

# CELL FORMATION DESIGN USING FUZZY RELATIONS BASED ON A SIMILARITY COEFFICIENT

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## Abstract:

Based on literature survey results, the cell formation design is one of the most important operations in cellular manufacturing systems (CMS), since it impacts various factors such as set-up times, batch sizes, queue times, inventory, etc. The purpose of this paper is to develop, apply, and illustrate a new mathematical approach to solve cell formation problems in CMS. Specifically, a procedure using the fuzzy relations concept and the production flow analysis is formulated and utilized to design cell formations; the application of this matrix formulation - based algorithm, employing a similarity coefficient between machines and components (parts), is illustrated through a numerical example. From a binary machine - component matrix, the machine groups and part families are determined. Based on the fuzzy component - cell relation, parts are assigned to appropriate cells. A unique advantage of the proposed method is that the cell formation obtained is very flexible because each part can be assigned to different cells with different degrees of relation. A significant original contribution of this research is the ability to design cell formation involving multiple criteria/attributes where the number of cells to be formed is unknown. This is very useful for production engineers and managers. Suggestions for future research include addressing cell formation in which the system information is a combination of deterministic, probabilistic, and fuzzy values; addressing cell formation through a combination of fuzzy set theory (to handle the imprecise and vague information), neural networks (to capture the pattern), and genetic algorithms (to reach the global optimum).

**Key Words:** Cell Formation, Fuzzy Relations, Similarity Coefficient

## 1. INTRODUCTION

The increased competition faced by manufacturers has made them very receptive to ideas leading to improved productivity. Various manufacturing systems such as just in time (JIT), flexible manufacturing systems (FMS), and cellular manufacturing systems (CMS), have been developed to achieve this purpose. The JIT, also called the pull system, improves productivity by controlling inventory in raw materials, work-in-process, and finished products. The FMS are integrated systems in which productivity improvement is obtained through automated machines with computerized systems. Productivity improvement in CMS is achieved by creating cells or groups such that a part can be made in a certain cell only. This eliminates travel to other cells, thus reducing setup time.

The design of CMS involves many different tasks. A major task is the design of cell formation, which includes the identification of part families and the formation of associated machine cells, such that all parts and machines in each cell have high similarity. The similarity between parts can be based on geometry, function, material, process, tools and operators. The main idea in cell formation design is that if parts having high similarity are put in a cell, the setup time of the machines in the cell is reduced. Consequently, many benefits can be achieved such as reducing material handling time and work-in-process inventory as

well as increasing machine and tool utilization, and improving operator's responsibility. Therefore, the objective of many approaches to design cell formation has been to maximize similarity coefficient in a cell. This similarity can be between parts (part similarity), between machines (machine similarity), and between parts and machines (part-machine similarity).

The cell formation design can be done by two techniques, part-coding based techniques and matrix formulation based techniques. The main difference between part-coding based techniques and matrix formulation based techniques, lies on the source of information used. The former uses part codes which represent part attributes, while the latter uses the part production flow information. Matrix formulation based techniques are divided into two groups: (1) array-based techniques and (2) similarity coefficient based techniques. The main principle of array-based techniques is to arrange the elements of a machine-component matrix such that a block diagonal matrix is formed, while similarity coefficient based techniques use part similarity, machine similarity, or part-machine similarity to form cell formation.

Similarity coefficient plays an important role in cell formation because the basic aim of cell formation is to group similar parts into a part family. The similarity between parts can be based on design features, functionalities, and processing requirements. According to Seifoddini and Djassemi (1995), among all approaches to cell formation design, the similarity coefficient method is most effective because of its capability in forming machine cells in the presence of exceptional parts and its flexibility in incorporating production volume, sequence of operations and operational times into the machine cell formation process.

## **2. PROBLEM DESCRIPTION**

The basic problem in the design of cell formation is to design part families and corresponding machine groups such that parts and machines in a cell have high similarity. Similarity between parts can be based on part geometry or part process flow. Cell formation design techniques which are based on production flow analysis use information of a part-machine relationship which is presented in a machine-component matrix. There are three types of machine-component matrices. They are a binary machine-component matrix, a weighted machine-component matrix, and a non-binary machine-component matrix. A binary machine-component matrix shows machines needed to process a part. This matrix does not present information of process sequence of the part. A weighted machine-component matrix shows both the information of machines needed to process the part and the degree of relation between parts and machines. The degree of relation between parts and machines can be machine loading, production volume, or processing time of parts on machines, etc. Information of process sequences of a part can be obtained from a non-binary machine component matrix.

Similarity coefficient between machines, between parts, and commonality scores between parts and machines can be calculated from a machine-component matrix. Many formulations to calculate similarity coefficient have been proposed for different attributes (features). The similarity between machines can be calculated based on the number of parts requiring the machines, production volume of the part, processing time of the part, etc. Similarity coefficient between machines may be different for different calculation criteria (features). Consequently, different cell formations may be obtained for different grouping criteria. A cell formation may yield better performance for one criterion but worse performance for other criteria.

## **3. LITERATURE SURVEY**

Various formulations for calculating similarity coefficient and similarity coefficient-based techniques have appeared in literature. Hyung et al. (1994) proposed a formulation to calculate the similarity between fuzzy sets, and similarity between elements in a fuzzy set. Chen (1995) suggested two similarity formulations for vague data – one with the same

weights, and the other with different weights on the elements of sets. Pappis and Karacapilidis (1993) proposed three formulations to calculate the similarity between two fuzzy sets – first based on the operations of union and intersection, second based on the maximum difference, and third based on the difference and the sum of grades of membership; Wang et al. (1995) made corrections to the above formulations.

Dimopoulos and Mort (2000) employed a genetic programming algorithm for the evolution of new similarity coefficients and tested them against the Jaccard's similarity coefficient. Yin and Yasuda (2005) conducted a comparative study of various similarity coefficients in the cell formation problem and found Jaccard to be the most stable similarity coefficient. They (2006) also developed a taxonomy to clarify the definition and usage of various similarity coefficients in designing cellular manufacturing systems.

Islam and Sarker (2000) utilized a heuristic procedure based on a similarity coefficient using a set of important characteristic properties for machine-parts grouping in cellular manufacturing. Garbie et al. (2008) also devised a heuristic approach to machine cell formation based on a new similarity coefficient incorporating properties such as alternative routings, processing time, machine capacity and capability, production volume, product demand, and number of operations on a machine.

Hwang and Oh (2003) formulated a p-median model in group technology using a modified similarity coefficient. Lei and Wu (2005) proposed a hybrid Tabu search algorithm by combining the similarity coefficient and the meta-heuristics to solve cell formation in group technology. Lozano et al. (2006) investigated the cell formation problem using symmetric sequence based similarity coefficients between each pair of machines, and neural networks.

Adenso-Diaz et al. (2005) grouped parts into families, and machines into cells, in two phases – first by computing weighted similarity coefficients and clustering machines through a Tabu search algorithm, and then assigning part types to the previously formed groups through a linear minimum cost network flow model. Hachicha et al. (2006) also solved the cell formation problem in cellular manufacturing in two phases – first by using a correlation matrix as a similarity coefficient matrix, and then applying a principal component analysis as a cluster analysis to make simultaneously machine groups and part families.

The above papers did not consider the fuzziness of data. In our paper, we propose an approach for cell formation design using fuzzy relations based on a similarity coefficient.

#### **4. PROPOSED APPROACH**

The concepts of fuzzy relations (Zadeh, 1965) and aggregation operations on fuzzy sets (Klir and Yuan, 1995) form the basis for our cell formation design approach.

Let  $A [a_{ij}]$  be a binary machine-component matrix, where  $a_{ij} = 1$  if machine  $i$  is needed by part  $j$ , and 0, otherwise, as shown in Figure 1. Similarity between machines  $i_1$  and  $i_2$  can be calculated using various formulations. For instance, using Jaccard's similarity coefficient (Sokal and Sneath, 1963), similarity between machines  $i_1$  and  $i_2$  can be formulated

$$S_{i_1, i_2} = \frac{N_{i_1, i_2}}{N_{i_1} + N_{i_2} + N_{i_1, i_2}} \quad (1)$$

where:

$N_{i_1, i_2}$  = number of parts requiring both machines  $i_1$  and  $i_2$

$N_{i_1}$  = number of parts requiring machine  $i_1$  only

$N_{i_2}$  = number of parts requiring machine  $i_2$  only

Formulation in Equation 1 holds the following properties:

- (i)  $0 \leq S_{i_1, i_2} \leq 1, \forall (i_1, i_2) \in i$
- (ii)  $S_{i_1, i_1} = 1, \forall i_1 \in i$
- (iii)  $S_{i_1, i_2} = S_{i_2, i_1}, \forall (i_1, i_2) \in i$

		<i>Component No.</i>						
$A =$	1	2	3	4	5	6	1	
	1	1	0	0	1	0	2	
	1	0	1	1	0	0	3	
	0	1	1	1	0	1	4	
	1	1	0	0	1	1	5	
	1	0	0	1	1	1	5	
							<i>Machine No.</i>	

Figure 1: A binary machine-component matrix.

The similarity between machines can also be interpreted as a degree of relation between machines. Let  $R(M, M)$  be a relation function between machines. If  $R(M, M) = S(M, M)$ , then relation between machines  $i$  and  $j$ , where  $s_{ij} = \alpha_{ij}$ , can be stated as the binary fuzzy relation given by

$$R(M, M) = \frac{\alpha_{11}}{M_1 M_1} + \frac{\alpha_{12}}{M_1 M_2} + \dots + \frac{\alpha_{mm}}{M_m M_m} \tag{2}$$

$R$  is reflexive, symmetric, and non-transitive. To determine the transitive closure of  $R(M, M)$  denoted by  $R_T(M, M)$ , the following algorithm can be used.

Algorithm to determine transitive closure: (3)

- Step 1:  $R' = R \cup (R \circ R)$ ; if the max-min composition and the max operator for set union are used, then  $R_T$  is called the transitive max-min closure.
- Step 2: if  $R' \neq R$ , make  $R = R'$  and go to Step 1.
- Step 3: Stop:  $R' = R_T$ .

Based on transitive closure,  $R_T(M, M)$  machine groups can be obtained for different relation levels ( $\alpha$ ). Machines  $i$  and  $j$  are grouped into a cell with  $\alpha$  – level, if  $R_T(M_i, M_j) \geq \alpha$ .

In order to determine the level of relation between a part and a machine, a normalized machine-component matrix is introduced and defined as  $A^* = [a^*_{ij}]$ , where:

$$a^*_{ij} = \frac{a_{ij}}{\sum_{i=1}^M a_{ij}} \tag{4}$$

A normalized machine-component matrix for Figure 1 can be calculated as shown in Figure 2.

		<i>Component No.</i>						
		1	2	3	4	5	6	
$A^* =$		0.25	0.33	0	0	0.33	0	1
		0.25	0	0.5	0.33	0	0	2
		0	0.33	0.5	0.33	0	0.33	3
		0.25	0.33	0	0	0.33	0.33	4
		0.25	0	0	0.33	0.33	0.33	5

*Machine No.*

Figure 2: A normalized machine-component matrix.

Part  $j$  then can be stated as an  $M$ -dimensional vector, where  $M$  is the number of machines, denoted by  $P_j = [a^*_{1j}, a^*_{2j}, \dots, a^*_{Mj}]^T$ . For instance, parts 1 and 2 from Figure 2, can be stated as  $P_1 = [0.25, 0.25, 0, 0.25, 0.25]^T$  and  $P_2 = [0.33, 0, 0.33, 0.33, 0]^T$ . From this example, we can see that both parts 1 and 2 need machine 1 but their levels of relation are different.

Suppose for level of desirability  $\alpha_i$ , we get three machine groups  $C_1, C_2,$  and  $C_3$  respectively. Then the fuzzy relation between a part and a cell can be formulated as

$$R(P, C) = \frac{\beta_{11}}{P_1 C_1} + \frac{\beta_{12}}{P_1 C_2} + \frac{\beta_{13}}{P_1 C_3} + \dots + \frac{\beta_{N1}}{P_N C_1} + \frac{\beta_{N2}}{P_N C_2} + \frac{\beta_{N3}}{P_N C_3} \quad (5)$$

where:

$$\beta_{jk} = \sum_{i \in C_k} a^*_{ij} \quad (6)$$

$i$  = index of machines ( $i = 1, 2, \dots, M$ )

$j$  = index of parts ( $j = 1, 2, \dots, N$ )

$k$  = index of cells ( $k = 1, 2, \dots, K$ )

Based on this relation, we can assign a part into a cell which has the highest level of relation.

If  $\max(\beta_{jk}) < 1$ , then part  $j$  is an exceptional element.

$k \in K$

#### 4.1 Algorithm

To design cell formation based on a binary machine-component matrix, an algorithm is proposed as follows:

- Stage 1: Machine Groups Design
  - Step 1: From a binary machine-component matrix, calculate similarity between machines, namely machine similarity matrix,  $S$ .
  - Step 2: Determine a transitive max-min closure of  $S$ , denoted by  $S_T$  by using the algorithm (Equation 3).
  - Step 3: Determine machine groups for different  $\alpha$  – levels.
- Stage 2: Part Families Design
  - Step 4: Determine a normalized machine-component matrix, called  $A^*$ .



$$S = \begin{array}{c} \begin{array}{c} \text{Machine No.} \\ 1 \quad 2 \quad 3 \quad 4 \quad 5 \\ \hline 1 \quad 0.5 \quad 0.25 \quad 0.5 \quad 0.25 \\ 0.5 \quad 1 \quad 0.25 \quad 0.5 \quad 0.25 \\ 0.25 \quad 0.25 \quad 1 \quad 0.25 \quad 0.5 \\ 0.5 \quad 0.5 \quad 0.25 \quad 1 \quad 0.25 \\ 0.25 \quad 0.25 \quad 0.5 \quad 0.25 \quad 1 \end{array} \end{array} \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \text{Machine No.}$$

– Step 3: Machine groups then can be designed for different  $\alpha$  - levels.

\*For  $\alpha = 0.25$ , we obtain

$$S_T(\alpha = 0.25) = \begin{array}{c} \begin{array}{c} \text{Machine No.} \\ 1 \quad 2 \quad 3 \quad 4 \quad 5 \\ \hline 1 \quad 1 \quad 1 \quad 1 \quad 1 \\ 1 \quad 1 \quad 1 \quad 1 \quad 1 \\ 1 \quad 1 \quad 1 \quad 1 \quad 1 \\ 1 \quad 1 \quad 1 \quad 1 \quad 1 \\ 1 \quad 1 \quad 1 \quad 1 \quad 1 \end{array} \end{array} \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \text{Machine No.}$$

\*For  $\alpha = 0.5$ , we obtain

$$S_T(\alpha = 0.5) = \begin{array}{c} \begin{array}{c} \text{Machine No.} \\ 1 \quad 2 \quad 3 \quad 4 \quad 5 \\ \hline 1 \quad 1 \quad 0 \quad 1 \quad 0 \\ 1 \quad 1 \quad 0 \quad 1 \quad 0 \\ 0 \quad 0 \quad 1 \quad 0 \quad 1 \\ 1 \quad 1 \quad 0 \quad 1 \quad 0 \\ 0 \quad 0 \quad 1 \quad 0 \quad 1 \end{array} \end{array} \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \text{Machine No.}$$

or it can also be stated

	1	2	4
1	1	1	1
2	1	1	1
4	1	1	1

	3	5
3	1	1
5	1	1



$$\begin{aligned}
 R(P, C) &= \frac{(a^*_{11} + a^*_{21} + a^*_{41})}{P_1 C_1 (M_1, M_2, M_4)} + \frac{(a^*_{31} + a^*_{51})}{P_1 C_2 (M_3, M_5)} + \frac{(a^*_{12} + a^*_{22} + a^*_{42})}{P_2 C_1 (M_1, M_2, M_4)} + \\
 &\frac{(a^*_{32} + a^*_{52})}{P_2 C_2 (M_3, M_5)} + \frac{(a^*_{13} + a^*_{23} + a^*_{43})}{P_3 C_1 (M_1, M_2, M_4)} + \frac{(a^*_{33} + a^*_{53})}{P_3 C_2 (M_3, M_5)} + \\
 &\frac{(a^*_{14} + a^*_{24} + a^*_{44})}{P_4 C_1 (M_1, M_2, M_4)} + \frac{(a^*_{34} + a^*_{54})}{P_4 C_2 (M_3, M_5)} + \frac{(a^*_{15} + a^*_{25} + a^*_{45})}{P_5 C_1 (M_1, M_2, M_4)} + \\
 &\frac{(a^*_{35} + a^*_{55})}{P_5 C_2 (M_3, M_5)} + \frac{(a^*_{16} + a^*_{26} + a^*_{46})}{P_6 C_1 (M_1, M_2, M_4)} + \frac{(a^*_{36} + a^*_{56})}{P_6 C_2 (M_3, M_5)} \\
 &= \frac{0}{P_1 C_1} + \frac{1}{P_1 C_2} + \frac{1}{P_2 C_1} + \frac{0}{P_2 C_2} + \frac{0.75}{P_3 C_1} + \frac{0.25}{P_3 C_2} + \\
 &\frac{0}{P_4 C_1} + \frac{1.0}{P_4 C_2} + \frac{1}{P_5 C_1} + \frac{0}{P_5 C_2} + \frac{0.5}{P_6 C_1} + \frac{0.5}{P_6 C_2}
 \end{aligned}$$

$$\begin{aligned}
 R(P) &= \frac{\max(0,1)}{P_1} + \frac{\max(1,0)}{P_2} + \frac{\max(0.75, 0.25)}{P_3} + \frac{\max(0, 1)}{P_4} + \frac{\max(1, 0)}{P_5} + \frac{\max(0.5, 0.5)}{P_6} \\
 &= \frac{1(= C_2)}{P_1} + \frac{1(= C_1)}{P_2} + \frac{0.75(= C_1)}{P_3} + \frac{1(= C_2)}{P_4} + \frac{1(= C_1)}{P_5} + \frac{0.5(= C_1 \text{ or } C_2)}{P_6}
 \end{aligned}$$

- Step 6: Based on the fuzzy component-cell relation, we can assign parts 1 and 4 into cell 1; parts 1, 3, and 5 into cell 1; and part 6 into either cell 1 or cell 2. It also shows that parts 3 and 6 are exceptional parts because the relation values are less than 1. Thus the cell formation is obtained as follows:

Cell No. 1 (C <sub>1</sub> )			Cell No. 2 (C <sub>2</sub> )		
Machine (M)	Parts (P)	μ(P,C <sub>1</sub> )	Machine (M)	Parts (P)	μ(P,C <sub>2</sub> )
1 2 4	2	1.00	3 5 6	1	1.00
	5	1.00		4	1.00
	3	0.75		6	0.50
	6	0.50		3	0.25
	1	0.00		2	0.00
	4	0.00		5	0.00

Machine Component No.

No.	2	5	3	1	4	6
1	1	1				
2	1		1			
4	1	1	1			1
3			1	1	1	
5				1	1	1

## 5. DISCUSSION

This paper considered the problem of cell formation design where the number of cells to be formed is unknown. An approach based on the fuzzy set concept is proposed and applied to

a numerical example. From a binary machine-component matrix, the machine groups and part families are determined. Based on the fuzzy component-cell relation, parts are assigned to appropriate cells. The cell formation obtained is the same as the results of applying the Rank Order Clustering ROC (King, 1980) and Rank Order Clustering Extension ROC-2 (King and Nakornchai, 1982) algorithms. By using fuzzy component-cell relation, we can see exceptional parts and the relation level of a part in a cell. Advantage of the proposed approach is that the cell formation obtained is very flexible because each part can be assigned to different cells with different degrees of relation.

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