

# Product quality improvement and air pollutant emission reduction in a mining metal three-stage supply chain under cap-and-trade regulation

Homaei, H.<sup>a,b,\*</sup>, Mahdavi, I.<sup>c</sup>, Tajdin, A.<sup>c</sup>, Khorram, E.<sup>d</sup>

<sup>a</sup>Mazandaran University of Science and Technology, Babol, Iran

<sup>b</sup>Golgozar Mining and Industrial Co., Sirjan, Iran

<sup>c</sup>Mazandaran University of Science and Technology, Faculty of Industrial Engineering, Babol, Iran

<sup>d</sup>Amirkabir University of Technology, Faculty of Mathematics and Computer Science, Tehran, Iran

## ABSTRACT

In today's competitive market, all industries such as mine industries try to increase their profit and keep their customers. Product quality improvement is the miner's most important key to success in competitive market because the mining metals price depends on their quality level. On the other hand, nowadays the management of air pollutant emissions with harmful environmental and health effects is one of the most pressing problems. This paper studies the decision behaviour and coordination issue of a mining metal three-level supply chain with one supplier (extractor), one mineral processor and one manufacturer in which product quality improvement cost at the processor level is higher than the supplier level and at the level of the manufacturer is more than the processor level. We compare the decentralized and centralized systems and identify the optimal product quality level for each supply chain member by designing a revenue sharing contract for decentralized supply chain under cap-and-trade regulation. Finally, numerical example shows that the designed contract not only provides a win-win condition for all supply chain members and increases whole supply chain profit but also increases the final product quality level and reduces harmful air pollutant emissions.

© 2019 CPE, University of Maribor. All rights reserved.

## ARTICLE INFO

### Keywords:

Mining metals;  
Supply chain;  
Quality improvement;  
Channel coordination;  
Emissions reduction;  
Cap-and-trade regulation

### \*Corresponding author:

[h.homaei@in.iut.ac.ir](mailto:h.homaei@in.iut.ac.ir)  
(Homaei, H.)

### Article history:

Received 16 May 2018  
Revised 31 January 2019  
Accepted 24 February 2019

## 1. Introduction

In today's globalized economy, supply chain management is one of the most useful management practices for industries to increase their profit and competitiveness. Nowadays product quality is one of the key competitive dimensions of industries. Industries are always trying to increase their profits, and mines are one of these industries whose profit depends on their product's quality level; but product quality level improvement in mine industries usually emits air pollutants that are serious threats to human health and environment. On the other hand, since rapid economic development brings huge amounts of pollutant emissions, governmental pressures such as cap-and-trade regulation are made to force companies to find new methods to reduce these emissions across all the stages of their supply chains. Under a cap-and-trade regulation, companies get predetermined free emission credits from the government [1]. They could sell/buy credits in the air pollutants trading market when they have surplus/lack credits; this emission credit price is determined by the market. There are two common practices for improv-

ing product quality in mine industries: (1) Technology changing and (2) Practical policies. Since the first method is very costly, the second method is a competitive advantage for miners; therefore, one of the most important concerns miners have is how to increase their profit through product quality level enhancement by operational approaches without increasing harmful pollutant emissions.

There are two decision making systems in a supply chain: centralized and decentralized. In the centralized system, supply chain members operate jointly as a single firm and make their decisions to maximize the total profit of the system; but in the decentralized system, supply chain members make their decisions separately to maximize their own profits. The decision making system in most supply chain models such as this study is assumed to be decentralized. To improve the overall performance of the supply chain, a coordination mechanism is needed. Different definitions and perspectives on the supply chain coordination exist in the literature (refer to [2, 3]) for the comprehensive review of supply chain coordination. A supply chain is coordinated when the members make the decisions that are optimal for the whole supply chain. For coordinating a supply chain, contracts are designed to reduce the difference between the outcome of a centralized system and a decentralized system. Different kinds of contracts such as commitment to purchase quantity, credit option, two-part tariff, revenue sharing [4, 5], buy back, sales rebate, and mail-in-rebate, have been used in supply chains as the ways improving supply chain performance. Revenue sharing is one of the widely used contracts in the supply chain that is between an upper and lower level of supply chain, where the upper level provides better selling condition to the lower level and then the lower level shares a fraction of its revenue with upper level.

There are three streams of literature related to the research in this paper. The first stream focuses on improving the quality level of products in the supply chain. Many efforts have been made to improve product quality in the supply chain. Radej *et al.* provided an overview of the quality tools and methods such as quality techniques and linked it to manufacturing process quality and manufacturing cost-effectiveness; [6]. Singer *et al.* studied a single product distribution channel and suggested a contract that simultaneously increases profit and improves quality [7]. Xiao *et al.* presented a game-theory model to show how the manufacturer coordinates the supply chain by revenue-sharing contract [8]. El Ouardighi discussed the potential coordinating power of revenue-sharing contracts in supply quality management [9]. Yan explored a joint pricing and product quality decision problem in a two level decentralized supply chain [10]. Zhu *et al.* investigated a supply chain, where the buyer has the option to invest in the supplier's quality improvement [11]. Gao *et al.* considered quality improvement effort coordination in a decentralized supply chain with a partial cost allocation contract [12].

The second part of literature explores operational decisions in the supply chain under the cap-and-trade regulation in order to reduce harmful gas emissions. The cap-and-trade regulation is a mechanism to control air pollutant emissions [13]. Many researches have studied the problems in supply chains considering the cap-and-trade regulation and it has been recommended by many senior researchers such as [14] and [15] and implemented in many parts of the world. Xu *et al.* studied the joint production and pricing problem of a manufacturer under cap-and-trade and carbon tax policies. [16]. Gong and Zhou proposed an optimal manufacturing strategy under carbon trading policy through a dynamic model [17]. Hua *et al.* explored how companies manage carbon footprints in inventory management under the carbon-trading regulation [14]. Xu *et al.* investigated the production and pricing problems in make-to-order supply chain under cap-and-trade regulation. [18]. He *et al.* considered the impact of cap-and-trade regulation on company's carbon emission decisions [19]. Zhang and Xu investigated a company's optimal manufacturing quantities under cap-and-trade regulation. [13]. Benjaafar *et al.* studied the multi-period operational decision-making of a company under cap-and-trade regulation [20].

The last subset of literature related to this research is the supply chain coordination under revenue sharing contract. Cachon provided a good survey on this contract [21]. Cachon and Lariviere proved that revenue sharing contracts for decentralized supply chains are beneficial in achieving coordination for various types of supply chains [22]. Qin and Yang used the Stackelberg game to model the revenue sharing contract problem. They showed that the party that

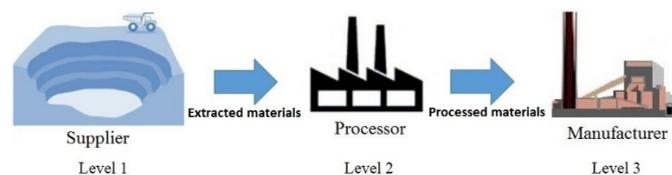
keeps more than half the revenue should serve as the leader of the Stackelberg game [23]. Hsueh presented a new revenue sharing contract embedding corporate social responsibility to coordinate a two level supply chain [24]. Yao *et al.* proposed a revenue sharing contract to coordinate a two stage supply chain. They illustrated that the provision of revenue sharing in the contract can increase supply chain performance more than a price-only contract [25]. Palsule-Desai proposed a game theory model for revenue-dependent revenue sharing contracts in which the supply chain revenue is shared among the members depending on the quantum of revenue generated [26]. Zhang *et al.* discussed the revenue sharing contracts for coordinating a supply chain in which demands are disrupted [27]. Hu *et al.* studied supply chain coordination via revenue sharing contracts in a three-stage supply and a two-stage supply chain [28].

However, a few researches have been done on the three level supply chain coordination with revenue sharing contract considering environmental aspects under cap-and-trade regulation; also the three-level supply chain coordination research literature mentioned above neither take the product quality improvement into account nor focuses on mining metal supply chain coordination. Therefore the main purpose of this study is to design a revenue sharing contract for a mining metal three level supply chain in order to: 1) coordinate supply chain and provide a win-win condition for all its members and decrease the difference between the outcome of a centralized system and a decentralized system, 2) reduce air pollutant emissions in the supply chain under cap-and-trade regulation, 3) improve the final product quality level of the supply chain.

The rest of this paper proceeds as follows. Section 2 presents the notations definition and the supply chain descriptions and assumptions used in this paper. We have analysed the decision behaviour the decentralized and centralized supply chain in Section 3. Section 4 develops a new revenue sharing contract for coordinating the decentralized supply chain. Section 5 provides a numerical example to illustrate the proposed contract performance. Conclusions are provided in Section 6.

## 2. Model description and assumptions

A decentralized mining metal three-stage supply chain in which minerals will convert to concentrate after extraction is assumed in this paper. The considered supply chain consists of a supplier (extractor), a processor, and a manufacturer. The first level extracts processed minerals and sells them to second level who processes minerals and sells the mineral concentrate to the manufacturer, who in turn produces mineral products such as pellets and ingots and sells them to the customers. The product price of each supply chain level depends on the quality of that product. Therefore all of these supply chain members try to increase their product quality.



**Fig. 1** Mining metal three level supply chain

The following notations are used to describe the proposed model:

- $i$  Index for supply chain levels;  $S$  for supplier,  $P$  for processor and  $M$  for manufacturer
- $j_0$  Minimum acceptable product quality level in considered supply chain
- $P_{ij}$  selling price of unit product produced at the supply chain level  $i$  with quality level  $j$
- $d_i$  The amount of product quality level improvement in supply chain level  $i$
- $C_i$  Constant production cost for a unit product in supply chain level  $i$
- $cd_{ij}$  Cost coefficient for increasing product quality level with quality level  $j$  in supply chain level  $i$
- $\alpha_i$  Price increasing coefficient for product produced in supply chain level  $i$  per unit product quality improvement in supply chain level  $i$

- $\beta_i$  Price increasing coefficient for product produced in supply chain level  $i$  per unit product quality improvement in supply chain levels before level  $i$
- $\gamma_i$  Quality improvement cost increasing coefficient in supply chain level  $i$  per unit product quality improvement in supply chain levels before level  $i$
- $\pi_i$  Supply chain level  $i$  profit
- $\phi_1$  Processor's revenue share,  $0 < \phi_1 < 1$
- $\phi_2$  Manufacturer's revenue share,  $0 < \phi_2 < 1$
- $cp_i$  Unit air pollutant emissions trading price for supply chain level  $i$
- $K_i$  Air pollutant emissions cap for supply chain level  $i$
- $g_i$  Amount of air pollutant emission for a unit product quality level improvement in supply chain level  $i$

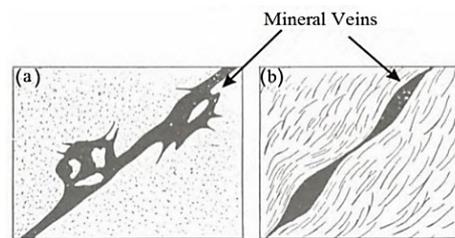
In this paper  $d_i$ ,  $\phi_1$  and  $\phi_2$  are decision variables. Product quality level improvement at the supplier level doesn't emit air pollutants because improving product quality at that level is done by some activities such as more samplings for accurate identification of underground mineral veins (Fig. 3) and performing explosive operation optimally ( $cp_S = 0$ ). But product quality improvement in supply chain levels 2 and 3 emit air pollutants; the emitted air pollutant type at the processor level is usually dust because of the physical processes at this level and the emitted air pollutant at the manufacturer level is of the chemical type, such as SO<sub>2</sub>, due to chemical processes. That is why the parameter  $cp$  for the manufacturer is higher than the processor ( $cp_M > cp_P$ ). The government monitors pollutant emissions of the supply chain members by online measuring equipment (Fig. 2). Product quality level improvement is not mandatory for supply chain members but supplier must supply raw material with minimum quality level  $j_0$ . Product quality improvement for each supply chain member requires more operating costs but these cost enhancements are different for each member because of different production processes in each supply chain level and it is assumed to be a nonlinear ascending.

The product quality improvement cost increases from supplier to manufacturer due to the increasing complexity of production processes from supplier to manufacturer ( $cd_{Sj} < cd_{Pj} < cd_{Mj}$ ). It is a mention worthy assumption that in this type of supply chain, increasing the quality of the product at each supply chain level creates an added value for both that level (with coefficient  $\alpha$ ) and the next levels (with coefficient  $\beta$ ), but increases next levels' product quality improvement costs (with coefficient  $\gamma$ ). For example the cost for increasing 5 quality levels from level 60 to level 65 is more than the cost for increasing 5 quality levels from level 30 to level 35.

In our study we assume that all members are in full capacity production and all their products will be sold. Therefore, the consideration of the demand parameter in the problem is neglected. Also the shipment costs are not considered in this model due to equality in centralized and decentralized system.



**Fig. 2** Air pollutant online monitoring equipment in Golgohar mining & Industrial Co.



**Fig. 3** Mineral Veins

### 3. Decision analysis

In this part, we propose the decision model for the decentralized and centralized systems. After solving both the system models, we obtain the quantitative relationships among the profits and the decision variables under the centralized and decentralized systems.

### 3.1 Analysis of the decentralized system

We assume that all members in the considered supply chain try to improve their products quality in order to increase their profit, but in the decentralized supply chain they try to maximize their own profit. Considering the model assumptions the supplier's profit for each unit product extraction in the decentralized system is

$$\pi_S = P_{Sj_0}(1 + \alpha_S d_S) - C_S - cd_{Sj_0} d_S^2 \quad (1)$$

where the first part denotes a unit extracted material selling price with minimum quality level plus the selling price enhancement due to product quality level improvement by the supplier ( $d_S$ ). The second part is the constant extraction cost for a unit product at the supplier level. Similar to previous studies [29, 30] the third part shows the supplier's cost for increasing product quality level. As mentioned before, at this supply chain level we don't have environmental costs for product quality improvement.

The processor's profit for processing one unit product in the decentralized system is

$$\pi_P = P_{Pj_0}(1 + \beta_P d_S + \alpha_P d_P) - P_{Pj_0}(1 + \alpha_S d_S) - C_P - cd_{Pj_0} d_P^2(1 + \gamma_P d_S) - cp_P(g_P d_P - K_P) \quad (2)$$

where similar to Eq. 1, the first part shows a unit processed material selling price with minimum quality level plus the selling price enhancement due to product quality level improvement by processor ( $d_P$ ) and supplier ( $d_S$ ). The second part is the purchasing price of a unit extracted product from the supplier. The third part is the constant processing cost for a unit product in processor and the fourth term shows the processor's cost for increasing product quality level and the last part is the cost (income) from buying (selling) extra dust emission permits for the processor.

The manufacturer's profit for manufacturing one unit product in the decentralized system is

$$\pi_M = P_{Mj_0}(1 + \beta_M(d_S + d_P) + \alpha_M d_M) - P_{Pj_0}(1 + \beta_P d_S + \alpha_P d_P) - C_M - cd_{Mj_0} cd_M^2(1 + \gamma_M(d_S + d_M)) - cp_M(g_M d_M - K_M) \quad (3)$$

where the first part represents a unit manufactured product selling price with minimum quality level plus the selling price enhancement due to product quality level improvement by the manufacturer ( $d_M$ ) and its previous levels ( $d_S + d_P$ ). The second part is the purchasing price of a unit processed product from the processor. The third part is constant manufacturing cost for a unit product at the manufacturer level. The fourth part shows the manufacturer's cost for increasing product quality level and the last term is the cost (income) from buying (selling) extra chemical pollutant emission permits for the manufacturer.

As mentioned before, all members in the decentralized supply chain try to maximize their own profit so the members' optimal decision will be as follows.

*Proposition 1:* The optimal product quality level improvement by the supplier in considered decentralized supply chain is

$$d_S^* = \frac{\alpha_S P_{Sj_0}}{2cd_{Sj_0}} \quad (4)$$

*Proposition 2:* The optimal product quality level enhancement by the processor in considered decentralized supply chain is

$$d_P^* = \frac{\alpha_P P_{Pj_0} - cp_P g_P}{2cd_{Pj_0}(1 + \gamma_P d_S^*)} \quad (5)$$

*Proposition 3:* The optimal product quality level improvement by the manufacturer in considered decentralized supply chain is

$$d_M^* = \frac{\alpha_M P_{Mj_0} - cp_M g_M}{2cd_{Pj_0}(1 + \gamma_M(d_S^* + d_P^*))} \quad (6)$$

Therefore the optimal value of the whole decentralized supply chain profit without coordination can be written as

$$\pi_T^* = \pi_S^* + \pi_P^* + \pi_M^* \tag{7}$$

And optimal product *quality* improvement for final product without coordination can be calculated as follows

$$d_T^* = d_S^* + d_P^* + d_M^* \tag{8}$$

### 3.2 Analysis of the centralized system

In the centralized system, all supply chain members operate jointly as a single company and determine the optimal value of product quality level improvement to maximize the total profit of the whole supply chain. In this scenario, the total supply chain profit function can be formulated as

$$\begin{aligned} \pi_{TC} = & -cd_{Sj_0}d_S^2 - cd_{Pj_0}d_P^2(1 + \gamma_P d_S) - cp_P(g_P d_P - K_P) \\ & + P_{Mj_0}(1 + \beta_M(d_S + d_P) + \alpha_M d_M) - C_M - C_S - C_P \\ & - cd_{Mj_0}d_M^2(1 + \gamma_M(d_S + d_M)) - cp_M(g_M d_M - K_M) \end{aligned} \tag{9}$$

*Proposition 4:* The optimal product quality level enhancement by the supplier in considered centralized supply chain is

$$d_S^* = \frac{\beta_M P_{Mj_0} - cd_{Pj_0} \gamma_P d_P^2 - cd_{Mj_0} d_M^2 \gamma_M}{2cd_{Sj_0}} \tag{10}$$

*Proposition 5:* The optimal product quality level improvement by the processor in considered centralized supply chain is

$$d_P^* = \frac{\beta_M P_{Mj_0} - cd_{Mj_0} \gamma_M d_M^2 - cp_P g_P}{2cd_{Pj_0}(1 + \gamma_P d_S)} \tag{11}$$

*Proposition 6:* The optimal product quality level improvement by the manufacturer in considered centralized supply chain is

$$d_M^* = \frac{\alpha_M P_{Mj_0} - cp_M g_M}{2cd_{Mj_0}(1 + \gamma_M(d_S + d_P))} \tag{12}$$

It is mention worthy that unlike the decentralized system, in centralized system we have to obtain optimal value of  $d_S$ ,  $d_P$  and  $d_M$  by solving the systems of three equations.

### 4. Supply chain coordination with revenue sharing contract

Since the product quality level improvement by the supplier increases the processor and manufacturer's profit, they share a portion of this profit enhancement with the supplier. Based on the designed revenue sharing contract, whenever the supplier increases the quality of his product ( $d'_S$ ), he will receive more profit from the processor. Considering this revenue sharing contract, the supplier's profit for each unit product extraction is

$$\pi'_S = \phi_1 P_{Pj_0} \beta_P d'_S + \phi_1 \phi_2 P_{Mj_0} \beta_M d'_S + P_{Sj_0}(1 + \alpha_S d'_S) - C_S - cd_{Sj_0} d'_S \tag{13}$$

where the first and second terms show a portion of processor's profit which the supplier receives from the processor due to increasing the product quality improvement ( $d'_S$ ). It is clearly understandable that if the supplier doesn't increase his product quality level he will receive no shared profit from the processor. The other parts of the Eq. 13 are similar to Eq. 1.

According to the presented revenue sharing contract, when the processor delivers product with higher quality to the manufacturer, he will share his profit more with the processor. But some percent of this product quality improvement is done by supplier and the rest of product quality improvement is done by the processor. Therefore the processor shares a portion of the profit received from the manufacturer which is related to the supplier's product quality improvement with the supplier. Hence, considering the above contract descriptions, the processor's profit for processing one unit product under the proposed revenue sharing contract is

$$\pi'_P = \phi_2 P_{Mj_0} \beta_M ((1 - \phi_1) d_S + d_P) + P_{Pj_0} (1 + (1 - \phi_1) \beta_P d_S + \alpha_P d_P) - P_{Sj_0} (1 + \alpha_S d_S) - C_P - cd_{Pj_0} d'_P (1 + \gamma_P d_S) - cp_P (g_P d_P - K_P) \tag{14}$$

where the first part is a portion of the manufacturer's profit which the processor receives from manufacturer minus a part of it that the processor gives to supplier. The second part denotes a unit processed material selling price with minimum quality level plus the selling price enhancement due to product quality level improvement by the processor ( $d'_P$ ) and the supplier ( $d'_S$ ), minus a part of it that the processor gives to the supplier proportionate to ( $d'_S$ ). Based on the designed contract even if the processor doesn't like to increase his product quality, it's beneficial for him to motivate the supplier to improve product quality. The other parts of Eq. 14 are similar to Eq. 2.

According to the proposed contract, the manufacturer will share a part of his profit caused by product quality improvement in previous supply chain levels with the processor, so the manufacturer's profit for manufacturing one unit product based on the presented contract can be written as

$$\pi'_M = P_{Mj_0} (1 + (1 - \phi_2) \beta_M (d_S + d_P) + \alpha_M d_M) - P_{Pj_0} (1 + \beta_P d_S + \alpha_P d_P) - C_M - cd_{Mj_0} d'^2_M (1 + \gamma_M (d_S + d_M)) - cp_M (g_M d_M - K_M) \tag{15}$$

where the first term shows a unit manufactured product selling price with minimum quality level plus the selling price enhancement because of product quality level improvement by the manufacturer ( $d'_M$ ) and its previous levels ( $d'_S + d'_P$ ) minus a part of it that manufacturer gives to the processor proportionate to ( $d'_S + d'_P$ ). The other parts of Eq. 15 are similar to Eq. 3.

After considering the proposed revenue sharing contract in the supply chain all members still try to maximize their own profit due to the decentralization of the supply chain. Therefore the members' optimal decisions can be written as follows.

*Proposition 7:* The optimal product quality level improvement by the supplier after considering revenue sharing contract in supply chain will be

$$d'^*_S = \frac{\phi_1 \beta_P P_{Pj_0} + \phi_1 \phi_2 \beta_M P_{Mj_0} + \alpha_S P_{Sj_0}}{2cd_{Sj_0}} \tag{16}$$

*Proposition 8:* The optimal product quality level enhancement by the processor in assumed decentralized supply chain after considering revenue sharing contract is

$$d'^*_P = \frac{\phi_2 \beta_M P_{Mj_0} + \alpha_P P_{Pj_0} - cd_P g_P}{2cd_{Pj_0} (1 + \gamma_P d'^*_S)} \tag{17}$$

*Proposition 9:* The optimal product quality level improvement by the manufacturer in considered decentralized supply chain based on designed contract is

$$d'^*_M = \frac{\alpha_M P_{Mj_0} - cp_M g_M}{2cd_{Pj_0} (1 + \gamma_M (d'^*_S + d'^*_P))} \tag{18}$$

*Proposition 10:* The other decision variables in considered contract are  $\phi_1$  and  $\phi_2$ , whose optimal values can be calculated as follows

$$\phi^*_1 = \frac{2\alpha_S P_{Sj_0} + cd_{Pj_0} d'^2_P \gamma_P - \phi_2 \beta_M P_{Mj_0} - \beta_P P_{Pj_0}}{-2(\phi_2 \beta_M P_{Mj_0} + \beta_P P_{Pj_0})} \tag{19}$$

and

$$\phi^*_2 = \frac{1}{2} - \frac{cd_{Mj_0} d'^2_M \gamma_M}{2\beta_M P_{Mj_0}} - \frac{cd_{Pj_0} (1 + \gamma_P d'_S) (2\phi_1 \beta_P P_{Pj_0} + \alpha_S P_{Sj_0}) + cd_{Sj_0} (2\alpha_P P_{Pj_0} - cp_P g_P)}{2cd_{Pj_0} (1 + \gamma_P d'_S) \phi_1 \beta_M P_{Mj_0} + 2cd_{Sj_0} \beta_M P_{Mj_0}} \tag{20}$$

It is necessary to note that we have to obtain the optimal values of  $\phi_1$  and  $\phi_2$  by solving the systems of two equations.

Therefore, the optimal value of total supply chain profit under revenue sharing contract can be written as

$$\pi_T^* = \pi_S^* + \pi_P^* + \pi_M^* \tag{21}$$

Also, optimal product quality improvement for final product can be calculated as follows

$$d_T^* = d_S^* + d_P^* + d_M^* \tag{22}$$

The amount of air pollutant emissions depends on product quality improvement at the processor and manufacturer levels. In other words, the more product quality improvement at the processor and manufacturer levels, the greater air pollutant emissions. The *Propositions 11* and *12* obtain the upper bound and the lower bound for the decision variable  $\phi_2$  respectively and guarantee that processor and manufacturer’s air pollutant emissions after considering proposed revenue sharing contract is less than that without the coordination case.

*Proposition 11:* The optimal product quality level improvement by the processor in considered decentralized supply chain without coordination is more than the coordinated with designed revenue sharing contract case ( $d_P^* < d_P^*$ ) if

$$\phi_2 < \frac{\phi_1 \beta_P P_{Pj_0} (\alpha_P P_{Pj_0} - cp_P g_P)}{\beta_M P_{Mj_0} (2cd_{Sj_0} + \alpha_S P_{Sj_0}) - \phi_1 \beta_M P_{Mj_0} (\alpha_P P_{Pj_0} - cp_P g_P)} \tag{23}$$

*Proposition 12:* The optimal product quality level improvement by the manufacturer in considered decentralized supply chain without coordination is more than the coordinated with proposed revenue sharing contract case ( $d_M^* < d_M^*$ ) if

$$\phi_2 > \frac{(2cd_{Sj_0} + \gamma_P (\phi_1 \beta_P P_{Pj_0} + \phi_1 \beta_M P_{Mj_0} + \alpha_S P_{Sj_0})) \left( \frac{2cd_{Sj_0}^2 (\alpha_P P_{Pj_0} - cp_P g_P)}{2cd_{Sj_0} + \gamma_P \alpha_S P_{Sj_0}} - cd_{Pj_0} \phi_1 \beta_P P_{Pj_0} \right) - 2cd_{Sj_0}^2 (\alpha_P P_{Pj_0} - cp_P g_P)}{\beta_M P_{Mj_0} (2cd_{Sj_0}^2 + \phi_1 cd_{Pj_0} (2cd_{Sj_0} + \gamma_P (\phi_1 \beta_P P_{Pj_0} + \phi_1 \beta_M P_{Mj_0} + \alpha_S P_{Sj_0})))} \tag{24}$$

### 5. Results and discussion

In this section, we provide a numerical example in order to illustrate the designed revenue sharing contract performance by using the parameters below:  $C_S = 150$ ;  $C_P = 250$ ;  $C_M = 350$ ;  $P_{Sj_0} = 20$ ;  $P_{Pj_0} = 30$ ;  $P_{Mj_0} = 40$ ;  $\alpha_S = 3$ ;  $\alpha_P = 5$ ;  $\alpha_M = 7$ ;  $\beta_P = 6$ ;  $\beta_M = 20$ ;  $\gamma_P = \gamma_M = 10$ ;  $cd_{Sj_0} = 4$ ;  $cd_{Pj_0} = 5$ ;  $cd_{Mj_0} = 6$ ;  $cp_P = 2$ ;  $cp_M = 5$ ;  $g_P = 3$ ;  $g_M = 4$ ;  $K_P = 50$ , and  $K_M = 40$ . The MATLAB software is used to solve the numerical example considering mentioned parameters and its results are presented in tables 1-3 and Figs. 4-6.

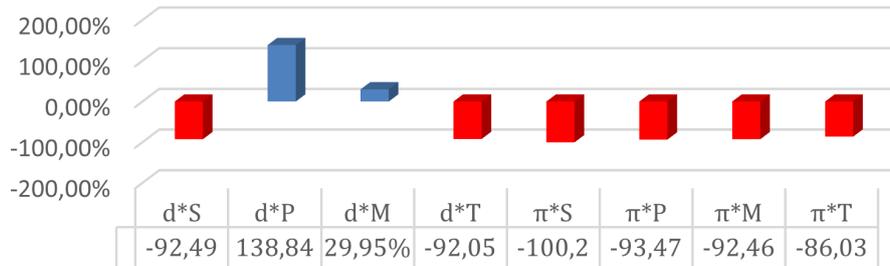
**Table 1** Optimum value of key variables for centralized supply chain

Key variables	$d_S^*$	$d_P^*$	$d_M^*$	$d_T^*$	$\pi_S^*$	$\pi_P^*$	$\pi_M^*$	$\pi_T^*$
Optimum value	99.92	0.0793	0.214	100.21	-34093	11834	61908	39649

Table 1 shows the optimum value of key variables for the centralized supply chain. As it can be observed from Table 1, the highest increase in product quality level in the centralized supply chain is done by the supplier ( $d_S^* \gg d_P^*$ ,  $d_S^* \gg d_M^*$ ) and that is why that his profit is negative. It is necessary to note that the negative profit of the supplier in the centralized supply chain is not important because all members in the centralized supply chain operate jointly as a single company and achieving the win-win condition for supply chain members is not important in this case.

**Table 2** Optimum value of key variables for decentralized supply chain without coordination

Key variables	$d_S^*$	$d_P^*$	$d_M^*$	$d_T^*$	$\pi_S^*$	$\pi_P^*$	$\pi_M^*$	$\pi_T^*$
Optimum value	7.5	0.1894	0.2781	7.9675	95	773	4669	5537

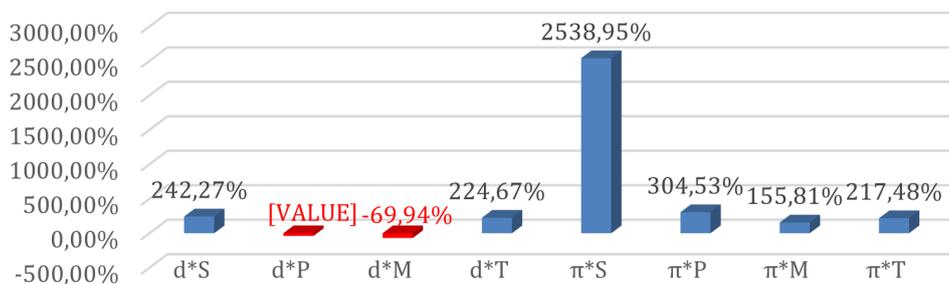


**Fig. 4** Key variables change percentage of the decentralized supply chain without coordination compared to centralized supply chain

From Table 2 and Fig. 4, we observe that the profit of all supply chain members, whole supply chain profit, product quality improvement by supplier and the final product quality in decentralized supply chain without coordination are much lower than that in the centralized case; But product quality improvement by the processor and manufacturer in decentralized supply chain is higher than that in the centralized system. As mentioned before, since the air pollutant emissions depend on the product quality improvement at the processor and manufacturer level of supply chain, so increasing product quality improvement by the processor and manufacturer leads to the enhancement of air pollution emissions; therefore we can say that air pollutant emissions in the decentralized supply chain are higher than that in the centralized system. As mentioned before, we obtain the optimal values of  $\phi_1$  and  $\phi_2$  by solving the systems of two equations using Eqs. 19 and 20. In this example, the lower bound and upper bound for the decision variable  $\phi_2$  are obtained -0.102 and 2.879, respectively and the conditions mentioned in Propositions 11 and 12 are satisfied because the optimal values of  $\phi_1$  and  $\phi_2$  are obtained 0.438 and 0.190 respectively.

**Table 3** Optimum value of key variables for coordinated decentralized supply chain with designed revenue sharing contract

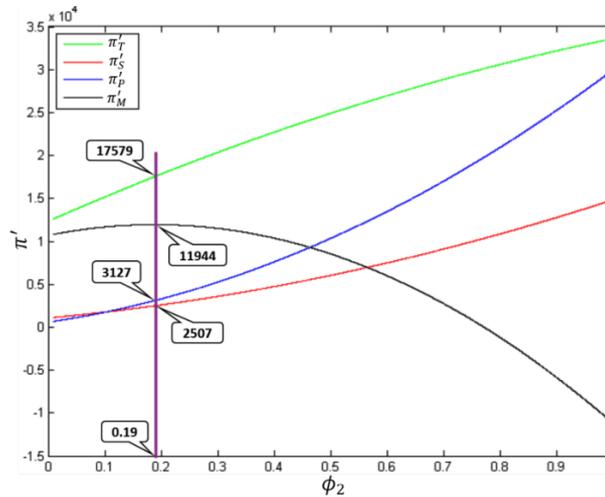
Key variables	$d_S^*$	$d_P^*$	$d_M^*$	$d_T^*$	$\pi_S^*$	$\pi_P^*$	$\pi_M^*$	$\pi_T^*$
Optimum value	25.67	0.1148	0.0836	25.8684	2507	3127	11944	17579



**Fig. 5** Key variables change percentage of the coordinated decentralized supply chain compared to without coordination case

As it is shown in Table 3 and Fig. 5, the designed revenue sharing contract increases the whole supply chain profit and total product quality improvement by 217.48 % and 224.67 % respectively; Also the proposed revenue sharing contract increases the supplier, processor and manufacturer's profit by 2538.95 %, 304.53 % and 155.81 % respectively so we can say that this contract provides a win-win condition for all supply members. It should be mentioned that we can never increase the whole supply chain profit and the final product quality of the decentralized supply chain to its centralized case due to the necessity of the win-win condition for all members in the decentralized supply chain. Also we can say that the designed revenue sharing contract decreases air pollutant emissions 39.39 % and 69.94 % at the processor and manufacturer supply chain level respectively.

Fig. 6 shows supply chain members and the total supply chain profit for different values of  $\phi_2$  by fixing the decision variable  $\phi_1$  to its optimum value 0.438 after coordinating supply chain with the proposed revenue sharing contract. The purple line in Fig. 6 indicates the value of  $\phi_2$  which maximizes the manufacturer's profit. Also the numbers on intersection points of this line with the other curves can be seen in Table 3.



**Fig. 6** Supply chain members and total supply chain profit for different values of  $\phi_2$  by fixing  $\phi_1$  to its optimum value

## 6. Conclusions

One of the main concerns of miners is to increase the quality level of their products because the mining metals price depends on their quality level; but increasing the quality level of these products has different costs at different levels of the supply chain. These costs usually increase after extractor level. The two main practices for increasing product quality in industries are technology changing and practical policies; the first method is rarely used by miners because it's very expensive, so miners try to increase their profit through product quality level improvement by operational approaches without increasing air pollutants emissions. This paper studied the coordination issue of a decentralized three-level mining metal supply chain with one supplier (extractor), one processor and one manufacturer under cap-and-trade regulation and compared it with the centralized system. Due to different product quality improvement costs of supply chain members, a revenue sharing contract designed and optimal product quality level for each of them was obtained. It is necessary to say that the proposed model is designed for some kinds of metals that have impurities and will be processed after extraction (such as Iron and Copper) Finally, the numerical example illustrated that the proposed revenue sharing contract can (a) increase the final product quality level, (b) provide a win-win condition for all supply chain members, (c) increase the whole supply chain profit, and (d) reduce harmful air pollutant emissions in the supply chain. The authors' suggestions for future research is to use other coordination mechanisms and consider the quality dependent demand in the model.

## References

- [1] Xu, J., Chen, Y., Bai, Q. (2016). A two-echelon sustainable supply chain coordination under cap-and-trade regulation, *Journal of Cleaner Production*, Vol. 135, 42-56, doi: [10.1016/j.jclepro.2016.06.047](https://doi.org/10.1016/j.jclepro.2016.06.047).
- [2] Arshinder; Kanda, A.; Deshmukh, S.G. (2009). A framework for evaluation of coordination by contracts: A case of two-level supply chains, *Computers & Industrial Engineering*, Vol. 56, No. 4, 1177-1191, doi: [10.1016/j.cie.2008.03.014](https://doi.org/10.1016/j.cie.2008.03.014).
- [3] Arshinder; Kanda, A.; Deshmukh, S.G. (2008). Supply chain coordination: Perspectives, empirical studies and research directions, *International Journal of Production Economics*, Vol. 115, No. 2, 316-335, doi: [10.1016/j.ijpe.2008.05.011](https://doi.org/10.1016/j.ijpe.2008.05.011).

- [4] Panda, S. (2013). Coordinating two-echelon supply chains under stock and price dependent demand rate, *Asia Pacific Journal of Operational Research*, Vol. 32, No. 2, Art. No. 1250051, doi: [10.1142/S0217595912500510](https://doi.org/10.1142/S0217595912500510).
- [5] Panda, S. (2014). Coordination of a socially responsible supply chain using revenue sharing contract, *Transportation Research Part E: Logistics and Transportation Review*, Vol. 67, 92-104, doi: [10.1016/j.tre.2014.04.002](https://doi.org/10.1016/j.tre.2014.04.002).
- [6] Radej, B., Drnovšek, J., Begeš, G. (2017). An overview and evaluation of quality-improvement methods from the manufacturing and supply-chain perspective, *Advances in Production Engineering & Management*, Vol. 12, No. 4, 388-400, doi: [10.14743/apem2017.4.266](https://doi.org/10.14743/apem2017.4.266).
- [7] Singer, M., Donoso, P., Traverso, P. (2003). Quality strategies in supply chain alliances of disposable items, *Omega*, Vol. 31, No. 6, 499-509, doi: [10.1016/j.omega.2003.08.006](https://doi.org/10.1016/j.omega.2003.08.006).
- [8] Xiao, T., Yang, D., Shen, H. (2011). Coordinating a supply chain with a quality assurance policy via a revenue-sharing contract, *International Journal of Production Research*, Vol. 49, No. 1, 99-120, doi: [10.1080/00207543.2010.508936](https://doi.org/10.1080/00207543.2010.508936).
- [9] El Ouardighi, F. (2014). Supply quality management with optimal wholesale price and revenue sharing contracts: A two-stage game approach, *International Journal of Production Economics*, Vol. 156, 260-268, doi: [10.1016/j.ijpe.2014.06.006](https://doi.org/10.1016/j.ijpe.2014.06.006).
- [10] Yan, X. (2015). Contract efficiency for a decentralized supply chain in the presence of quality improvement, *International Transactions in Operational Research*, Vol. 22, No. 4, 713-734, doi: [10.1111/itor.12106](https://doi.org/10.1111/itor.12106).
- [11] Zhu, K., Zhang, R.Q., Tsung, F. (2007). Pushing quality improvement along supply chains, *Management Science*, Vol. 53, No. 3, 421-436, doi: [10.1287/mnsc.1060.0634](https://doi.org/10.1287/mnsc.1060.0634).
- [12] Gao, C., Cheng, T.C.E., Shen, H., Xu, L. (2016). Incentives for quality improvement efforts coordination in supply chains with partial cost allocation contract, *International Journal of Production Research*, Vol. 54, No. 20, 6216-6231, doi: [10.1080/00207543.2016.1191691](https://doi.org/10.1080/00207543.2016.1191691).
- [13] Zhang, B., Xu, L. (2013). Multi-item production planning with carbon cap and trade mechanism, *International Journal of Production Economics*, Vol. 144, No. 1, 118-127, doi: [10.1016/j.ijpe.2013.01.024](https://doi.org/10.1016/j.ijpe.2013.01.024).
- [14] Hua, G., Cheng, T.C.E., Wang, S. (2011). Managing carbon footprints in inventory management, *International Journal of Production Economics*, Vol. 132, No. 2, 178-185, doi: [10.1016/j.ijpe.2011.03.024](https://doi.org/10.1016/j.ijpe.2011.03.024).
- [15] Du, S., Tang, W., Song, M. (2016). Low-carbon production with low-carbon premium in cap-and-trade regulation, *Journal of Cleaner Production*, Vol. 134, Part B, 652-662, doi: [10.1016/j.jclepro.2016.01.012](https://doi.org/10.1016/j.jclepro.2016.01.012).
- [16] Xu, X., Xu, X., He, P. (2016). Joint production and pricing decisions for multiple products with cap-and-trade and carbon tax regulations, *Journal of Cleaner Production*, Vol. 112, Part 5, 4093-4106, doi: [10.1016/j.jclepro.2015.08.081](https://doi.org/10.1016/j.jclepro.2015.08.081).
- [17] Gong, X., Zhou, S.X. (2013). Optimal production planning with emissions trading, *Operations Research*, Vol. 61, No. 4, 908-924, doi: [10.1287/opre.2013.1189](https://doi.org/10.1287/opre.2013.1189).
- [18] Xu, X., Zhang, W., He, P., Xu, X. (2017). Production and pricing problems in make-to-order supply chain with cap-and-trade regulation, *Omega*, Vol. 66, Part B, 248-257, doi: [10.1016/j.omega.2015.08.006](https://doi.org/10.1016/j.omega.2015.08.006).
- [19] He, P., Zhang, W., Xu, X., Bian, Y. (2015). Production lot-sizing and carbon emissions under cap-and-trade and carbon tax regulations, *Journal of Cleaner Production*, Vol. 103, 241-248, doi: [10.1016/j.jclepro.2014.08.102](https://doi.org/10.1016/j.jclepro.2014.08.102).
- [20] Benjaafar, S., Li, Y., Daskin, M. (2013). Carbon footprint and the management of supply chains: Insights from simple models, *IEEE Transactions on Automation Science and Engineering*, Vol. 10, No. 1, 99-116, doi: [10.1109/TASE.2012.2203304](https://doi.org/10.1109/TASE.2012.2203304).
- [21] Cachon, G.P. (2003). Supply chain coordination with contracts, In: Graves, S., de Kok, T. (eds.), *Handbooks in Operations Research and Management Science: Supply Chain Management*, North Holland, Vol. 11, 227-339, doi: [10.1016/S0927-0507\(03\)11006-7](https://doi.org/10.1016/S0927-0507(03)11006-7).
- [22] Cachon, G.P., Lariviere, M.A. (2005). Supply chain coordination with revenue-sharing contracts: Strengths and limitations, *Management Science*, Vol. 51, No. 1, 30-44, doi: [10.1287/mnsc.1040.0215](https://doi.org/10.1287/mnsc.1040.0215).
- [23] Qin, Z., Yang, J. (2008). A revenue-sharing contract in supply chain management, *International Journal of Logistics: Research and Applications*, Vol. 11, No. 1, 17-29, doi: [10.1080/13675560701380354](https://doi.org/10.1080/13675560701380354).
- [24] Hsueh, C.-F. (2014). Improving corporate social responsibility in a supply chain through a new revenue sharing contract, *International Journal of Production Economics*, Vol. 151, 214-222, doi: [10.1016/j.ijpe.2013.10.017](https://doi.org/10.1016/j.ijpe.2013.10.017).
- [25] Yao, Z., Leung, S.C.H., Lai, K.K. (2008). Manufacturer's revenue-sharing contract and retail competition, *European Journal of Operational Research*, Vol. 186, No. 2, 637-651, doi: [10.1016/j.ejor.2007.01.049](https://doi.org/10.1016/j.ejor.2007.01.049).
- [26] Palsule-Desai, O.D. (2013). Supply chain coordination using revenue-dependent revenue sharing contracts, *Omega*, Vol. 41, No. 4, 780-796, doi: [10.1016/j.omega.2012.10.001](https://doi.org/10.1016/j.omega.2012.10.001).
- [27] Zhang, W.-G., Fu, J., Li, H., Xu, W. (2012). Coordination of supply chain with a revenue-sharing contract under demand disruptions when retailers compete, *International Journal of Production Economics*, Vol. 138, No. 1, 68-75, doi: [10.1016/j.ijpe.2012.03.001](https://doi.org/10.1016/j.ijpe.2012.03.001).
- [28] Hu, B., Meng, C., Xu, D., Son, Y.-J. (2016). Three-echelon supply chain coordination with a loss-averse retailer and revenue sharing contracts, *International Journal of Production Economics*, Vol. 179, 192-202, doi: [10.1016/j.ijpe.2016.06.001](https://doi.org/10.1016/j.ijpe.2016.06.001).
- [29] Gavious, A., Lowengart, O. (2012). Price-quality relationship in the presence of asymmetric dynamic reference quality effects, *Marketing Letters*, Vol. 23, No. 1, 137-161, doi: [10.1007/s11002-011-9143-4](https://doi.org/10.1007/s11002-011-9143-4).
- [30] Kopalle, P.K., Winer, R.S. (1996). A dynamic model of reference price and expected quality, *Marketing Letters*, Vol. 7, No. 1, 41-52, doi: [10.1007/BF00557310](https://doi.org/10.1007/BF00557310).

## Appendix A

*Proof of Proposition 1:*

Since the  $\pi_S$  is concave in  $d_S$ , there exists a unique optimal product quality level improvement  $d_S$  that maximizes supplier's profit because the second derivative of equation  $\pi_P$  is negative

$$\frac{\partial^2 \pi_S}{\partial d_S^2} = -2cd_{Sj_0} < 0$$

Therefore the optimal value of  $d_S$  can be obtained as follows

$$\frac{\partial \pi_S}{\partial d_S} = 0 \rightarrow \alpha_S P_{Sj_0} - 2cd_{Sj_0} d_S = 0 \rightarrow d_S^* = \frac{\alpha_S P_{Sj_0}}{2cd_{Sj_0}}$$

This completes the proof.

*Proof of Proposition 2:*

Since the  $\pi_P$  is concave in  $d_P$ , there exists a unique optimal product quality level improvement  $d_P$  that maximizes the processor's profit because second derivative of the function  $\pi_P$  is negative

$$\frac{\partial^2 \pi_P}{\partial d_P^2} = -2cd_{Pj_0}(1 + \gamma_P d_S) < 0$$

Therefore the optimal value of  $d_P$  can be obtained as follows

$$\frac{\partial \pi_P}{\partial d_P} = 0 \rightarrow \alpha_P P_{Pj_0} - 2cd_{Pj_0} d_P (1 + \gamma_P d_S) - cp_P g_P = 0 \rightarrow d_P^* = \frac{\alpha_P P_{Pj_0} - cp_P g_P}{2cd_{Pj_0}(1 + \gamma_P d_S^*)}$$

This completes the proof.

*Proof of Proposition 3:*

Since the  $\pi_M$  is concave in  $d_M$ , there exists a unique optimal product quality level improvement  $d_M$  that maximizes the manufacturer's profit because second derivative of the function  $\pi_M$  is negative

$$\frac{\partial^2 \pi_M}{\partial d_M^2} = -2cd_{Mj_0}(1 + \gamma_M(d_S + d_P)) < 0$$

Therefore the optimal value of  $d_M$  can be obtained as follows

$$\frac{\partial \pi_M}{\partial d_M} = 0 \rightarrow \alpha_M P_{Mj_0} - 2cd_{Mj_0} d_M (1 + \gamma_M(d_S + d_P)) - cp_M g_M = 0 \rightarrow d_M^* = \frac{\alpha_M P_{Mj_0} - cp_M g_M}{2cd_{Mj_0}(1 + \gamma_M(d_S^* + d_P^*))}$$

This completes the proof. Proof of *Propositions 4* and *7* is similar to proof of *Proposition 1*. Proof of *Propositions 5* and *8* is similar to proof of *Proposition 2*. Proof of *Propositions 6* and *9* is similar to proof of *Proposition 3*.

*Proof of Proposition 10:*

Since the  $\pi'_P$  is concave in  $\phi_1$  and there exists a unique optimal value for  $\phi_1$  that maximizes the processor's profit because second derivative of the function  $\pi'_P$  is negative. It should be noted that before the derivation of  $\pi'_P$  we should replace Eq. 16 in Eq. 14.

$$\frac{\partial^2 \pi'_P}{\partial \phi_1^2} = -2(\phi_2 \beta_M P_{Mj_0} + \beta_P P_{Pj_0}) < 0$$

Therefore, the optimal value of  $\phi_1$  can be obtained as follows

$$\begin{aligned} \frac{\partial \pi'_P}{\partial \phi_1} = 0 &\rightarrow 2\alpha_S P_{Sj_0} + cd_{Pj_0} d_P'^2 \gamma_P - \phi_2 \beta_M P_{Mj_0} + \beta_P P_{Pj_0} + 2\phi_1(\phi_2 \beta_M P_{Mj_0} + \beta_P P_{Pj_0}) = 0 \rightarrow \phi_1^* \\ &= \frac{2\alpha_S P_{Sj_0} + cd_{Pj_0} d_P'^2 \gamma_P - \phi_2 \beta_M P_{Mj_0} - \beta_P P_{Pj_0}}{-2(\phi_2 \beta_M P_{Mj_0} + \beta_P P_{Pj_0})} \end{aligned}$$

This proofs Eq. 19.

In the following,  $\pi'_M$  is concave in  $\phi_2$  and there exists a unique optimal value for  $\phi_2$  that maximizes the processor's profit because second derivative of the function  $\pi'_M$  is negative. It should be noted that before the derivation of  $\pi'_M$  we should replace Eqs. 16 and 17 in Eq. 15.

$$\frac{\partial \pi_M'^2}{\partial^2 \phi_2} = -\beta_M P_{Mj_0} \left( \frac{\phi_1 \beta_M P_{Mj_0}}{cd_{Sj_0}} + \frac{\beta_M P_{Mj_0}}{cd_{Pj_0}(1 + \gamma_P d'_S)} \right) < 0$$

Therefore the optimal value of  $\phi_2$  can be calculated as follows

$$\begin{aligned} \frac{\partial \pi'_M}{\partial \phi_2} = 0 &\rightarrow \left( \frac{\phi_1 \beta_M P_{Mj_0}}{2cd_{Sj_0}} + \frac{\beta_M P_{Mj_0}}{2cd_{Pj_0}(1 + \gamma_P d'_S)} \right) (\beta_M P_{Mj_0} - cd_{Mj_0} d_M'^2 \gamma_M) - \frac{\phi_1 P_{Pj_0} \beta_P \beta_M P_{Mj_0}}{2cd_{Sj_0}} - \\ &\frac{P_{Pj_0} \alpha_P \beta_M P_{Mj_0}}{2cd_{Pj_0}(1 + \gamma_P d'_S)} - 2\phi_2 \beta_M P_{Mj_0} \left( \frac{\phi_1 \beta_M P_{Mj_0}}{2cd_{Sj_0}} + \frac{\beta_M P_{Mj_0}}{2cd_{Pj_0}(1 + \gamma_P d'_S)} \right) - \beta_M P_{Mj_0} \left( \frac{\phi_1 \beta_P P_{Pj_0}}{2cd_{Sj_0}} + \frac{\alpha_P P_{Pj_0} - cp_P g_P}{2cd_{Pj_0}(1 + \gamma_P d'_S)} \right) \\ &= 0 \rightarrow \phi_2^* \\ &= \frac{1}{2} - \frac{cd_{Mj_0} d_M'^2 \gamma_M}{2\beta_M P_{Mj_0}} - \frac{cd_{Pj_0}(1 + \gamma_P d'_S)(2\phi_1 \beta_P P_{Pj_0} + \alpha_S P_{Sj_0}) + cd_{Sj_0}(2\alpha_P P_{Pj_0} - cp_P g_P)}{2cd_{Pj_0}(1 + \gamma_P d'_S)\phi_1 \beta_M P_{Mj_0} + 2cd_{Sj_0}\beta_M P_{Mj_0}} \end{aligned}$$

This completes the proof.

*Proof of Proposition 11:*

$$d_P^* < d_P^* \rightarrow \frac{\phi_2 \beta_M P_{Mj_0} + \alpha_P P_{Pj_0} - cp_P g_P}{2cd_{Pj_0}(1 + \gamma_P d_S^*)} < \frac{\alpha_P P_{Pj_0} - cp_P g_P}{2cd_{Pj_0}(1 + \gamma_P d_S^*)}$$

By replacing Eq. 16 in Eq. 17 and replacing Eq. 4 in Eq. 5 we have:

$$\begin{aligned} \frac{\phi_2 \beta_M P_{Mj_0} + \alpha_P P_{Pj_0} - cp_P g_P}{2cd_{Sj_0} + \phi_1 \beta_P P_{Pj_0} + \phi_1 \phi_2 \beta_M P_{Mj_0} + \alpha_S P_{Sj_0}} &< \frac{\alpha_P P_{Pj_0} - cp_P g_P}{2cd_{Sj_0} + \alpha_S P_{Sj_0}} \\ \rightarrow \phi_2 &< \frac{\phi_1 \beta_P P_{Pj_0} (\alpha_P P_{Pj_0} - cp_P g_P)}{\beta_M P_{Mj_0} (2cd_{Sj_0} + \alpha_S P_{Sj_0}) - \phi_1 \beta_M P_{Mj_0} (\alpha_P P_{Pj_0} - cp_P g_P)} \end{aligned}$$

This completes the proof.

*Proof of Proposition 12:*

$$d_M^* < d_M^* \rightarrow \frac{\alpha_M P_{Mj_0} - cp_M g_M}{2cd_{Pj_0}(1 + \gamma_M (d_S^* + d_P^*))} < \frac{\alpha_M P_{Mj_0} - cp_M g_M}{2cd_{Pj_0}(1 + \gamma_P (d_S^* + d_P^*))}$$

Comparing Eqs. 18 and 6 it is clear that numerator in both equations is the same so we have

$$\begin{aligned} \rightarrow 2cd_{Pj_0}(1 + \gamma_M (d_S^* + d_P^*)) &> 2cd_{Pj_0}(1 + \gamma_P (d_S^* + d_P^*)) \rightarrow (d_S^* + d_P^*) > (d_S^* + d_P^*) \rightarrow (d_S^* - d_S^*) > (d_P^* - d_P^*) \\ \rightarrow \frac{\phi_1 \beta_P P_{Pj_0} + \phi_1 \phi_2 \beta_M P_{Mj_0} + \alpha_S P_{Sj_0}}{2cd_{Sj_0}} - \frac{\alpha_S P_{Sj_0}}{2cd_{Sj_0}} &> \frac{\alpha_P P_{Pj_0} - cp_P g_P}{2cd_{Pj_0}(1 + \gamma_P d_S^*)} - \frac{\phi_2 \beta_M P_{Mj_0} + \alpha_P P_{Pj_0} - cp_P g_P}{2cd_{Pj_0}(1 + \gamma_P d_S^*)} \rightarrow \\ \phi_2 &> \frac{\left( 2cd_{Sj_0} + \gamma_P (\phi_1 \beta_P P_{Pj_0} + \phi_1 \beta_M P_{Mj_0} + \alpha_S P_{Sj_0}) \right) \left( \frac{2cd_{Sj_0}^2 (\alpha_P P_{Pj_0} - cp_P g_P)}{2cd_{Sj_0} + \gamma_P \alpha_S P_{Sj_0}} - cd_{Pj_0} \phi_1 \beta_P P_{Pj_0} \right) - 2cd_{Sj_0}^2 (\alpha_P P_{Pj_0} - cp_P g_P)}{\beta_M P_{Mj_0} \left( 2cd_{Sj_0}^2 + \phi_1 cd_{Pj_0} (2cd_{Sj_0} + \gamma_P (\phi_1 \beta_P P_{Pj_0} + \phi_1 \beta_M P_{Mj_0} + \alpha_S P_{Sj_0})) \right)} \end{aligned}$$

This completes the proof.