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PETRI NET VERSUS QUEUING THEORY FOR EVALUATION OF FLEXIBLE MANUFACTURING SYSTEMS

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Abstract:
Petri net and Queuing theory are used in performance analysis of a Flexible Manufacturing System (FMS). They are used to determine the FMS measures of performance. These measures of performance include optimal Work-In-Process (WIP), lead time, rate of production, machine utilization and number of servers at each station. The measures of performance given by both the tools are then compared.

Petri net model with dual kanban and closed Queuing network are used for analysis and performance evaluation of a FMS. Integrated Network Analyzer (INA) is used for performance evaluation of FMS. CANQ is used for performance evaluation of a FMS by Queuing network. In both the approaches, the throughput is bounded by the utilization of the bottleneck machines.

The comparison shows that for the given number of servers Petri net gives better values of performance measures for FMS. The Petri net optimization gives minimum Work-In-Process corresponding to the maximum production rate. Minimum Work-In-Process leads to minimum lead time.

In the Petri net model, the transportation times are included in the transitions times. In the future research, the proposed model can be extended for inclusion of transportation times for AGVs.

Using the Petri net model, the production manager may design, analyze, evaluate, and even optimize the layout of the production system for minimum Work-In-Process, maximum throughput, and reduced lead time. The determination of the total Work-In-Process, total number of stations in the production system, and the number of servers at each station may be helpful in factory floor management. It may result in more efficiency along with ease of management and supervision.

This paper presents a comparison between Petri net and Queuing network to determine the optimum values for FMS measures of performance.

Key Words: Petri Net, Queuing Theory, INA, CANQ, Optimization, Performance Measures

1. INTRODUCTION

1.1 Petri Net model with dual kanban for flexible manufacturing system

The design and operation of modern industrial systems require modeling and analysis in order to select optimal design alternative and operational policy. Petri nets (PNs) are graphical and mathematical modeling techniques developed as effective modeling tools for concurrent system operations. Since then, they have been extended and applied to a wide variety of systems [8]. PN provides a uniform environment for modeling, formal analysis, and design of discrete systems. A PN is a bipartite directed graph consisting of four primitive elements (i.e., tokens, places, transitions, and arcs) along with rules that govern their operation. Tokens (represented by dots) are conceptual entities used to represent objects moving in an abstract network. Places (represented by circles) show the states of the objects. Places may represent resources such as machines, AGVs, computer code, or parts in a buffer. The existence of a token in a place represents the availability of resource, while no token indicates that the resource is unavailable. Transitions (denoted by bars) represent activities. Places and Transitions together represent conditions and precedence relations in the system's operation. The nodes (places or transitions) are connected by directed arcs. The arcs represent the sequence of operations. Tokens reside in places and travel along arcs. The existence of tokens in the system at a given time describe the state of that net [5].

A transition fires provided there is at least one token in each of the input places of the transition. An elementary circuit represented by the symbol is defined as a directed path that starts from one node (place or transition) back to this node such that no other nodes are repeated [6].

Liveness and boundedness are the two properties that are very important for the application of PNs to controller design and performance analysis. The concept of a strongly connected PN is also very important because in a strongly connected PN, there exists a directed path from every node to every other node in the net. When the PN is a strongly connected event graph the system will reach a periodic regime after a finite time interval [1].

A kanban is a card, labeled container, computer order, or other device that contains information on the exact product or component specifications that are required for the subsequent process step. In a dual kanban, each work cell has input and output inventory buffers. Two kinds of kanbans are employed in a dual kanban system: Production kanban and Movel kanban. Production kanban is used to authorize production at a workstation. Move kanban authorizes movement of a part from the output buffer point of one workstation to the input buffer point of the successor workstation [5].

The initial marking is determined in such a way that there is at least one token in each elementary circuit. This condition guarantees the liveness of the PN. Physically, this corresponds to a deadlock free FMS. The rule of thumb is to put a token in a kanban place instead of a parts place because it does not increase WIP [7].

The total transition time in each elementary circuit $\tau(\gamma)$ is determined as the sum of the transition firing times in that elementary circuit. The total number of tokens M(γ) in each elementary circuit γ is obtained as the sum of the number of tokens in that circuit. The cycle time C(γ) of each elementary circuit is the ratio between the total transition time of the circuit $\tau(\gamma)$ and the total number of tokens M(γ). The utilization U_j of each station j can be calculated as the ratio of the cycle time of the server circuit j and the cycle time of the critical circuit. The lead-time can be determined by using Little's law [6].

1.2 Queuing theory approach for flexible manufacturing system

Queuing theory is considered to be a branch of operations research. It is used as powerful tool in modeling and performance analysis of many complex systems, such as telecommunication systems, call centers, computer networks, flexible manufacturing systems and service systems. A Queuing system consists of inputs, queue and servers as service centers. Generally, it consists of one or more servers for serving customers arriving in some manner and having some service requirements. The customers (the flow of entities) represent users, jobs, transactions or programs. They arrive at the service facility for service, waiting for service if there is a waiting room, and leave the system after being served [3].

Closed queueing networks are often used to model production systems [2, 4]. In closed queueing network models, the numbers of jobs (N) and machines (M) are fixed, with the jobs cycling around and visiting the machines in sequence.

The paper is structured as follows: section 2 presents statement of a case study. Section 3 mentions system optimization using Petri net model. It is followed by system optimization using Queuing theory in section 4. Section 5 provides a comparison between the results of Petri net and Queuing theory. At the end, conclusion is provided.

2. CASE STUDY

Problem statement

An FMS to manufacture the engine of a car will be modeled to evaluate the design and performance. The system will be modeled as a closed CANQ and also as a Petri Net. The estimated demand for the engine blocks about 100,000 per year. The engine block consists of three pieces:

- Top Cover,
- Engine Block,
- Base Cover.

The operations to be performed on workpieces are:

- Loading/unloading- manned loading and unloading system (Station 1),
- Face milling-CNC Horizontal Machinig Center (Station 2),
- Drilling and Tapping- CNC Drilling and Threading Machining Center (Station 3),
- Boring- CNC Horizontal Boring Machine (station 4).

The parts will be manufactured on these four stations as given above. The process planning and routings are given in Table I below for one server in each station.

Table I: Process planning for the given problem.

Process Plan	Station 1	Station 2	Station 3	Station 4
Top Cover	5 Min	5 Min	6 Min	5 Min
Engine Block	8 Min	15 Min	8 Min	20 Min
Base cover	5 Min	5 Min	4 Min	4 Min
Cost per Sever	4,000 US$	200,000 US$	250,000 US$	300,000 US$

The cost of the AGV system including 3 AGVs is 120,000 US$, each additional AGV cost 30,000 US$.
The labor cost per hour is 10 US$.
For the rate of return use 20% and for the amortization of the machine use 10 years.

The workpieces are mounted on pallets at the loading station. An AGV system transports the full and empty pallets between all the stations. Only station 1 is manned. Determine the optimal system.
Part 1: Use a Petri Net model to optimize the system.
Part 2: Use a CANQ to optimize the system.
Part 3: Compare the Petri Net solution with CANQ solution.

3. SYSTEM OPTIMIZATION USING PETRI NET MODEL

Petri net model for part 1 is shown in Figure 1. The places P_{ij2} and P_{ij1} connected by the outer big circle represent parts. Places P_{ij1} represent storage for parts i to be processed on machine T_{ij} with a production card. Places P_{ij2} represent storage for parts i to be processed on machine T_{ij} with a move card authorizing move to station j attached. Places MC_{ij} and PC_{ij} connected by the inner small circle show kanban cards, i-e, Move Card and Production Card respectively. Places C_{ij} at each transition T_{ij} denote control places. Tokens in these places indicate the availability of a server of station j to process part i. Transitions T_{ij} show deterministic processing time of part i on machine j. Transitions K_{ij1} and K_{ij2} are called kanban transitions. Transitions K_{ij1} represent time to detach Move Kanban & attach Production Kanban. Transitions K_{ij2} represent time to detach Production Kanban & attach Move Kanban. Petri net models similar to Figure 1 can also be drawn for part 2 and part 3.

The transitions on each station are linked in the circuits connecting the places C_{ij}. These circuits are referred to as the sequencing circuits. These sequencing circuits will contain a number of tokens equal to the number of identical servers (machines) for each station j. The way places Cij are connected in the sequencing loop determines the sequence in which the parts are processed on this station j. To draw the sequencing circuits, the sequence of operations for parts on machines is assumed as: M1: [P1 P3 P2]; M2: [P1 P3 P2]; M3: [P3 P1 P2]; and M4: [P1 P3 P2]. The sequencing circuit for M1: [P1 P3 P2] is shown in Figure 2. For further study on Petri net, the readers are referred to [1, 6, 7, 9].

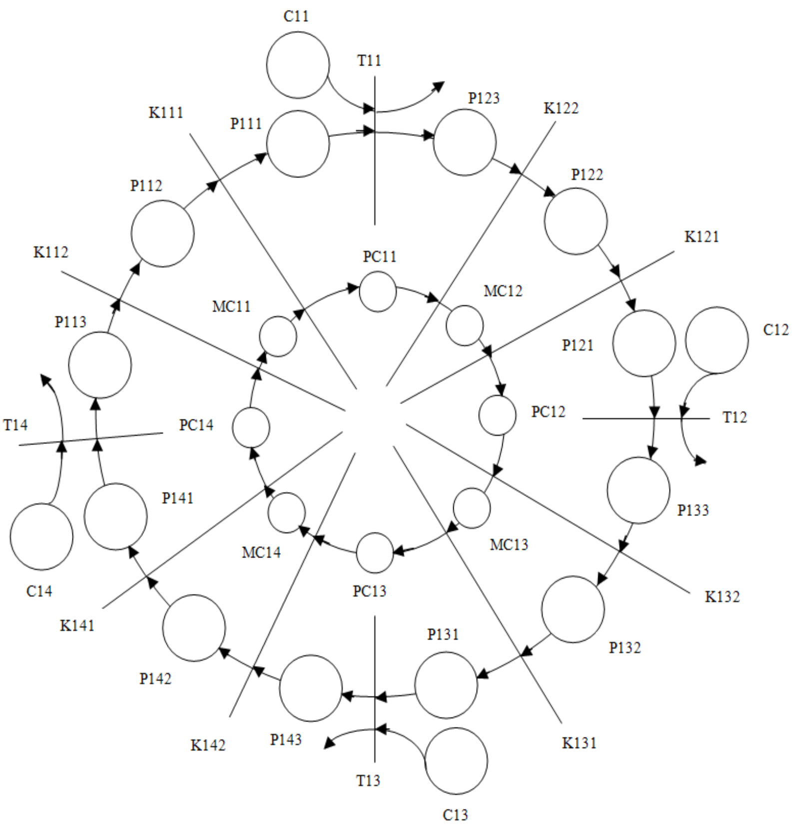

Figure 1: Modeling of part 1 using Petri net with dual kanban.

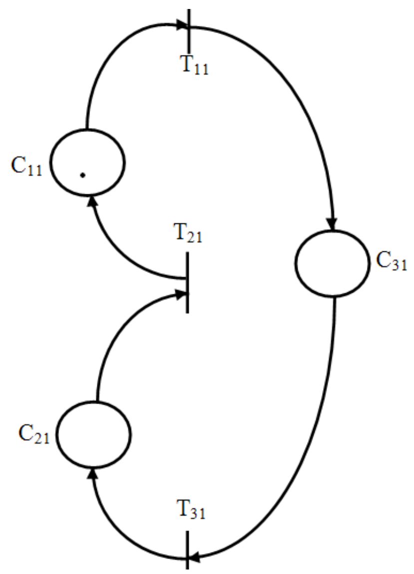

Figure 2: Petri net modeling of sequencing circuit for the given parts on machine M1.

The INA (Integrated Network Analyzer) software by Starke [10] is used to determine the elementary circuits for the PN model. INA is a tool package supporting the analysis of Place/Transition Nets (Petri Nets) and Colored Petri nets. The analysis can be carried out under different transition rules (normal, safe, under capacities), with or without priorities or time restrictions (three types), and under firing of single transitions or maximal sets of concurrently enabled transitions. The analysis part contains a small expert system which infers - from the known ones - further properties of the given net. The structural information computed include: conflicts (static, dynamic) and their structure (e.g. free choice property), deadlocks and traps (deadlock-trap-property), and state machine decomposition and covering. For certain subclasses of nets, these properties can be used to deduce dynamic properties. Invariant analysis can be done by computing generator sets of all place/transition invariants and of all non negative invariants. For bounded nets, the reachability graph can be computed and analysed for liveness, reversability, dynamic conflicts, realisable transition invariants, livelocks etc. Furthermore, minimal paths can be computed, and the nonreachability of a marking can be decided [11].

The optimal distribution of the tokens in parts places and kanban places given by LINGO is shown in Table II. The number of move cards and production cards in kanban places MC_{ij} and PC_{ij} is determined by the following formulas.

The number of move cards in place $MC_{ij} = \left(P_{ij2} + MC_{ij}\right)$ (1)

The number of assembly cards in place $PC_{ij} = \left(P_{ij1} + PC_{ij}\right)$ (2)

Where, the parts places P_{ij2} and P_{ij1} stand for the number of parts attached with move cards and production cards respectively, while, the kanban place MC_{ij} and PC_{ij} show the number of move cards (put in the move card containers) and the number of production cards (put in the production card containers) respectively.

Table II: Number of tokens in each part places and Kanban places after optimal marking by LINGO.

Part Places					
PLACE	**TOKENS**	**PLACE**	**TOKENS**	**PLACE**	**TOKENS**
P113	0	P213	0	P313	0
P112	1	P212	1	P312	1
P111	2	P211	3	P311	0
P123	0	P223	0	P323	0
P122	0	P222	0	P322	0
P121	1	P221	9	P321	0
P133	0	P233	0	P333	0
P132	3	P232	0	P332	0
P131	2	P231	0	P331	7
P143	0	P243	0	P343	0
P142	0	P242	0	P342	0
P141	1	P241	6	P341	1
Kanban Places					
MC11	0	MC21	0	MC31	0
PC11	1	PC21	1	PC31	3
MC12	1	MC22	1	MC32	1
PC12	2	PC22	1	PC32	3
MC13	1	MC23	1	MC33	1
PC13	1	PC23	4	PC33	1
MC14	1	MC24	1	MC34	1
PC14	2	PC24	1	PC34	1

Table II provides the following information about the total WIP, the total number of Move Cards and Production Cards.

Total WIP = 38.
Total No. of Move Cards = 15.
Total No. of Production Cards = 53.

3.1 Optimum values of WIP, production rate, and lead time

Optimum Values of WIP, Production rate, and Lead Time for the system are given below.

$WIP)_{Total}$ = 38
$WIP)_{Part1}$ = 10
$WIP)_{Part2}$ = 19
$WIP)_{Part3}$ = 09

Critical Cycle time = 3.22 Min
Production rate = No. of parts/critical cycle time = 3/3.22 = 0.932 Parts per Min
= **56 parts per Hour**

Lead time for Part1, LT1 = $(WIP)_1$ * critical cycle time = 10 * 3.22 = 32.20 Min
Lead time for Part2, LT2 = $(WIP)_2$ * critical cycle time = 19 * 3.22 = 61.18 Min
Lead time for Part3, LT2 = $(WIP)_3$ * critical cycle time = 09 * 3.22 = 28.98 Min

Lead time for the system, LT_S = $WIP)_{Total}$/ TH = 38/0.931 = **40.78 Min.**

3.2 Optimum values of machine utilization

The optimal values of machine utilization for each station are calculated by dividing cycle-time-of-station by critical-cycle-time.

1. Load/Unload = cycle-time-of-station1/critical-cycle-time =3.00/3.22 = 93%
2. Face Milling = cycle-time-of-station2/critical-cycle-time =3.13/3.22 = 97%
3. Drilling & tapping = cycle-time-of-station3/critical-cycle-time =3.00/3.22 = 93%
4. Boring = cycle-time-of-station4/critical-cycle-time =3.22/3.22 = 100%

3.3 Optimum values of number of servers

Optimum values of number of servers at each station are given below.

1. Load/Unload: 6
2. Face Milling: 8
3. Drilling & tapping: 6
4. Boring: 9

4. SYSTEM OPTIMIZATION USING QUEUING THEORY

The stations in FMS are modeled as the machines M_1, M_2, M_3, and M_4 in Figure 3. In the closed Queuing network, the jobs will proceed from machine M_1 (Loading/unloading) to machine M_2 (Face milling), get “processed” at machine M_2, move on to machine M_3 (Drilling and Tapping), get processed at machine M_3, move on to machine M_4 (Boring), get processed at machine M_4. Jobs are processed on machines with a processing time mentioned in the process plan. Processing time of jobs on machines is represented by μ.

CANQ is used for evaluating the system performance measures. A complete CANQ analysis produces the overall system production rate and the rates of production of each part type in the system. Another important measure of performance that is produced by CANQ is the average time spent in the system. This measure of performance can be used to estimate product lead times. The most important measure of performance for an individual station is its utilization.

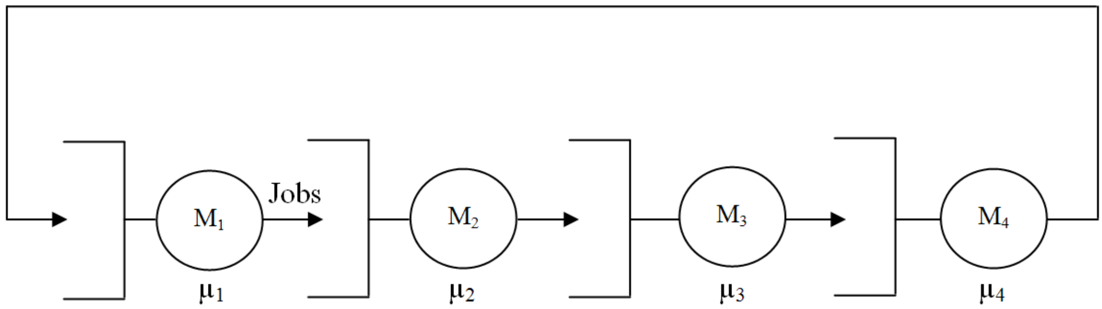

Figure 3: Closed Queuing network for the system.

CANQ gives a minimal system, with WIP = 4 * 3 = 12 and 1 server on each station. When CANQ is run, system 29 is selected for the required capacity, taking into account the average cost per part to be minimum. Final value of WIP is obtained by multiplying the sum of number of servers with 3, i.e., WIP = (6+8+6+9) * 3 = 87. However, this value of WIP = 87 gives more lead time (95.15 minutes) without substantial increase in production rate. It also gives machine utilization above 90%. Therefore, in order to get realistic machine utilization (i.e., machine utilization in between 80% and 90%), and also to get reduced value of the lead time, the WIP was kept on decreasing till the optimal values of performance measures (WIP, lead time, production rate, and average cost/part) given in Table III are obtained.

Table III: FMS performance measures obtained form CANQ.

Performance Measures		**Optimal value**
WIP		42
Lead Time		50.14 Min
Production Rate		50.26 pieces per hour 16.75 parts per hour
Average Cost/Part		18.82 US$
Machine Utilization	Load/Unload	83.7 %
	Face Milling	87.2 %
	Drill. & Tapping	83.7 %
	Boring	89.9 (Bottleneck station)
	Transportation	83.7 %
No. of Servers	Load/Unload	6
	Face Milling	8
	Drill. & Tapping	6
	Boring	9
	Transportation	4

5. COMPARISON BETWEEN RESULTS OF PETRI NET AND QUEUING NETWORK

Table IV shows a comparison of FMS performance measures obtained from Petri net and Queuing network.

Table IV: Comparison of FMS performance measures between Petri net and Queuing network performance measures CANQ Petri Net.

Performance Measures		**CANQ**	**Petri Net**
WIP		42	43
Lead Time		50.14 Min	46.15 Min
Production Rate		52.26 Pieces per hour	56.0 Pieces per hour
Average Cost/Part		18.82 US$	--------------------------
Machine Utilization	Load/Unload	83.7%	93%
	Face Milling	87.2%	97%
	Drilling & Tapping	83.7%	93%
	Boring	89.9% (Bottleneck)	100% (Bottleneck)
	Transportation	83.7%	--------------------------
No. of Servers	Load/Unload	6	6
	Face Milling	8	8
	Drilling & Tapping	6	6
	Boring	9	9
	Transportation	4	--------------------------

6. CONCLUSION

The comparison shows that, for the given number of servers, Petri net gives better values of performance measures for FMS compared to values of performance measures given by CANQ. It is seen from the table of comparison, that in the case of Petri net, lead time is reduced, while production rate is increased, and this is something required in FMS, i.e., more production rate and less lead time for the given number of servers.

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