# ENERGETIC ANALYSIS OF NONLINEAR METAL CUTTING DYNAMICS UNDER REGENERATIVE CHATTER

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#### Abstract:

Identification of nonlinear metal cutting dynamics with respect to machine tool response is important for predicting the occurrence of chatter vibration onset (limit of stability).

The present paper deals with constitutive modelling of nonlinear metal cutting dynamics with respect to machine tool response. An adopted approach to predict the occurrence of threshold of chatter vibration from the view point of energy transfer in regenerative chatter is to use an analytical formulation which is concerned with the analysis of chatter vibration. Results of this investigation reveal, that machining process develop some energy which is absorbed by the considering structure. The limit of chatter vibration is achieved only when the energy received by the vibrating process is exactly equal to the energy dissipated by the structural damping.

The relationship between the oscillating cutting forces and tool vibration is developed. The present analysis reveals that this relationship exhibits an active hysteresis which is proportional to the square of vibration amplitude, and both transmitted and dissipated energies are represented by parabolic curves.

The present work reveals that the amplitude, at which the work changes sign, corresponds to the onset of stability. In other words, when chatter vibration (instability) occurs its amplitude is greater than the critical value. In this case, there is a phase lead between cutting force and tool motion. Experimental investigation was conducted to demonstrate the efficacy of this energetic approach.

Key Words: Chatter Vibration, Energy Transfer, Damping, Dynamic Stability

#### 1. INTRODUCTION

Chatter is a self excited vibration which is induced and maintained by forces generated by cutting process itself. It is highly detrimental to tool life, dimensional accuracy, surface finish and limits severely the performance of the machine tool.

It is well known that chatter appears as a result of the interaction between the cutting process Tu and the dynamic characteristics of the machine tool structure Tm.

For regenerative chatter, this interaction is represented in the form of a block diagram with negative feedback (Figure 1).

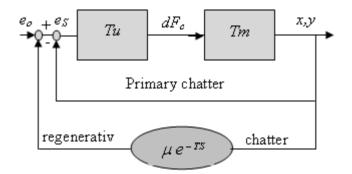


Figure 1: Block diagram of regenerative chatter.

The structure of the machine tool is a continuous system with complex form, and could be described by partial differential equations, with undefined conditions of limits. The vibratory modes are then very difficult to estimate without previous experimental tests.

An experimental study was undertaken at the "Ecole Nationale des Arts et Métiers de Lille France" in order to identify the dynamic cutting process Tu, from the point of view of relationship between oscillating cutting forces and tool vibrations and its velocity. This paper presents some results of this investigation.

# 2. EXPERIMENTAL PROCEDURE

In order to make in evidence the transmitted energy by machining process to the system which allow to chatter vibrations to be sustained or increased, the thrust force and cutting force acting on the tool were sensed with a three dimensional dynamometer. The horizontal tool motion was sensed by a vibrations pickup. The obtained signals were amplified and recorded by a vibration analyser. The experimental setup used in this investigation shown in Figure 2 is as follow.

- **Workpiece:** Inside tapered test pieces were used in order to obtain a continuous increase of the depth of cut (5% of conicity, 135mm of diameter and 50mm of thickness). The workpiece is rigidly linked to the machine-tools.
- **Cutting tool:** Centered cutting tool "sandvik" type (TmaxP CNMG 120412) is used and rigidly fixed on a circular boring bar fixed with an overhang.
- Displacement measurement: Proximity transducer, Bently Névada type.
- **Analysis equipment:** This equipment is constituted by a signal analyzer (HP 3562A) and a memory drawing table (HP 7090A).

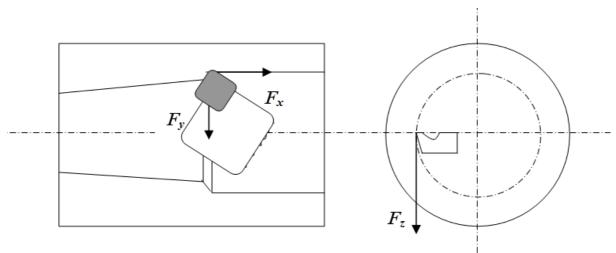


Figure 2: Experimental test piece specimen.

A typical record is shown in Figure 3. As can be seen, the horizontal and vertical tool displacements seem to be a perfect sine wave and are in opposite phase.

As can be seen from Figures 3 and 4, the fluctuating thrust component and cutting component are periodic but not sinusoidal while the horizontal tool displacement resembles a perfect sine wave. It may be observed from this figures that the waveform of both cutting components lead the horizontal tool displacement with a phase angle of about 0.0026 seconds. This phenomenon of phase lead show clearly that the cutting process is nonlinear, and the relationship between the cutting force and the instantaneous tool motion exhibits closed active hysteresis as shown in Figures 5a and 5b. The encountered area is proportional to the transmitted energy. However the stability of the cutting process can be analysed as a function of the area of this hysteresis. As expected, if the area enclosed by the hysteresis is null, the machining process is then stable (Figure 4a and 4b). The variation of the transmitted energy per cycle against the amplitude of tool vibrations was evaluated by integrating the area encountered in one cycle using the relationship between the oscillation in

the dynamic cutting force and the horizontal tool motion ( $W = \oint dF_c dx$ ). As the period of the

oscillations vary, it was more convenient to calculate for each cycle the corresponding period, and evaluate separately the exact values of the energy gain during each cycle of oscillation (Figure 6a and 6b). It is seen from this figures that the transmitted energy posses positive values for high amplitudes of vibrations and negative values for smaller amplitudes indicating that chatter will not occur under these conditions. However it must exist a critical value of the amplitude at which the work changes sign. This amplitude corresponds to the threshold of stability. In other terms our experimental results indicate that when chatter occurs, the amplitude of vibrations must be grater than this critical value.

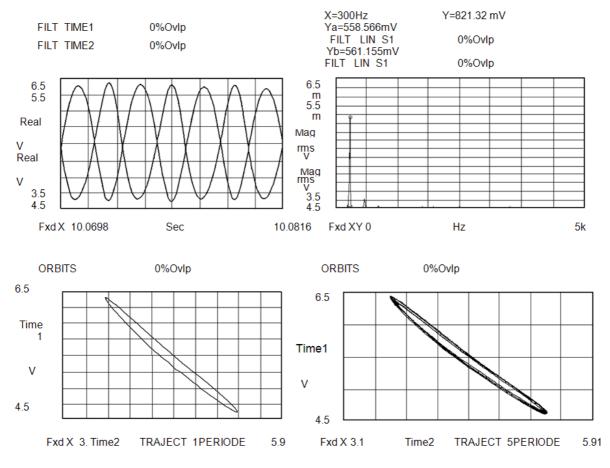


Figure 3: Vibration frequency spectrum produced by the cutting tool.

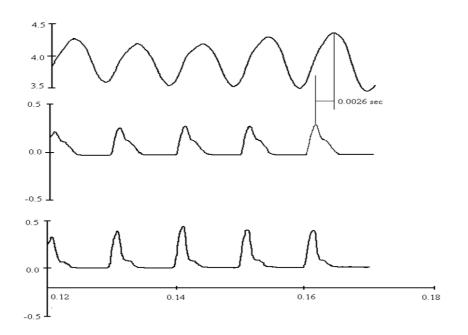


Figure 4: Waveforms of cutting and thrust force fluctuations and horizontal oscillations.

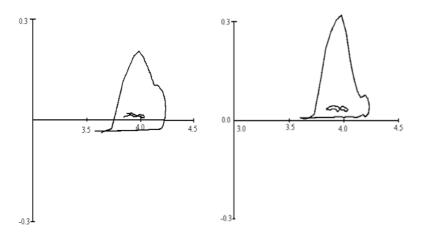


Figure 5: Experimental relationships between cutting and thrust force fluctuations and horizontal oscillations (hysteresis).

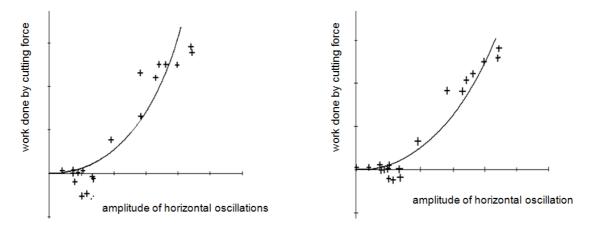


Figure 6: Experimental relationships between work done by cutting and thrust force and amplitude of horizontal oscillations.

### 3. PREVIOUS WORK

A majority of the previous metal cutting dynamic investigation are often controversial, with conflicting data as well as theories. H.E.Merrit [1], J.Tlusty [2] and others have developed a linear relationship between the dynamic cutting force and the undeformed chip thickness. But the experimental results indicate that the chip thickness and forces in cutting are related by a nonlinear relationship. The most interesting observation is that the oscillations in the dynamic cutting forces are periodic but not purely sinusoidal and lead the oscillations in the undeformed chip thickness. It is observed that the phase lead angles is the same for the two components (cutting force and thrust force) of the resultant cutting force and is increased with exciting frequency. Kato and Mariu [3] noticed that the cutting force oscillations lag the imposed oscillation in the undeformed chip thickness and this phase angles are different for the two components. Smith and Tobias [4], covered a wide range of frequencies (0-400cps) and observed the same result as [3] for low frequencies (<100Hz), but for high frequencies (>100Hz) they showed that the cutting force oscillations lead the oscillation in the tool displacement. The same results have been obtained by P.Albrecht [5], Smith [6], Hanna & Tobias [7], Altintas Y [8] and Lago TL, Olsson S, Ha° kansson L, Claesson [9]. These results correspond to our experimental investigation.

This paper presents some results of this experimental investigation. The relationships between the oscillating cutting force and tool vibrations are discussed from the point of view of energy transfer in vibrations.

# **4. ENERGETIC STUDY**

In most physical system, the mechanism of auto-excitation and structural damping coexists and take action separately.

Experimental results indicate that under chatter conditions, the relative displacement of the cutting tool tip with respect to the workpiece usually is not confined to a straight line but that the cutting edge describes an approximately elliptical path (Figure 7).

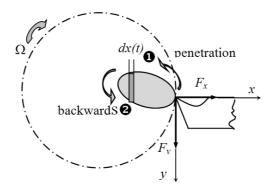


Figure 7: Relative displacement of the cutting tool tip with respect to the workpiece.

1 - In upward position (1), the thrust force dF<sub>t</sub> and tool motion are in opposite direction, the work done by dF<sub>t</sub> on the vibrating tool is then negative, and allow the structure to relaxed. In this case the amplitude of vibration is decreasing.

2 - In downward position<sup>2</sup>, the thrust force dF<sub>t</sub> and tool motion are in the same direction, the work done by dF<sub>t</sub> on the vibrating tool is then positive. The potential energy is then accumulated in the system and the amplitude of vibration increase until dissipation of this energy.

The tool displacement in the direction of the cutting force was negligible and hence the work done in that direction is considered to be zero.

The work done by the thrust force  $dF_t$  on the vibrating tool is given by:

$$W_{t} = \int_{t_{1}}^{t_{1}+\tau} dF_{t}(t)\dot{x}(t)dt$$
(1)

Were  $W_t$  is the work per cycle and  $\omega$  is the frequency of oscillation.

For primary chatter were  $\mu x(t-\tau)=0$ , and de(t)=-x(t). As seen in the experimental results (Figure 3 and 4), the tool displacement is sinusoidal and can be represented by:

$$x = X \cos(\omega t)$$
 then  $\dot{x}(t) = X\omega \sin(\omega t)$  (2)

The resulting skewed waveform of the thrust force can be expressed by

$$dF_t = dF_0 \sin(\omega t + \phi) \tag{3}$$

By substituting Equations 2 and 3 into Equation 1, we obtain

 $t_1 + \tau$ 

$$W_{t} = \int_{t_{1}}^{t_{1}} dF_{0}\omega X \sin(\omega t) \sin(\omega t + \phi) dt$$

$$W_{t} = \int_{t_{1}}^{t_{1}+\tau} dF_{0}\omega X \sin(\omega t) \left\{ \sin(\omega t) \cos\phi + \sin\phi \cos(\omega t) \right\} dt$$

$$W_{t} = F_{0}X\omega \cos\phi \int_{t_{1}}^{t_{1}+\tau} \sin(\omega t)^{2} dt + \int_{t_{1}}^{t_{1}+\tau} \left\{ \sin(\omega t) + \cos(\omega t) \right\} dt$$

$$W_{t} = F_{0}X\omega \cos\phi \int_{t_{1}}^{t_{1}+\tau} \sin(\omega t)^{2} dt = \pi \cdot F_{0}X \cos\phi \qquad (4)$$
Where  $\tau = \frac{2\pi}{\omega}$ .

The static thrust force  $F_0$  is proportional to the cutting width *b* and it is linked to the tool displacement by a constant of proportionality  $\lambda$  called « cutting stiffness ».

$$F_0 = b\lambda.X$$
(5)

Substituting Equations 5 into Equation 4, we obtain

$$W_{t} = \pi . b\lambda . X^{2} \cos\phi$$
(6)

This quantity of energy is absorbed by the structure which became unstable. In order to rend the structure stable, we must bring some energy to it. This dissipation of energy could be done only in the form of damping. The damped force is related to the vibrating motion velocity  $\dot{x}$  by the damping coefficient C.

$$F_{a} = -C\dot{x}$$

$$W_{a} = -\int_{n}^{n+\tau} C\dot{x}^{2}dt = -C\omega^{2}X^{2}\int_{n}^{n+\tau} \{\sin^{2}(\omega t)\}dt$$

$$W_{a} = -C\omega^{2}X^{2}\left[\frac{\tau}{2} - \sin(2\pi)\right] = -C\omega^{2}X^{2}\frac{\tau}{2}$$

$$W_{a} = -\pi C\omega X^{2} \qquad (7)$$

Two cases are present:

- When C > 0, Wa is negative (Wa<0), because F<sub>2</sub> and x' are always in opposite direction. In this case, the mechanical energy is transformed to heat in the dashpot. This energy comes from the vibrating system. The amplitude of tool motion oscillation and the kinetic energy decreased.
- 2. When C < 0, Wa est positive (Wa>0), because  $F_2$  and x' are always in the same direction. In this case, the dashpot gives additional Kinetic energy, and the vibrating amplitude increase.

$$W_{t} = \pi b\lambda X^{2} \cos \phi \qquad (transmitted energy) W_{a} = -\pi c\omega X^{2} \qquad (dissipated energy)$$

Machining is stable only if:

$$\frac{W_{t}}{W_{a}} < 0 \Rightarrow \frac{b\lambda . \cos\phi}{C\omega} < 1$$
(8)

As seen in Figure 8a, transmitted energy Wt and dissipated energy Wa per cycle are both represented by parabolic curves since they are both proportional to the square of amplitude of vibration  $(X^2)$  as indicated by Equation 7.

The final vibration of the system is therefore self-excited or simply damped following which one of this parabola is over the other.

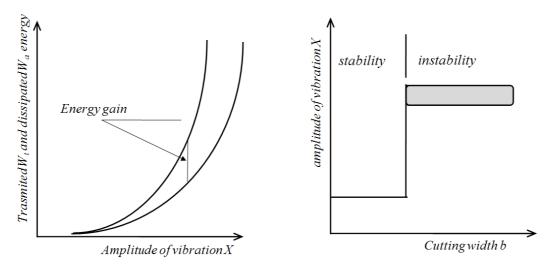


Figure 8*a*: Relation between variation of transmitted and dissipated energy and amplitude of vibration.

Figure 8*b*: Relation between amplitude of vibration *X* and cutting width b.

- If Wt > Wa (energy gain <0), amplitude of vibration is increased, and the vibrating system is susceptible to instability (chatter will occur).
- If Wt < Wa(energy gain >0), there is dissipation of energy, and amplitude of vibration is decreased and the vibrating system becomes stable (chatter will not occur).
- If Wt = Wa(energy gain =0), both curves coincide and the final movement becomes a free vibration without free damping in permanent state. Hence the vibrating system is at the limit of stability.

It is seen from these figures that for a given amplitude Xo, the energy gain per cycle (Wt-Wa) supplies information on the dynamic behaviour of the vibrating system. These mean that there is a critical value of the cutting width at which the chatter occurs suddenly (Figure 3b).

According to the energetic study, the stability of cutting tool is simply characterised by its damping, in other terms how much and how this damping appears in a machining process while varying cutting parameters.

### 5. CONCLUSION

The fundamental cause of primary chatter is ascertained experimentally. It is clearly shown that relationship between the oscillating cutting and thrust force and the tool motion exhibits a closed hysteresis which is active for small amplitudes and passive for large amplitudes of vibrations. The amplitude, at which the work changes sign, corresponds to the onset of chatter. In other words when chatter vibration occurs its amplitude is greater than this critical value. In this case there is a phase lead between cutting force and tool motion.

It has been proved that machining process brought some energy which is absorbed by the structure, and chatter is maintained only when the energy received by the vibrating process is exactly equal to the energy dissipated by the structural damping.

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