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Simultaneous determination of production and shipment decisions for a multi-product inventory system with a rework process

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A B S T R A C T

In a turbulent and highly competitive business environment, management always pursues options to reduce overall operating costs. The vendor-buyer integrated system has recently drawn attention from managers, because it can benefit both parties of the supply chain and it is suitable to be applied to a so-called intra-supply chain system within the present-day globalized enterprise. This study attempts to simultaneously determine production and shipment decisions for a multi-product vendor-buyer integrated inventory system with a rework process, wherein multiple products are fabricated in sequence by a single machine under a rotation cycle time policy. All defective items produced in regular production are assumed repairable, and are reworked right after the regular production ends. Finished goods of each product are transported to sales offices/customers after rework. A multi-delivery policy is applied, wherein a fixed quantity of *n* instalments of the finished batch is delivered at fixed intervals during the delivery timeframe. Mathematical modelling and optimization techniques are used to help simultaneously determine the optimal production and shipment decisions that minimize the expected overall system costs. A numerical example is used to show the applicability of our research results.

A R T I C L E I N F O

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1. Introduction

This study attempts to simultaneously determine production and shipment decisions for a multi-product vendor-buyer integrated inventory system with a rework process. Higher machine utilization and minimum total production-inventory-delivery costs are two important operating goals, among others, for present-day producers in supply-chain environments. In order to achieve higher machine utilization, the management of manufacturing firms often proposes making multiple products in sequence on a single machine [1-3]. Gaalman [4] proposed a multi-item production smoothing model using an aggregation technique that uses structural properties of the inventory-production model. Leachman and Gascon [5] studied a multi-product single-machine manufacturing system where demand is stochastic and time-varying. Heuristic scheduling policy is proposed and it can integrate feedback control according to inventory levels of economic production cycles. The policy can be applied to decision making involving the type and quantity of items to be produced during the next time period. Zipkin [6] studied the performance of a multi-item production-inventory system. Two alternative policies representing different modes of collecting and utilizing information are considered and compared. He derived a closed-form measure of performance for one of them, namely the first-come-first-served (FCFS) policy, and proposed a comparable approximation for the other, namely the longest-queue policy. These results were illustrated and tested via simulations, and used to address several basic managerial issues. Muramatsu et al. [7] studied a multi-product, multi-process dynamic lot-size scheduling problem with setup time, lot sizing, lot sequencing, and dispatching features. A near-optimal solution method along with computational procedure was presented for the proposed problem. They also solved sub problems with known values of Lagrange multiplier. Jodlbauer and Reitner [8] explored a stochastic multi-product, make-to-order fabrication system under common cycle policy. The effects of demand, cycle time, safety stock, processing time and setup time on service levels, and total system cost were determined. Algorithms for calculating the cycle time that leads to maximum service levels at constant safety stocks was introduced. Additional studies [9-15] are related to different aspects of the multi-item production management and optimization issues.

Unlike conventional economic production quantity (EPQ) model [16] that assumes a continuous inventory issuing policy, multiple or periodic product delivery policies are often used in real vendor-buyer integrated production-delivery systems. Hahm and Yano [17] derived optimal frequency of production and delivery for a single-product vendor-buyer integrated inventory model, with the objective of minimizing the long-run average cost per unit time. Production setup costs and inventory holding costs for both vendor and buyer, and transportation costs are considered. They proved that in an optimal solution, the ratio between production interval and delivery interval must be an integer. Eben-Chaime [18] studied the effect of discreteness in vendor-buyer relationships. An analytical methodology was developed to characterize the effect of the cycle ratio on inventory levels. Sarmah et al. [19] explored a coordination problem in a situation where there is a single producer and multiple heterogeneous customers. Two cases were studied: (i) an ex-site distribution case that considered vendor dominance, where a vendor transports end product to a group of customers at a common replenishment time and (ii) an ex-factory distribution case considering customer dominance and a common replenishment time for distribution. They developed a coordination mechanism to improve supply chain performance and focused on the ways of negotiations to obtain a due share of extra savings for business parties. Other studies that addressed various aspects of periodic or multi-delivery issues in vendor-buyer integrated systems can also be found in [20-27].

Product quality assurance is another critical success factor for most present-day manufacturing firms. In real-world production environments, the generation of random nonconforming products is almost inevitable. Reworking these defective items can be an alternative to not only assuring product quality but also lowering the quality costs in production [28]. Consequently, it helps to minimize production-inventory costs. For example, the production of plastic goods in the plastic injection process, printed circuit board assemblies (PCBAs) in PCBA manufacturing, and so on. Zargar [29] explored the effects of two different reworking policies on the cycle time. One is that the "mother" lot is held back, while the "child" sub-lots are reworked, after rework is completed both members are reunited for the next process; the other is that the mother lot is permitted to proceed to the next process, while the child is held back. Queuing models for these policies were developed and a simulation of a wafer production model is used to demonstrate the effectiveness and impacts of the proposed policies. Inderfurth et al. [30] examined a production system with a rework process using the same facility. They assumed that the defective items deteriorate while waiting for rework. There is a given deterioration time limit and deterioration increases in time. A polynomial dynamic programming algorithm was proposed for resolving the problem and the objective was to derive lot sizes and aspects of items to be reworked that minimizes overall costs. Chiu et al. [31] explored the optimal common cycle time for multiple products finite production rate (FPR) system with rework and multiple shipments polices. Their study focused on derivation of an optimal cycle time for the *producer* to minimize producer's overall production-inventory costs. Chiu et al. [32] studied a *single-product* intra-supply chain system with multiple sales offices and quality assurance. They considered that *a single product* is fabricated by the production unit of a firm, and upon completion of the quality assurance tasks, the entire lot is transported to multiple sales locations of the firm. Their objective is to decide the optimal production-shipment policy that minimizes total costs for the intra-supply chain of the firm. Additional studies [33-40] address different aspects of imperfection issues in production systems.

Since the vendor-buyer integrated type of system can benefit both parties of the supply chain, and it is suitable to be applied to an intra-supply chain system within the present-day globalized enterprise to assist managers in achieving the goal of lowering overall operating costs. Motivated by this concept [32], the present study extends the multi-product FPR problem [31] to a so-called *multi-product intra-supply chain problem*, and attempts to simultaneously determine production and shipment decisions for such a practical multi-product vendor-buyer integrated inventory system with a rework process. As little attention has been paid to this specific research area, the present study is intended to fill the gap.

2. The proposed model and formulation

This study attempts to simultaneously determine the production and shipment decisions for a multi-item vendor-buyer integrated inventory system with a rework process. Fabricating multiple products on a single machine with the aim of maximizing machine utilization is an operating goal of most manufacturing firms. In the proposed multi-product intra-supply chain system, the production rate is P_{1i} per year and the annual demand rate is λ_i, where i = 1, 2, …, L. All products made are checked for their quality, and the unit screening cost is included in the unit production cost C_i. It is also assumed that the production process can randomly produce x_i portion of nonconforming items at a rate d_i, where d_i can be expressed as $d_i = x_i P_{1i}$, and $(P_{1i} - d_i - \lambda_i) > 0$ must be satisfied in order to sustain regular operations (i.e., avoid the occurrence of shortage). All defective items produced are reworked and fully repaired at the rate of P_{2i} at the end of each production cycle, with additional rework cost C_{Ri} per item. After the rework process, the entire quality assured lot of each product i are transported to sales offices/customers under a multi-delivery policy, in which n fixed quantity instalments of the lot are shipped at fixed intervals of time in t_{3i} (see Figs. 1 and 2 [31]).

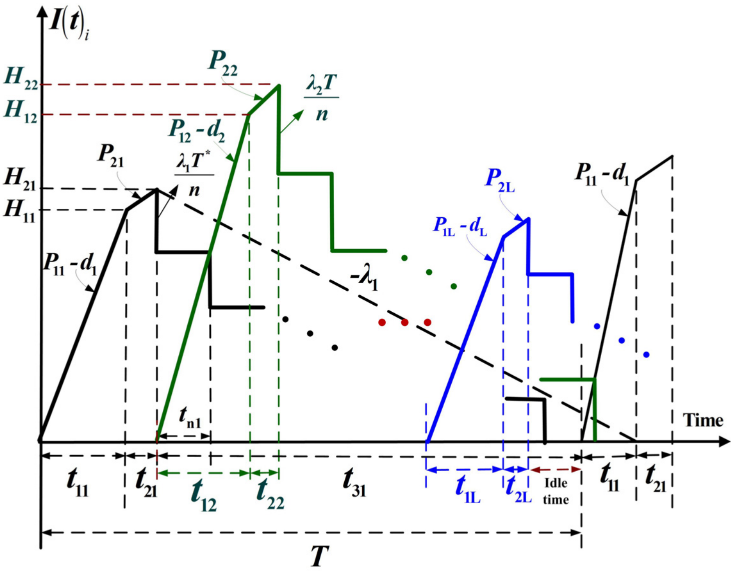

Fig. 1 On-hand inventory level of perfect quality product i at time t in the proposed system [31]

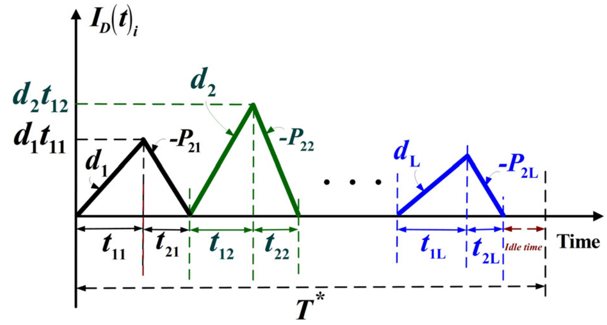

Fig. 2 On-hand inventory level of defective product i at time t in the proposed system [31]

The on-hand inventory of product i stored at the sales offices/customers' side is illustrated in Fig. 3. Accordingly, the sales offices' holding cost along with delivery cost for all L products are included in the proposed cost analysis. Moreover, in order to ensure the production equipment has sufficient capacity in regular and rework processes so as to meet demands for all L products, the following formula must hold: $\sum_{i=1}^{L}\left((\lambda_i/P_{1i}) + (x_i\lambda_i/P_{2i})\right) < 1$.

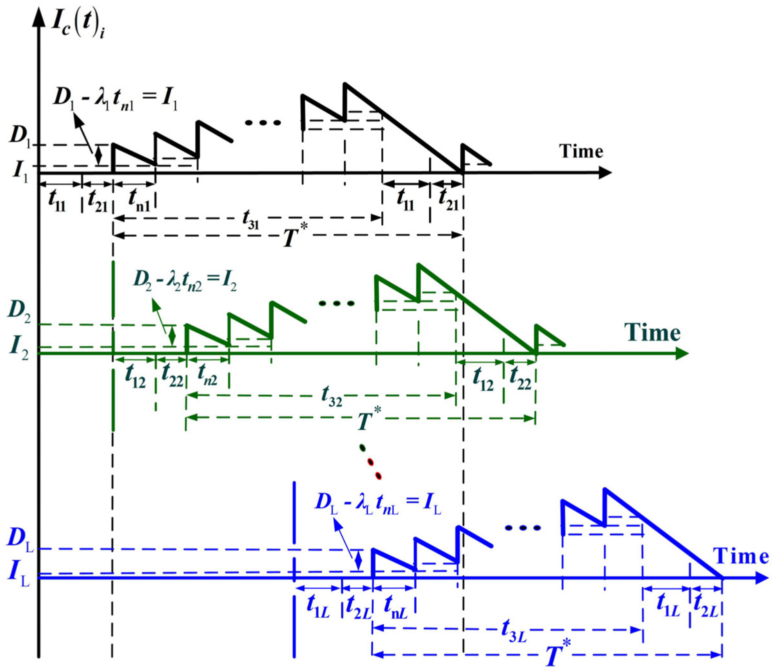

Fig. 3 On-hand inventory level of product i stored at the sales offices at time t in the proposed system

In the proposed mathematical analysis, for each product i the following cost-correlated parameters are used: producer's production setup cost K_i, unit inventory holding cost h_i, holding cost h_{1i} per item undergoing rework, sales offices' unit holding cost h_{2i}, fixed transportation cost K_{1i} per shipment, and unit delivery cost C_{Ti}. Other notations used are listed below:

H_{1i} – maximum on-hand inventory in units of product i when regular production finishes
H_{2i} – maximum on-hand inventory in units of product i when rework process terminates
t_{1i} – production uptime of product i in the proposed system
t_{2i} – rework time of product i in the proposed system
t_{3i} – delivery time of product i in the proposed system
t_{ni} – fixed interval of time between each delivery of product i in t_{3i}
n – number of shipments transported to sales offices per cycle (a decision variable)
T – the common production cycle time (the other decision variable)
Q_i – production batch size per cycle for product i

$I(t)_i$ – level of on-hand inventory of perfect quality product *i* at time *t*
$I_D(t)_I$ – level of on-hand inventory of defective product *i* at time *t*
$I_c(t)_i$ – level of on-hand inventory of product *i* stored at the sales offices at time *t*
D_i – fixed quantity of finished items of product *i* transported to sales offices/customers per shipment
I_i – left over items of product *i* per shipment at the end t_{ni}
$TC(Q_i,n)$ – overall production-inventory-transportation costs per cycle for product *i*
$E[TCU(Q_i,n)]$ – overall expected production-inventory-transportation costs per year for producing *L* products in the proposed system
$E[TCU(T,n)]$ – overall expected production-inventory-transportation costs per year for producing *L* products in the proposed system using the common cycle time rather than lot size as decision variable.

By observing Figs. 1 and 2, the following formulas can be obtained:

$$H_{1i} = (P_{1i} - d_i)t_{1i} \tag{1}$$

$$H_{2i} = H_{1i} + P_{2i}t_{2i} \tag{2}$$

$$t_{1i} = \frac{Q_i}{P_{1i}} = \frac{H_{1i}}{P_{1i} - d_i} \tag{3}$$

$$t_{2i} = \frac{x_i Q_i}{P_{2i}} \tag{4}$$

$$t_{3i} = nt_{ni} = T - (t_{1i} + t_{2i}) \tag{5}$$

$$T = t_{1i} + t_{2i} + t_{3i} \tag{6}$$

$$d_i t_{1i} = x_i Q_i \tag{7}$$

Total delivery costs of *n* shipments of product *i* at t_{3i} is

$$nK_{1i} + C_{Ti}Q_i \tag{8}$$

From Fig. 1, the holding cost of the finished items of product *i* at t_3 is

$$h_i \left(\frac{n-1}{2n}\right) H_{2i} t_{3i} \tag{9}$$

According to the proposed multi-delivery policy, when *n* fixed quantity (i.e., *D*) instalments of finished lot of product *i* are transported to sales offices at a fixed time interval t_{ni}, the following formulas are obtained:

$$t_{ni} = \frac{t_{3i}}{n} \tag{10}$$

$$D_i = \frac{H_{2i}}{n} \tag{11}$$

$$I_i = D_i - \lambda_i t_{ni} \tag{12}$$

The sales offices' stock holding cost of product *i* is [36]

$$h_{2i}\left[n\frac{(D_i - I_i)}{2}t_{ni} + \frac{nI_i}{2}(t_{1i} + t_{2i}) + \frac{n(n+1)}{2}I_i t_{ni}\right] \tag{13}$$

Therefore, $TC(Q_i, n)$ for $i = 1, 2, \ldots, L$, comprises the variable fabrication cost, setup cost, variable reworking cost, production units' inventory holding cost during the periods t_{1i}, t_{2i}, and t_{3i} (including holding cost of nonconforming items in t_{1i}), inventory holding cost of reworked items in t_{2i}, fixed and variable transportation costs, and the stock holding cost from the sales offices/customers, is

$$\sum_{i=1}^{L} TC(Q_i,n) = \sum_{i=1}^{L} \left\{ C_iQ_i + K_i + C_{Ri}(x_iQ_i) \right.$$
$$+ h_i\left[\frac{H_{1i}+d_{1i}t_{1i}}{2}(t_{1i}) + \frac{H_{1i}+H_{2i}}{2}(t_{2i}) + \frac{(n-1)}{2n}(H_{2i}t_{3i})\right]$$
$$+ h_{1i}\frac{d_{1i}t_{1i}}{2}(t_{2i}) + nK_{1i} + C_{Ti}Q_i$$
$$\left. + h_{2i}\left[n\frac{(D_i-I_i)}{2}(t_{ni}) + \frac{nI_i}{2}(t_{1i}+t_{2i}) + \frac{n(n+1)}{2}I_it_{ni}\right]\right\} \tag{14}$$

Substituting relevant parameters from Eqs. 1 to 13 in Eq. 14, using the expected values of x to take randomness of defective rate into account, and applying the renewal reward theorem, $E[TCU(Q_i, n)]$ is obtained as follows:

$$E[TCU(Q_i,n)] = \\ = \sum_{i=1}^{L}\left\{\begin{array}{l} \left[C_i\lambda_i + \frac{K_i\lambda_i}{Q_i} + C_{Ri}\lambda_iE[x_i]\right] + \frac{h_iQ_i\lambda_i}{2}\left[-\frac{1}{\lambda_in} + \frac{1}{nP_{1i}} + \frac{E[x_i]}{nP_{2i}} + \frac{1}{\lambda_i} + \frac{E[x_i]}{P_{2i}} + \frac{E[x_i]^2}{P_{2i}}\right] \\ + \frac{h_{1i}Q_i\lambda_iE[x_i]^2}{2P_{2i}} + \left[C_{Ti}\lambda_i + \frac{nK_{1i}\lambda_i}{Q_i}\right] + \frac{h_{2i}Q_i\lambda_i}{2}\left[\frac{1}{\lambda_in} - \frac{1}{nP_{1i}} - \frac{E[x_i]}{nP_{2i}} + \frac{1}{P_{1i}} + \frac{E[x_i]}{P_{2i}}\right] \end{array}\right\} \tag{15}$$

Since $Q_i = T\lambda_i$, Eq. 15 becomes

$$E[TCU(T,n)] = \\ = \sum_{i=1}^{L}\left\{\begin{array}{l} C_i\lambda_i + \frac{K_i}{T} + C_{Ri}\lambda_iE[x_i] + \frac{h_i{\lambda_i}^2T}{2}\left[\frac{1}{\lambda_i} + \frac{E[x_i]}{P_{2i}} - \frac{E[x_i]^2}{P_{2i}}\right] + \frac{h_{1i}{\lambda_i}^2E[x_i]^2T}{2P_{2i}} \\ +C_{Ti}\lambda_i + \frac{nK_{1i}}{T} + \frac{h_{2i}{\lambda_i}^2T}{2}\left[\frac{1}{P_{1i}} + \frac{E[x_i]}{P_{2i}}\right] + \frac{{\lambda_i}^2T}{2n}\left[\frac{1}{\lambda_i} - \frac{1}{P_{1i}} - \frac{E[x_i]}{P_{2i}}\right](h_{2i}-h_i) \end{array}\right\} \tag{16}$$

2.1 Deriving optimal production-shipment policy

In this section, Hessian matrix equations (Rardin [41]) are used to help determine the optimal operating policy of the common production cycle time T^* and the number of deliveries n^*. In order to prove that the expected system cost function is convex, we must first verify that Eq. 17 holds:

$$[T \quad n]\cdot\begin{bmatrix} \frac{\partial^2E[TCU(T,n)]}{\partial T^2} & \frac{\partial^2E[TCU(T,n)]}{\partial T\partial n} \\ \frac{\partial^2E[TCU(T,n)]}{\partial T\partial n} & \frac{\partial^2E[TCU(T,n)]}{\partial n^2} \end{bmatrix}\cdot\begin{bmatrix} T \\ n \end{bmatrix} > 0 \tag{17}$$

The following equations can be obtained from Eq. 16:

$$\frac{\partial E[TCU(T,n)]}{\partial n} = \sum_{i=1}^{L}\left\{\frac{K_{1i}}{T} - \frac{\lambda_i^2T}{2n^2}\left[\frac{1}{\lambda_i} - \frac{1}{P_{1i}} - \frac{E[x_i]}{P_{2i}}\right](h_{2i}-h_i)\right\} \tag{18}$$

$$\frac{\partial^2E[TCU(T,n)]}{\partial n^2} = \sum_{i=1}^{L}\left\{\frac{\lambda_i^2T}{n^3}\left[\frac{1}{\lambda_i} - \frac{1}{P_{1i}} - \frac{E[x_i]}{P_{2i}}\right](h_{2i}-h_i)\right\} \tag{19}$$

$$\frac{\partial E[TCU(T,n)]}{\partial T}=\sum_{i=1}^{L}\left\{\begin{aligned}&-\frac{K_i}{T^2}-\frac{nK_{1i}}{T^2}+\frac{h_i\lambda_i{}^2}{2}\left[\frac{1}{\lambda_i}+\frac{E[x_i]}{P_{2i}}-\frac{E[x_i]^2}{P_{2i}}\right]+\frac{h_{1i}\lambda_i^2E[x_i]^2}{2P_{2i}}\\&+\frac{h_{2i}\lambda_i{}^2}{2}\left[\frac{1}{P_{1i}}+\frac{E[x_i]}{P_{2i}}\right]+\frac{\lambda_i{}^2}{2n}\left[\frac{1}{\lambda_i}-\frac{1}{P_{1i}}-\frac{E[x_i]}{P_{2i}}\right](h_{2i}-h_i)\end{aligned}\right\} \tag{20}$$

$$\frac{\partial^2 E[TCU(T,n)]}{\partial T^2}=\sum_{i=1}^{L}\left\{\frac{2(K_i+nK_{1i})}{T^3}\right\} \tag{21}$$

$$\frac{\partial^2 E[TCU(T,n)]}{\partial T\partial n}=\sum_{i=1}^{L}\left\{-\frac{K_{1i}}{T^2}-\frac{\lambda_i^2}{2n^2}\left[\frac{1}{\lambda_i}-\frac{1}{P_{1i}}-\frac{E[x_i]}{P_{2i}}\right](h_{2i}-h_i)\right\} \tag{22}$$

Substituting Eqs. 19, 21, and 22 in Eq. 17, the following equation is obtained:

$$[T\quad n]\cdot\begin{bmatrix}\dfrac{\partial^2 E[TCU(T,n)]}{\partial T^2} & \dfrac{\partial^2 E[TCU(T,n)]}{\partial T\partial n}\\ \dfrac{\partial^2 E[TCU(T,n)]}{\partial T\partial n} & \dfrac{\partial^2 E[TCU(T,n)]}{\partial n^2}\end{bmatrix}\cdot\begin{bmatrix}T\\n\end{bmatrix}=\sum_{i=1}^{L}\frac{2K_i}{T}>0 \tag{23}$$

Since K_i and T are all positive, Eq. 23 is positive. Therefore, $E[TCU(T, n)]$ is strictly convex for all T and n not equal to zero, and $E[TCU(T, n)]$ has a minimum value. In order to determine the optimal operating production-shipment policy (i.e., T^* and n^*), we set the first derivatives of $E[TCU(T, n)]$ with respect to T and with respect to n equal to zeros, and solve the linear system (i.e., Eqs. 18 and 20). With further derivations, the following equation is obtained:

$$T^*=\sqrt{\frac{2\sum_{i=1}^{L}(K_i+nK_{1i})}{\sum_{i=1}^{L}\left\{h_i\lambda_i^2\left[\frac{1}{\lambda_i}+\frac{E[x_i]}{P_{2i}}-\frac{E[x_i]^2}{P_{2i}}\right]+h_{2i}\lambda_i{}^2\left[\frac{1}{P_{1i}}+\frac{E[x_i]}{P_{2i}}\right]+\frac{h_{1i}\lambda_i^2E[x_i]^2}{P_{2i}}+\frac{\lambda_i^2}{n}\left[\frac{1}{\lambda_i}-\frac{1}{P_{1i}}-\frac{E[x_i]}{P_{2i}}\right](h_{2i}-h_i)\right\}}} \tag{24}$$

and

$$n^*=\sqrt{\frac{\sum_{i=1}^{L}K_i\cdot\sum_{i=1}^{L}\left[\lambda_i^2(h_{2i}-h_i)\left(\frac{1}{\lambda_i}-\frac{1}{P_{1i}}-\frac{E[x_i]}{P_{2i}}\right)\right]}{\left(\sum_{i=1}^{L}K_{1i}\right)\cdot\sum_{i=1}^{L}\left[h_i\lambda_i^2\left(\frac{1}{\lambda_i}+\frac{E[x_i]}{P_{2i}}-\frac{E[x_i]^2}{P_{2i}}\right)+\frac{h_{1i}\lambda_i^2E[x_i]^2}{P_{2i}}+h_{2i}\lambda_i^2\left(\frac{1}{P_{1i}}+\frac{E[x_i]}{P_{2i}}\right)\right]}} \tag{25}$$

Eq. 25 results in a real number, but in reality the number of deliveries should be represented as an integer. Two adjacent integers to n are examined, respectively, to determine the integer value of n^* that minimizes $E[TCU(T, n)]$. Let n^- denote the largest integer less than or equal to n and n^+ denote the smallest integer greater than or equal to n (as derived from Eq. 25). First, we apply n^- and n^+ in Eq. 24 to obtain their corresponding T values, respectively. Next, we plug each pair in $E[TCU(T, n)]$ and choose the production-shipment policy that has the minimal system costs [13].

3. Numerical example

Consider five products being manufactured in sequence on a machine under the common cycle time policy in a multi-product inventory system with a rework process. Their annual production rates P_{1i} are 58,000, 59,000, 60,000, 61,000, and 62,000, respectively, and their annual demand rates λ_i are 3,000, 3,200, 3,400, 3,600, and 3,800, respectively. For each product, the production units has experienced the random nonconforming rates that follow the uniform distribution over intervals of [0, 0.05], [0, 0.10], [0, 0.15], [0, 0.20], and [0, 0.25], respectively. All nonconforming products are assumed to be repairable and are reworked at the end of the regular production, at annual rates P_{2i} of 46,400, 47,200, 48,000, 48,800, and 49,600, respectively. Additional costs for rework are \$50, \$55, \$60, \$65, and \$70 per nonconforming product, respectively. Other values of system variables used in this example are listed below:

K_i – production setup costs are \$17,000, \$17,500, \$18,000, \$18,500, and \$19,000, respectively.
C_i – fabrication cost per item are \$80, \$90, \$100, \$110, and \$120, respectively.
h_i – inventory holding cost per item are \$10, \$15, \$20, \$25, and \$30, respectively.
h_{1i} – unit holding costs during rework are \$30, \$35, \$40, \$45, and \$50, respectively.
K_{1i} – fixed cost per delivery are \$1,800, \$1,900, \$2,000, \$2,100, and \$2,200, respectively.
h_{2i} – stock holding cost per item at sales offices are \$70, \$75, \$80, \$85, and \$90, respectively.
C_{Ti} – transporting cost per item are \$0.1, \$0.2, \$0.3, \$0.4, and \$0.5, respectively.

First, in order to determine the number of deliveries, one can apply Eq. 25 and obtain n^* = 4.4278. As stated in section 2.1, practically, n^* should be an integer number only, and to find the integer value of n^* one can plug n^+ = 5 and n^- = 4 in Eq. 24 and obtain (T = 0.6666, n^+ = 5) and (T = 0.6193, n^- = 4), respectively. Next, apply Eq. 16 with these two different policies to obtain $E[TCU(0.6666, 5)]$ = \$2,229,865 and $E[TCU(0.6193, 4)]$ = \$2,229,658, respectively. By choosing a policy with minimum cost, the optimal production-shipment policy for the proposed system is determined as n^* = 4, T^* = 0.6193, and $E[TCU(T^*, n^*)]$ = \$2,229,658. The effect of the variation in the rotation cycle time T and the number of shipments n on $E[TCU(T, n)]$ is illustrated in Fig. 4.

Further analysis indicates that for the same system without considering rework process (i.e., treating all nonconforming items as scrap), $E[TCU(T, n)]$ = \$2,352,622. This cost is \$122,964 higher than our proposed model, or 25.91 % of other related costs (i.e., total system costs exclude the variable manufacturing costs) when considering the rework of nonconforming items.

Another interesting finding from our numerical analysis is the rate of the rework process. The effect of the variation in the ratio of rework and regular production rates (i.e., P_{2i}/P_{1i}) on the optimal rotation cycle time T^* and on the expected system cost $E[TCU(T^*, n^*)]$, is illustrated in Figs. 5 and 6, respectively. It can be noted that as the P_{2i}/P_{1i} ratio decreases, T^* decreases slightly, and when the P_{2i}/P_{1i} ratio drops below 0.5, T^* starts to decrease significantly (i.e., when the time required to rework a nonconforming item is twice or more than twice as much as the regular time needed to produce an item).

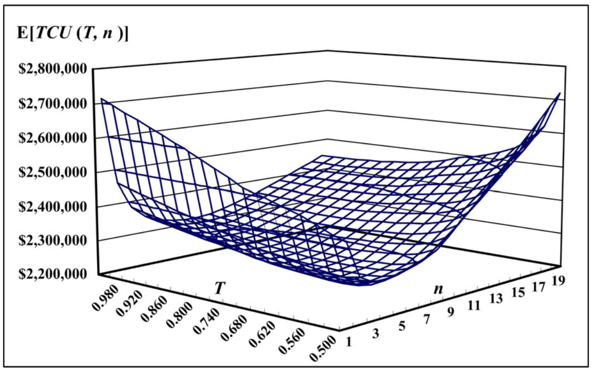

Fig. 4 Effects of the variations in common production cycle time T and number of deliveries n on $E[TCU(T, n)]$

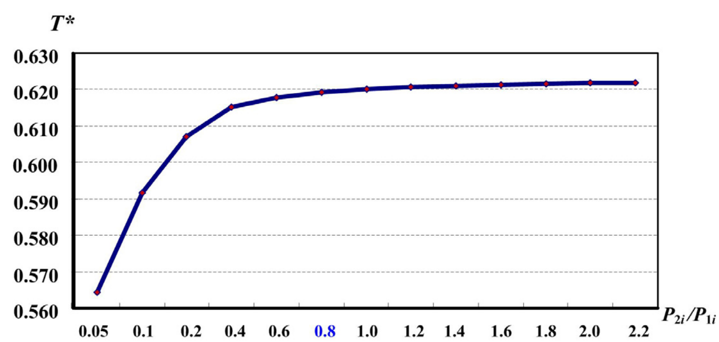

Fig. 5 Effects of the variation in the P_{2i}/P_{1i} ratio on optimal common production cycle time T^*