in Table III.

Table III: H controller transfer functions for overlapping decomposition.

K1	21.63 s^3 + 1.524e004 s^2 + 2371 s + 84.68 --- s^4 + 3068 s^3 - 41.83 s^2 - 100.4 s - 6.555
K2	5.025e004 s^4 + 1.306e006 s^3 + 6.46e006 s^2 + 9.205e006 s + 4e006 --- s^5 + 1610 s^4 + 3.18e004 s^3 - 4.151e-007 s^2 - 7.967e-006 s
K3	5.838 s^7 + 1684 s^6 + 1.374e004 s^5 + 7.441e004 s^4 + 2.523e005 s^3 + 5.954e005 s^2 + 8.818e004 s + 3107 --- s^8 + 412.5 s^7 + 119.8 s^6 + 8059 s^5 + 3.595e004 s^4 + 5448 s^3 + 193.3 s^2 + 3.444e-010 s + 1.27e-011
K4	5.062e004 s^4 + 1.314e006 s^3 + 6.475e006 s^2 + 9.212e006 s + 4e006 -- s^5 + 669.6 s^4 + 1.299e004 s^3 - 1.669e-008 s^2 - 3.118e-007 s

The controller design for the complete system may be designed like in figure 10, considering that *N* has the inverse of the matrix introduced in equation (7).

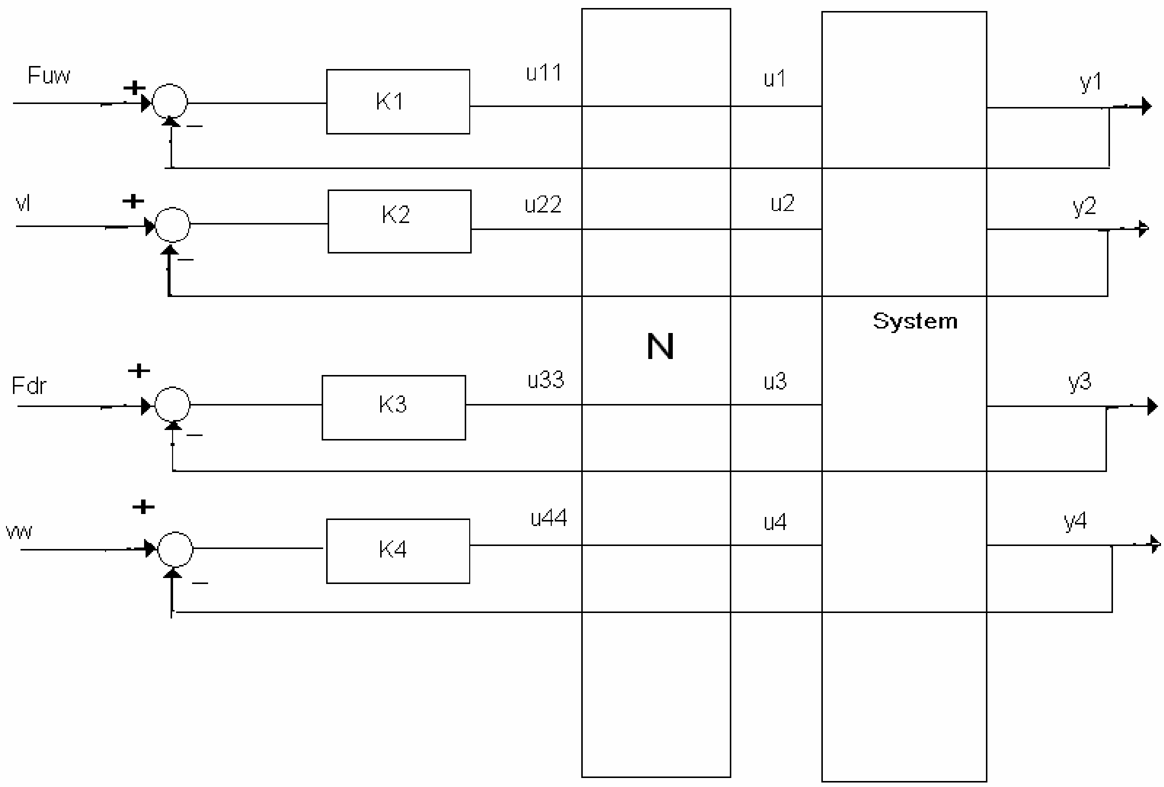

Figure 10: Controller scheme for the overlapping decomposed system.

The performances of the robust controller with overlapping decomposition in the same conditions of the previous tests are shown in Figures 11-14. The performances are excellent, in Figures 12 and 14 it is not possible to distinguish between setpoint and controlled variable; in Figures 11 and 13 the tension control is remarkably improved respect to the previous decomposition (Figs. 5 and 7).

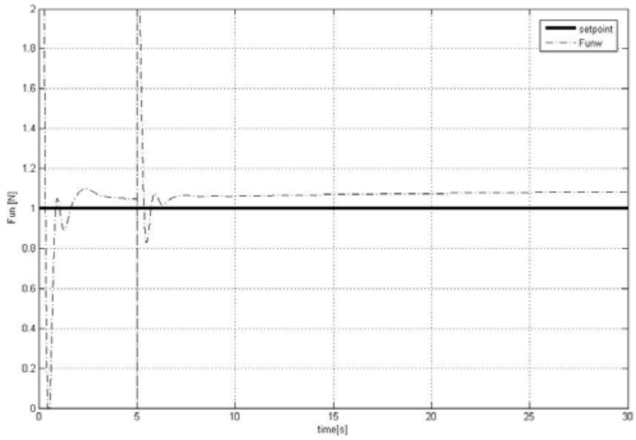

Figure 11: $H\infty$ controller performance with overlapping decomposition for output F_{un}.

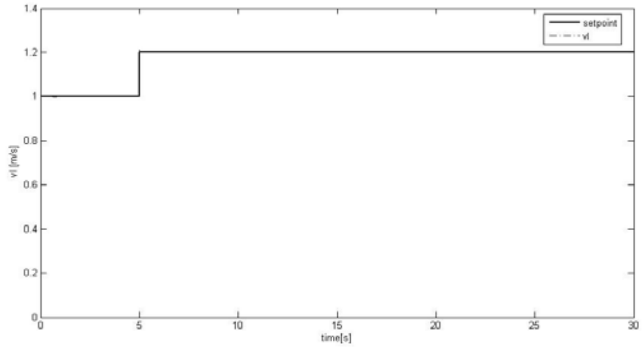

Figure 12: $H\infty$ controller performance with overlapping decomposition for output v_l.

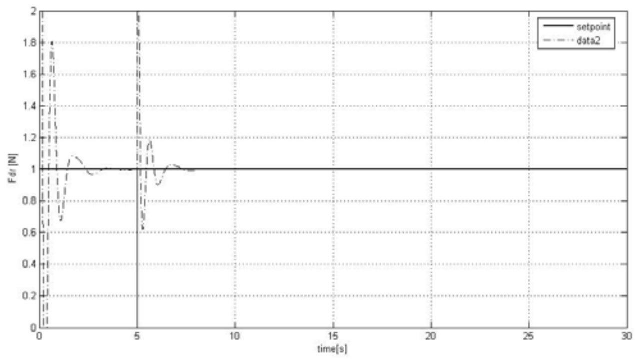

Figure 13: $H\infty$ controller performance with overlapping decomposition for output F_{dr}.

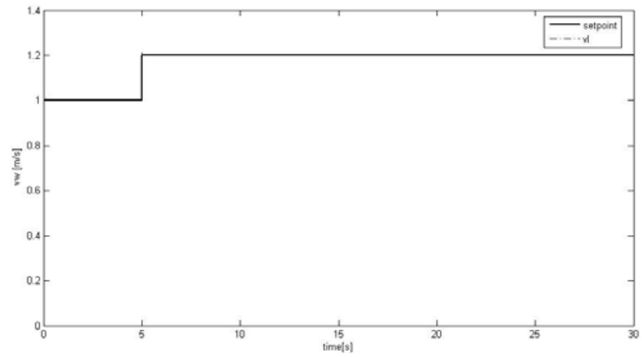

Figure 14: H∞ controller performance with overlapping decomposition for output v_w.

6. COMPARISON AND RESULTS

The application of the overlapping decomposition improve dramatically (Figs 11-15) the controller performances in the same conditions, same weight matrices, and with same quantity of noise added compared to the disjoint decomposition method (Figs 5-8).
All the simulations were carried out using Dormand-Prince method for resolution of integration (ode45 command) and using a variable integration step.

The application of overlapping decomposition method to the experimental system is surely attractive compared to the classical disjoint decomposition based methods. The research is now focusing on the difficulties of applying this method on the experimental system; in fact, due to the feedback sensors noise, there is, at the moment, a limit on the sampling time of the real-time controller.

7. CONCLUSION

A classical web tension system model applied at a new experimental system [11] realized by the authors was tested for designing a speed and tension controller. The simulation results using classical disjoint decomposition method and new overlapped decomposition method were shown demonstrating the significant control improvements of the overlapping method.

Considering the interesting simulation results of this paper, the following step of the research will try to apply the controller equations in the experimental system testing the real experimentally reliability of the controller design proposed. Moreover further research steps could be realized by improving the accuracy of the system model and adding secondary effects neglected in the present treatment.

8. ACKNOWLEDGMENT

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REFERENCES

[1] Sakamoto, T. (1998). PI control of Web tension control system based on overlapping decomposition, *Proceeding of IEEE Nordic Workshop on Power and Industrial Electronics,* 158-163.
[2] Benlatreche, A.; Knittel, D.; Osterbag, E. (in press). Decentralised control strategies for large-scale web handling systems, Control Engineering Practice.
[3] Sakamoto, T (1999). Decentralized controller design of web tension control system in terms of interactions, *Industrial Electronics*, 1466–71.
[4] Sakamoto, T.; Tanaka, S. (1998). Overlapping Decentralized Controller Design for Web Tension Control System, *Trans. IEE Japan*, vol. 118-D, N. 11 1272–78.

[5] Okada, K.; Sakamoto, T. (1998). An Adaptive Fuzzy Control for Web Tension Control System, *Proceeding of IECON*, 11 1762–68.
[6] Koc, H.; Knittel, D.; de Mathelin, M.; Abba, G. (2002). Modeling and robust control for winding systems for elastic webs, *Trans on Control Systems Technology*, vol. 10, N. 2 197–207.
[7] The Math Works (2005). Robust Control Toolbox User Guide.
[8] Green, M.; Limebeer, D. J. N. (1995). Linear robust control, Prenctice-Hall.
[9] Mita, T.; Hirata, M.; Vilas, S. (1995). H_∞ servo-controller design for plants having poles on the $j\omega$ axis, *Proceeding. Of Conf. on Decision & Control*, New Orleans, 2568–73.
[10] Kim J. (2006). Development of hardware simulator and controller for web, *Trans. of ASME*, vol. 128, 378–81.
[11] Giannoccaro N.I.; Oishi K.; Sakamoto T. (2007). An Experimental Web Tension Control System: System Set-Up, Submitted Paper