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ELECTRO HYDRAULIC LOAD-SENSING WITH CLOSED-LOOP CONTROLLED ACTUATORS - THEORETICAL BACKGROUND

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Abstract:
Hydraulic drives with variable supply pressure have become more and more important over recent years. Electro-hydraulic load sensing is one of these energy saving systems in which pressure adapts to real demand, depending on the actuator's load. The actuators in these systems can be controlled as both, open- and closed-loop. The design of the latter is a rather demanding task in terms of control engineering.

This paper presents a load-sensing system on the basis of a hydro motor, treated as a system with multiple variables, at which different control structures and de-coupling strategies are highlighted and tested. The problem of selecting appropriate controllers for the two main variables (velocity and pressure), as well as the two decoupling controllers, is shown by mathematical modelling and simulation.

Key Words: Electro hydraulic system, Load-sensing, Close loop controlled actuator, Modelling of system, Multiple control, Control system design

1. INTRODUCTION

Hydraulic drives with variable and adapting supply pressure gained importance over recent years. Controlling several valve-controlled consumers with a single-pump hydraulic load-sensing has become standard. Adaptation of supply pressure to the maximum load pressure-level leads to a significant decrease in throttle loss in the feeds of hydraulic consumers - actuators. In this way, efficiency can definitely be increased ([1], [2]). Energy savings from these kinds of circuits versus classical supply by constant pumps and a pressure-limit valves, is displayed schematically in Figure 1.

The principle of load sensing (LS) is widely used in mobile hydraulics (e.g. [3], [4]). Real systems are almost exclusively based on hydraulic-mechanical signal transmission which has some disadvantage, such as design restrictions due to signal pipe work, unfavourable dynamic behaviour, and addiction to oscillation [5], [6], [7].

These drawbacks can be avoided by the use of electro-hydraulic control-loops. Besides reduction in weight, this highly-dynamic signal transmission results in flexibility when locating components. A major advantage of digital control is in its free programmability. The performance characteristics of the system are thus shifted to the signal side. This principle of electro hydraulic load-sensing even offers an opportunity in stationary hydraulics to obtain energy-saving circuitry. Application fields include drives with differently-loaded actuators, such as complex production equipment or plastic processing machines.

Generally, in such energy-saving systems actuators can be driven in open- or closed-loop control. In the latter case, the design is more complicated, because the minimum of two control loops are both active and influence each other – this is 'interactive multivariable control'. In this instance, it is of great importance to select those appropriate controllers and strategies associated with good reference and disturbance behaviour.

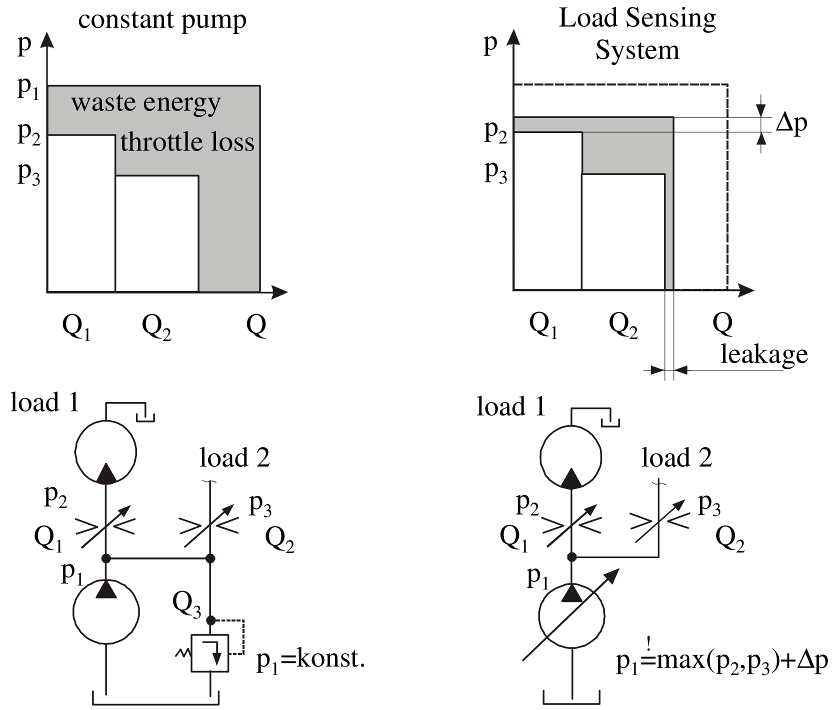

Figure 1: A classical supply with a constant pump (left side) and the principle of Load Sensing System to the right.

In an electro-hydraulic load-sensing system (ELS) with a controlled hydraulic motor as actuator, the two control loops are those of pressure and rotational speed. Any change in an input variable has an influence on the overall system's behaviour due to the nature of the controlled system in such a way that, besides desired changes in the directly-controlled variable, unwanted changes can also be observed on the other controlled variables. The layout of appropriate controllers is a demanding task from the control engineering point of view, especially because of the often desirable high dynamics. The focus of this paper is on this set of problems.

Understanding the behaviour of this kind of system can be accomplished over two stages. Firstly a model-based theoretical investigation and, secondly, followed by experiments. In line with theoretical analysis, a non-linear mathematical model was developed which, in turn, had to be linearised and simplified due to its complexity. Different transfer functions of the ELS-system can be obtained on the basis of this model, forming the base for synthesizing individual controllers.

This paper shows a comprehensive controller design for ELS-systems as multiple variable controls comprising the modelling of such a coupled-system, derivation of equations exactly specifying this double control, a search for the controlled process' transfer functions, necessary for controller selection, and for treating stability problems of such a system.

2. MODELLING OF THE SYSTEM

The principal layout of the ELS-system is shown in Figure 2. It can be see that these is a variable pump, as well as the motor controlled by a valve as an actuator which is mechanically coupled to a loading unit. Thereby the rotational speed $\dot{\varphi}$ would be controlled by the actuating variable u_y using a control valve, and the pressure difference $\Delta p = p_1 - p_L = p_1 - (p_3 - p_4)$ by the actuating signal u_α via the controlled pump. A detailed non-linear mathematical model is developed in [12]. For further investigation it was linearised and simplified due to its complexity.

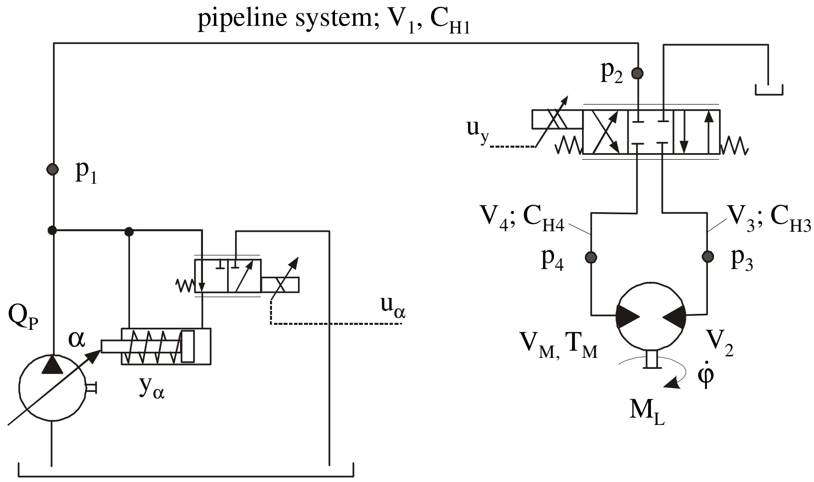

Figure 2: Simplified hydraulic scheme of the ELS-system with a hydraulic motor as a controlled actuator.

The non-linear mathematical model of the controlled system factually provides a precise characterisation of the process but, because of its complexity, is only suitable for making general statements about process dynamics. For the task of controller synthesis has only limited applicability. In order to obtain a model with increased manageability, the non-linearities of the motor control-valve are linearised and any coherence is merged into a new mathematical-physical model. The linearised block diagram corresponds to a 7th order transfer function which is still bulky. Certain simplifications have to be made in order to make the equations more concise and to reduce them to the essentials. When neglecting the pressure-dependency of the equivalent compressibility in rotary drives, the two hydraulic capacities can be set to the same size C_H. The external leakage of the motor is also neglected, and the joint between valve and motor can considered as non-elastic. The acceleration of the motor depends on the working pressure p_L (which in classic load sensing systems is subject to continuous measurement by means of two pressure transducers $p_L = p_3 - p_4$.The pressure difference Δp caused by flow across the motor valve is controlled by the load-sensing controller, and is defined as $\Delta p = p_1 - p_L$.

The signal flow-chart simplified according to the above, is shown in Figure 3.

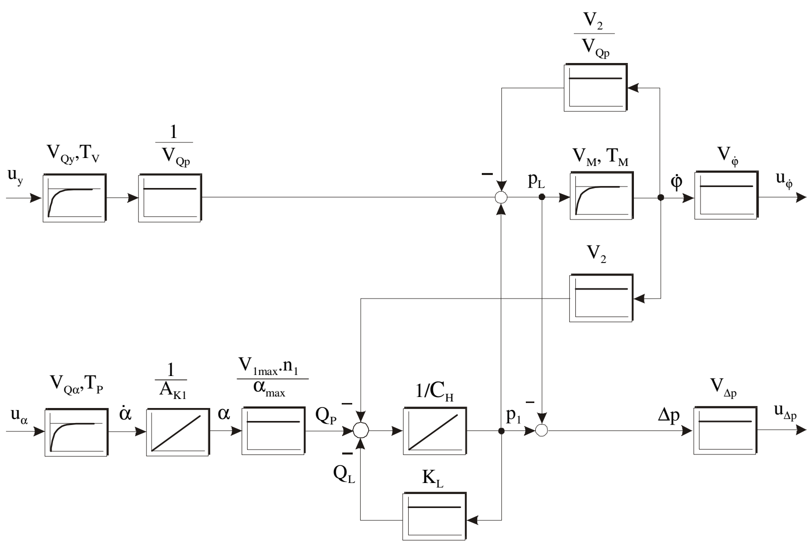

Figure 3: Linearised and simplified signal flow diagram of the ELS process.

The actuating variables u_y at the motor control-valve and u_α at the variable displacement pump, act as input signals of the system. The output variables are the rotational speed $\dot{\varphi}$ of the motor and the pressure difference Δp. The rotational speed is controlled by the motor's control-valve, and the pressure difference of the variable pump. The signal-flow diagram points-out, that both control paths are tightly coupled, hence it is a real multivariable process.

3. THE LOAD-SENSING SYSTEM AS A MULTIVARIABLE PROCESS

As previously mentioned, there are two control loops, one for pressure and one for rotational speed, which interact together. Due to the nature of the process, any change in one input variable leads, besides any wanted change in the corresponding output, to an unwanted variation in the other output(s). It is a coupled multivariable process; whose basics can be gathered from [10].

The dependencies of the output variables on the inputs can be converted into matrix – form, with transfer functions as elements:

$$\underline{X}(s) = \underline{G}(s) \cdot \underline{Y}(s) \tag{1}$$

Where :

$$\underline{G}(s) = \begin{bmatrix} G_{y\dot{\varphi}}(s) & G_{yp}(s) \\ G_{\alpha\dot{\varphi}}(s) & G_{\alpha p}(s) \end{bmatrix} \quad , \ \underline{X} = \begin{bmatrix} u_{\dot{\varphi}} \\ u_{\Delta p} \end{bmatrix} \ \text{, and} \quad \underline{Y} = \begin{bmatrix} u_y \\ u_a \end{bmatrix} \tag{2}$$

stands for the transfer function matrix.

The elements of this matrix ($G_{y\dot{\varphi}}, G_{\alpha p}$ as main transfer functions and $G_{\alpha\dot{\varphi}}, G_{yp}$ as cross-coupling functions) are complex rational functions. For clarity, in those cases of general analysis, and in the case where the total transfer behaviour is of interest, it can be presented as a canonical structure.

In the case of dual-control, an equal amount of in- and outputs results in a square matrix which can be depicted as a so called P-canonical or a V-canonical structure. They differ in the positions of summing and branching junctions. The P-type is characterised by summing junctions on the outputs while, in the V-type the coupling paths are routed from the outputs to the inputs.

A P-canonical structure is preferred because of effecting directional points from actuation towards the controlled-variables (as is usual for physical processes). Basically, both structural types can be converted into each other. Figure 4 shows the ELS-system as a P-canonical structure, together with its transfer functions.

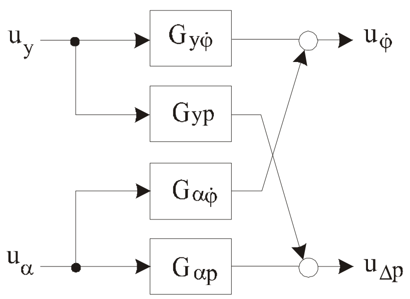

Figure 4: ELS as a dual control system depicted as a P-canonical structure.

If coupling paths have the same sign (both positive or both negative), it is called 'positive feedback' and in the case of different signs 'negative feedback'. The structure presented here has negative feedback: If the pump is slewing-out, the motor's rotational speed is increasing - $G_{\alpha\dot{\varphi}}$ is acting positively. If the control valve of the motor is opened to a greater extent, the differential pressure decreases - G_{yp} therefore has a negative sign. This negative coupling reduces any damping of particular control loops, and this is why differential pressure control tends to oscillation when combined with an engine's speed-control.

4. DETERMINING TRANSFER FUNCTIONS OF THE ELS-CONTROL PATH

If nothing else is known about the coupled system than its structure, particular transfer functions can be obtained by the measurement of step responses [11] or they can be gained by evaluating the relationship between input and output variables, due to physical phenomena acting in with the system.

In the considered example, the relationship between inputs u_y, u_α and outputs $u_{\dot\varphi}$, $u_{\Delta p}$ are set up mathematically. In this way, the transfer functions of both the two main and the coupling paths are evaluated, which together determine the behaviour of the total process.

The main transfer function $G_{y\dot\varphi}$ and one of the couplings $G_{\alpha\dot\varphi}$ can be derived from Figure 3. The result is the first characteristic equation:

$$\left(1+\frac{V_M\cdot V_2}{K_L}+\frac{V_M\cdot V_2}{V_{Qp}}\right)\left[\left(\frac{T_M\cdot C_H}{K_L\left(1+\frac{V_M\cdot V_2}{K_L}+\frac{V_M\cdot V_2}{V_{Qp}}\right)}\right)s^2+\left(\frac{T_M+\frac{C_H}{K_L}+\frac{V_M\cdot V_2\cdot C_H}{V_{Qp}\cdot K_L}}{1+\frac{V_M V_2}{K_L}+\frac{V_M\cdot V_2}{V_{Qp}}}\right)s+1\right]\frac{u_{\dot\varphi}}{V_{\dot\varphi}}=$$

$$\frac{\frac{V_M\cdot V_{Qy}}{V_{Qp}}\left(1+\frac{C_H}{K_L}\cdot s\right)}{1+T_V\cdot s}\cdot u_y+\frac{\frac{V_M\cdot V_P}{K_L}}{s\cdot(1+T_P\cdot s)}\cdot u_\alpha \tag{3}$$

from which the transfer functions $G_{y\dot\varphi}$ and $G_{\alpha\dot\varphi}$ can be extracted (coefficients a_i and b_i are derived according to above equation):

$$G_{y\dot\varphi}=\frac{u_{\dot\varphi}}{u_y}=\frac{b_1\cdot s+b_0}{a_3\cdot s^3+a_2\cdot s^2+a_1\cdot s+a_0} \quad \text{and} \tag{4}$$

$$G_{\alpha\dot\varphi}=\frac{u_{\dot\varphi}}{u_\alpha}=\frac{b_0}{a_4\cdot s^4+a_3\cdot s^3+a_2\cdot s^2+a_1\cdot s+a_0} \tag{5}$$

The two remaining functions, the main transfer function $G_{\alpha p}$ and the coupling function G_{yp}, can be determined using the differential pressure equation (15), by considering the equations for the two pressures (17), (18). This leads to the formula:

$$p_1-p_L=\Delta p=-\frac{\frac{V_{Qy}}{V_{Qp}}}{(1+T_V\cdot s)}\cdot u_y+\frac{V_2}{V_{Qp}}\cdot\dot\varphi \tag{6}$$

into which for $\dot\varphi$, the characteristic equation (24) solved after this variable has to be inserted. This results in the second characteristic equation:

$$\mathcal{K}\cdot u_{\Delta p}=\left(\frac{V_2\cdot V_M\cdot V_{Qy}\cdot V_{\Delta p}}{V_{Qp}\cdot V_{Qp}}\cdot\left(1+\frac{C_H}{K_L}\cdot s\right)-\frac{\cdot V_{Qy}\cdot V_{\Delta p}\cdot\mathcal{K}}{V_{Qp}}\right)\cdot\frac{u_y}{1+T_V\cdot s}+\frac{V_2\cdot V_{\Delta p}}{V_{Qp}}\cdot\frac{\frac{V_M\cdot V_P}{K_L}}{s\cdot(1+T_P\cdot s)}\cdot u_\alpha \quad . \tag{7}$$

K denotes the coefficient of $\dot\varphi$ ($u_{\dot\varphi}$ respectively) in the characteristic equation (3). Using characteristic equation (7), both wanted transfer functions can be obtained (coefficients a_i and b_i are derived respectively – see [12]):

$$G_{yp}=\frac{u_{\Delta p}}{u_y}=-\frac{b_2\cdot s^2+b_1\cdot s+b_0}{a_3\cdot s^3+a_2\cdot s^2+a_1\cdot s+a_0} \quad \text{in} \tag{8}$$

$$G_{\alpha p} = \frac{u_{\Delta p}}{u_{\alpha}} = \frac{b_0}{a_4 \cdot s^4 + a_3 \cdot s^3 + a_2 \cdot s^2 + a_1 \cdot s + a_0} \quad . \tag{9}$$

A more-detailed description of the processes small signal performances can be determined by means of frequency responses by experiment (see [11]) or theoretically as in the present case. If the performance of the entire control-loop has to be evaluated, all four transfer functions have to be set-up (both main paths $G_{y\dot{\varphi}}$, $G_{\alpha p}$ and both couplings $G_{\alpha\dot{\varphi}}$ and; G_{yp} see Figure 4).

The theoretical frequency responses of the different control paths at a medium operating point, are shown in Figure 5.

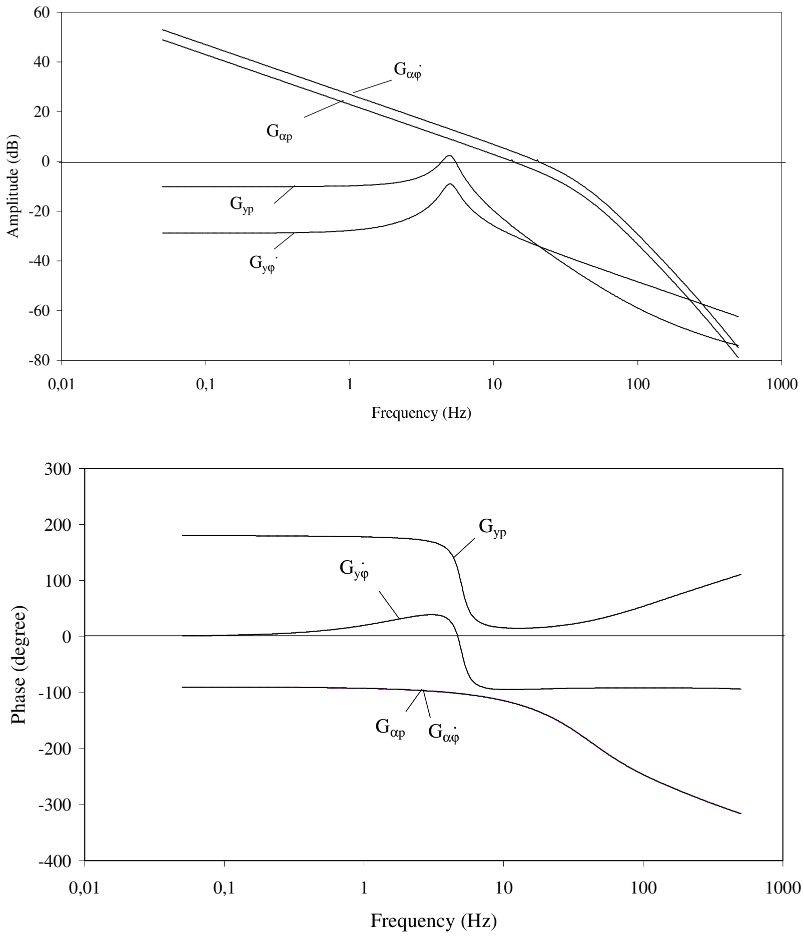

Figure 5: Frequency responses of the individual control paths.

The operating point is the same for all Bode plots: The variable pump is tilted to such an amount that, at a delivery rate of $Q = 75\% \cdot Q_{\max}$ and an control valve which is 100% open, the rotational speed is $\dot{\varphi} = 50\% \cdot \dot{\varphi}_{\max}$. The pressure drop in the supply of the control valve is set to 25 bar by valve piston displacement, and supply pressure is 100 bar.

The frequency response $G_{y\dot{\varphi}}$ describes the way the rotational speed of the motor reacts to valve-piston positional changes. The speed hardly changes when moving the valve piston at low frequencies. There is a constant volume flow because of the constant tilt angle of the pump. Therefore, the motor speed is predefined to a large extent. Displacing the valve piston only results in short acceleration and deceleration phases. The steady-state, motor speed remains constant. At higher frequencies, the frequency response drops again due to time constants representing inertia within the system.

The frequency response G_{yp} exhibits a more or less constant differential pressure on the valve piston. Above a certain frequency, depending on the capacity of the supply-line (which isn't modelled here in more detail), there is a remarkable decay in amplitude gain, which is a result of the control valves' tendency to exceed the natural frequency.

The frequency responses of the coupling path $G_{\alpha\dot{\varphi}}$ and the main path $G_{\alpha p}$ of the differential pressure control exhibit similar characteristics. In both cases the amplitude gain decreases with growing frequency.

All the frequency responses achieved by simulation, on the basis of the simplified model, are shown for a single combination of (concentrated) hydraulic capacity and mass inertia. Therefore, certain effects which can be observed at the measured frequency response couldn't be simulated within such as fluctuations in the delivery rate or resonance occurrences at those frequencies at which the motor oscillates alone with two directly connected oil-columns.

Nevertheless the obtained frequency responses of the particular control paths act as tools when discussing the influences of parameter changes or for defining the controllers. In order to ease controller synthesis, transfer functions can be reduced in such a way, that the simplified functions $G'_{y\dot{\varphi}}$, $G'_{\alpha\dot{\varphi}}$ and G'_{yp}, $G'_{\alpha p}$ (of reduced order) show similar frequency responses to the original ones ($G_{y\dot{\varphi}}$, $G_{\alpha\dot{\varphi}}$ and G_{yp}, $G_{\alpha p}$) within the low frequency region - see equations (10), (1), Figure 6.

$$G'_{y\dot{\varphi}} = \frac{K_{y\dot{\varphi}}}{1+T_{y\dot{\varphi}}s}, \; G'_{\alpha\dot{\varphi}} = \frac{K_{\alpha\dot{\varphi}}}{s} \quad \text{and} \tag{10}$$

$$G'_{yp} = -\frac{K_{yp}}{1+T_{yp}s}, \quad G'_{\alpha p} = \frac{K_{\alpha p}}{s} \tag{11}$$

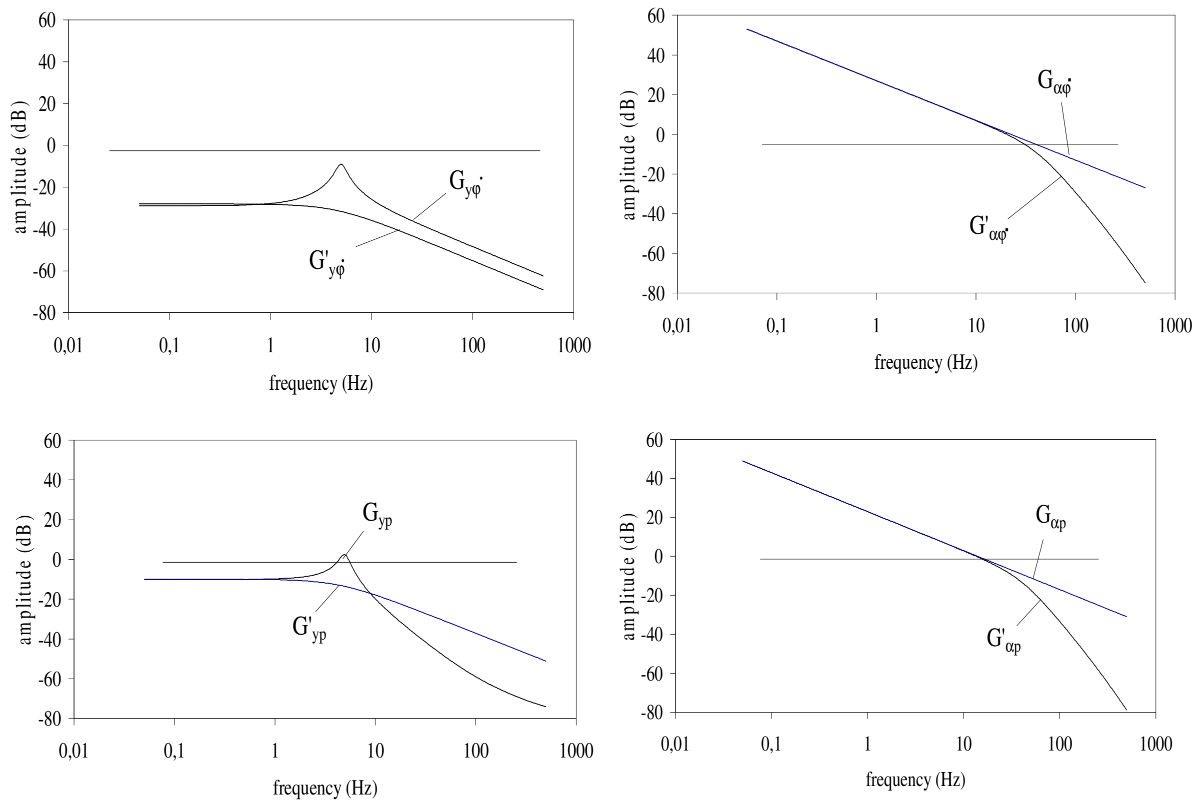

Figure 6: Bode plots of original and simplified transfer functions.

5. CONTROLLER SYNTHESIS AND STABILITY OF THE CONTROL LOOP

As both output variables of the cross-coupled ELS-system treated in this paper should be controlled, the paths have to be decoupled. Because this can't be done within the process itself (because it is determined by physical law), additional control elements have to be added in addition to the usual controllers, thus introducing additional couplings. This, in turn, compensate for the given coupling paths within the process, so that the resulting overall system behaves as if it consisted of isolated individual control loops [10].

This method of serial decoupling (which, in principle, works every time and is applied most frequently) can be summarised in a matrix equation:

$$\underline{X} = \underline{G}\,\underline{R}\,\underline{E} \tag{12}$$

in the case of the treated ELS system this is:

$$\begin{bmatrix} u_{\dot\varphi} \\ u_{\Delta p} \end{bmatrix} = \begin{bmatrix} G_{y\dot\varphi} & G_{\alpha\dot\varphi} \\ G_{yp} & G_{\alpha p} \end{bmatrix} \begin{bmatrix} R_{\dot\varphi\dot\varphi} & R_{p\dot\varphi} \\ R_{\dot\varphi p} & R_{pp} \end{bmatrix} \begin{bmatrix} e_{\dot\varphi} \\ e_{\Delta p} \end{bmatrix} \quad . \tag{13}$$

Where X is the output variable vector, G the matrix of main and coupling path's transfer functions, while E stands for the vector of the Laplace transformed control deviations and R is the matrix of those correcting elements (controllers), which have to be suitably defined. Therein R_{pp} and $R_{\dot\varphi\dot\varphi}$ are the main controllers (the pressure-controller acting on the tilting system of the pump and the rotational speed-controller driving the control valve of the motor) and $R_{\dot\varphi p}$, $R_{p\dot\varphi}$ the so-called decoupling controllers, having task of counteracting the cross-couplings of the process. Figure 9 illustrates the goal of decoupling.

On the left-side of Figure 7, the structure, including decoupling elements, is displayed as it should be realised, in which the main controllers $R_{\dot\varphi\dot\varphi}, R_{pp}$ and the decoupling controllers $R_{p\dot\varphi}, R_{\dot\varphi p}$ are chosen in such a way that the entire structure behaves as the one shown to its right. $G_{Ki}(s)$ are the transfer functions of the correction elements which have to be inserted into the assumed single variable control-loops to let them perform as desired.

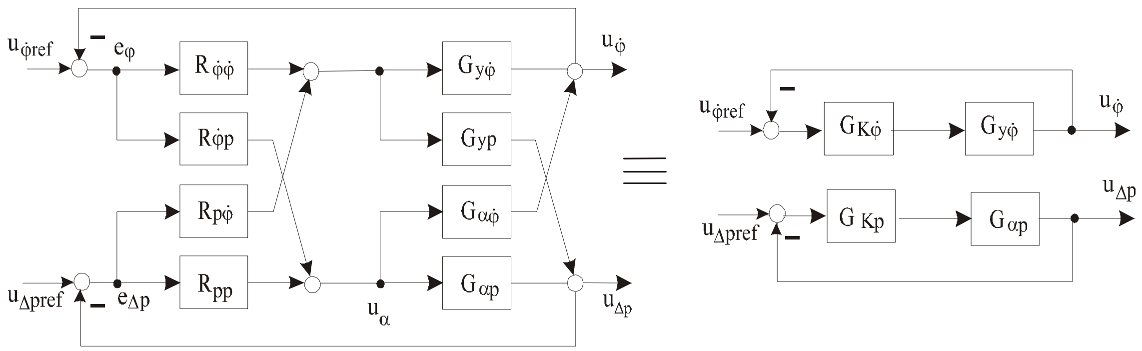

Figure 7: Serial-decoupling of the dual output ELS system.

Each of the both correction elements $G_{K\dot\varphi}$ and G_{Kp} should be determined by means of the well-known methods of control engineering on the basis of transfer functions $G_{y\dot\varphi}$, $G_{\alpha p}$ and their frequency responses (Figure 7), respectively. For the ELS system treated in this paper, correction elements in the subsequent form prove best:

$G_{K\dot\varphi}$ as a PI element: $G_{K\dot\varphi} = \dfrac{K_{R1}(T_{R1}s+1)}{s}$ and (14)

G_{Kp} as a PD element: $G_{Kp}(s) = K_{R2}(1+T_{R2}s)$. (15)

After having defined the correction elements, the individual controllers can be designed:

$$R_{\dot{\varphi}\dot{\varphi}}(s) = F(s)G_{K\dot{\varphi}}(s) \qquad R_{pp}(s) = F(s)G_{Kp}(s) \tag{16}$$

and the decoupling controllers:

$$R_{\dot{\varphi}p}(s) = -\frac{G_{yp}(s)}{G_{\alpha p}(s)}R_{\dot{\varphi}\dot{\varphi}}(s) \qquad R_{p\dot{\varphi}}(s) = -\frac{G_{\alpha\dot{\varphi}}(s)}{G_{y\dot{\varphi}}(s)}R_{pp}(s) \tag{17}$$

with: $F(s) = \left(1-\frac{G_{yp}(s)\,G_{\alpha\dot{\varphi}}(s)}{G_{y\dot{\varphi}}(s)\,G_{\alpha p}(s)}\right)^{-1}$. (18)

The mathematical problem of synthesis is solved if decoupling is done in this way. Due to substantial transfer functions, problems may arise when realising particular controllers according to equations (16) and (17) – the realisation effort may be too much. So, by itself, it reasons to abandon exact decoupling (by original transfer functions) and implement incomplete or partial decoupling instead.

The simplest case, stationary decoupling in which the controllers are designed so that $G_{ij}(s)$ is replaced by $G_{ij}(0)_{s\to 0}$ and (therefore F(s) by $F(0)_{s\to 0}$ as well) doesn't yield reasonable results due to the shape of the transfer functions.

For this reason the so-called 'improved stationary decoupling' [10] would be applied, using the same main controllers as in the case of the steady state procedure:

$$R_{\dot{\varphi}\dot{\varphi}}(s) = F(0)G_{K\dot{\varphi}}(s) \qquad R_{pp}(s) = F(0)G_{Kp}(s) \quad . \tag{19}$$

However, the decoupling controllers are designed more exactly:

$$R_{\dot{\varphi}p}(s) = -F(0)\frac{G_{yp}(s)}{G_{\alpha p}(s)}G_{K\dot{\varphi}}(s) \qquad R_{p\dot{\varphi}}(s) = -F(0)\frac{G_{\alpha\dot{\varphi}}(s)}{G_{y\dot{\varphi}}(s)}G_{Kp}(s) \quad . \tag{20}$$

Thereby factor F(0) remains unchanged, while the remaining transfer functions of equations (20) would be approximated using their simplified counterparts.

$$\frac{G_{yp}(s)}{G_{\alpha p}(s)}G_{K\dot{\varphi}}(s) \Rightarrow \frac{G'_{yp}(s)}{G'_{\alpha p}(s)}G_{K\dot{\varphi}}(s) \tag{21}$$

$$\frac{G_{\alpha\dot{\varphi}}(s)}{G_{y\dot{\varphi}}(s)}G_{Kp}(s) \Rightarrow \frac{G'_{\alpha\dot{\varphi}}(s)}{G'_{y\dot{\varphi}}(s)}G_{Kp}(s) \quad . \tag{22}$$

The principle of approximation lies in the fact of substituting the given frequency response by a simpler one, in such a way as to achieve good compliance with the original in the lower frequency area. Is indeed proves to be improved stationary decoupling.

Controller synthesis according to this method leads to the following controller types:

- for both main controllers

$$R_{\dot{\varphi}\dot{\varphi}}(s) = \frac{K_{P},_{\dot{\varphi}\dot{\varphi}}(T_{I},_{\dot{\varphi}\dot{\varphi}}\cdot s+1)}{s} \quad R_{pp}(s) = \frac{K_{P,pp}(T_{D,pp}\cdot s+1)}{(T'_{D,pp}\cdot s+1)} \tag{23}$$

- and for both decoupling controllers

$$R_{\dot{\varphi}p}(s) = \frac{K_{P,\dot{\varphi}p}(T_{D,\dot{\varphi}p} \cdot s + 1)}{(T'_{D,\dot{\varphi}p} \cdot s + 1)} \quad R_{p\dot{\varphi}}(s) = -\frac{K_{P,p\dot{\varphi}}(T_{I,p\dot{\varphi}} \cdot s + 1)}{s} \tag{24}$$

in which only the occurring control parameters have to be determined using well-known methods of control engineering and with the focus on stable and dynamically-optimal behaviour (control path compensation i.e. $T_{I,\dot{\varphi}\dot{\varphi}} \cong T_{y\dot{\varphi}}$, according to the given phase- and amplitude margins or according to tolerable overshoot of the control variable…).

In addition, it should be noted (especially in case of a LS-system) that the performances of both control variables should be weighted differently.

When the reference or the disturbance variables are changing, motor speed has to be adjusted quickly. There are other requirements to control performance in the pressure control-loop. The steady state differential pressure should be low to minimize throttle-loss in the control valves while, during speed changes, increased pressure differences are even desirable.

Stability, an essential aspect for all controls, has been skipped until now. The reason is, that compared to single variable controls there are no significant new phenomena has appeared.

Briefly condensed, even for multiple variable systems, it is necessary and sufficient that:

- All roots have real negative parts.
- All subsystems of the open-loop process are either stable by themselves or, in the case of instable subsystems, all of them have to be covered by a characteristic equation.

Analogous to single-loop systems, characteristic equation has to be determined for stability appraisal (see [10]):

$$\det\left[\underline{I} + \underline{S}(s)\right] = 0 \tag{25}$$

or in the case of dual variable control

$$\begin{vmatrix} 1+S_{11} & S_{12} \\ S_{21} & 1+S_{22} \end{vmatrix} = 0 \quad \text{or} \quad 1 + S_{11}(s) + S_{22}(s) + S_{11(s)}S_{22}(s) - S_{12}(s)S_{21}(s) = 0 \tag{26}$$

Therein, the transfer functions $S_{ik}(s)$ with $i = k$ are, an example, of the main paths, $S_{ik}(s)$ with $i \neq k$ representing the coupling paths.

As the characteristic equation for multiple control is the same shape when compared to single control, all the known methods can be used.

Having performed total decoupling, the transfer functions $S_{ik}(s)$ with $i \neq k$ are identical to zero and, hence, the characteristic equation, in its general style, is simplified to:

$$\det(\underline{I} + \underline{S}) = \begin{vmatrix} 1+S_{11}(s) & 0 \\ 0 & 1+S_{22} \end{vmatrix} \ . \tag{27}$$

Thus, it disintegrates into characteristic equations of particular single-loop controls:

$$1 + S_{ii}(s) = 0, \quad i = 1,2$$

whereas the total system is stable, if all individual control loops are stable.

In the case of ELS treated in this paper as a dual variable control, determination of matrix S in the characteristic equation (46) can be based on the already-known matrix equation (12).

$$\underline{X} = \underline{G}\,\underline{R}\,\underline{E} \text{ resp. } \underline{X} = \underline{S}\,\underline{E} \quad . \tag{28}$$

Therein

$$\underline{S} = \underline{G}\,\underline{R} = \underline{G}_D\,\underline{G}_K \tag{29}$$

is that matrix, which represents the interrelationship between control error and control variables in a decoupled system. In equation (29), GD is the matrix of the transfer function for the main paths and GK for the correction elements.

$$\underline{G}_D = \begin{bmatrix} G_{y\dot{\varphi}} & 0 \\ 0 & G_{\alpha p} \end{bmatrix}, \qquad \underline{G}_K = \begin{bmatrix} G_{K\dot{\varphi}} & 0 \\ 0 & G_{Kp} \end{bmatrix} \quad . \tag{30}$$

Thus matrix S is determined as:

$$\underline{S} = \begin{bmatrix} G_{y\dot{\varphi}} & 0 \\ 0 & G_{\alpha p} \end{bmatrix} \begin{bmatrix} G_{K\dot{\varphi}} & 0 \\ 0 & G_{Kp} \end{bmatrix} = \begin{bmatrix} G_{y\dot{\varphi}}\,G_{K\dot{\varphi}} & 0 \\ 0 & G_{\alpha p}\,G_{Kp} \end{bmatrix} \quad . \tag{31}$$

Simplified transfer functions $G'_{y\dot{\varphi}}$ and $G'_{\alpha p}$, equations (10) and (11), can be used for the main control paths regarding initial insight into the behavior of the control loops and to provide some early predictions on stability. Applying the correcting elements according to equations (14) and (15), leads to the equation (32):

$$\underline{S} = \begin{bmatrix} \dfrac{K_{y\dot{\varphi}}}{(1+\mathrm{T}_{y\dot{\varphi}}s)} \cdot \dfrac{K_{R1}(1+T_{R1}s)}{s} & 0 \\ 0 & \dfrac{K_{\alpha p} \cdot K_{R2}(1+T_{R2}s)}{s} \end{bmatrix} \quad . \tag{32}$$

After compensation of the control paths dynamic $T_{R1} \cong T_{y\dot{\varphi}}$, the characteristic equation of the ELS with the speed-controlled hydraulic motor simplifies to:

$$\det(\underline{I} + \underline{S}) = \det \begin{vmatrix} 1 + \dfrac{K_{y\dot{\varphi}}\,K_{R1}}{s} & 0 \\ 0 & \dfrac{K_{\alpha p} \cdot K_{R2}(1+T_{R2}s)}{s} \end{vmatrix} = 0 \quad . \tag{33}$$

The original transfer functions should be inserted into equation (31) for an in-depth theoretical stability analysis when determining stability regions against transfer factors or controller parameters, and investigations into the influences of couplings and drive parameters, such as pipeline capacity or motor inertia.

6. CONCLUSION

Due to its structure, the electro-hydraulic load sensing system ELS, with a hydraulic motor as controlled actuator, is a typical example of multi-variable or cross-coupled control systems. Therefore, the system treated in this paper was considered as a classical dual variable control with the emphasis of modelling of such a system, and the choice of a suitable controller.

Firstly, for the purpose of theoretical analysis and further experimental research, a nonlinear mathematical model was designed, which was then linearised and simplified. The model is the base for determining the particular transfer functions of the ELS process and is furthermore used for the design of all control elements (both main and decoupling controllers).

A method of improved stationary decoupling was chosen for this purpose. These determined components form the basis for further investigations into simulation and experiments, as well as for the selection and application of more-sophisticated non-linear controllers or control structures, respectively.

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