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Optimization of process performance by multiple pentagon fuzzy responses: Case studies of wire-electrical discharge machining and sputtering process

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A B S T R A C T

This research developed mathematical models to optimize process performance for multiple pentagon fuzzy quality responses. Initially, each quality response was represented by a pentagon membership function. Then, the combination of optimal factor levels was obtained for each response replicate. Those optimal combinations were then used to construct pentagon regression models for each response. A pentagon fuzzy optimization model was formulated and solved to determine the combination of optimal factor levels at each element of pentagon response's fuzzy number. Two real case studies, i.e. wire-electrical discharge machining and sputtering process, were provided for illustration. Optimal results of the two case studies revealed that the proposed procedure effectively optimized performance under uncertainty and provided larger improvement in multiple quality characteristics. In conclusion, the proposed procedure may enhance the process engineer's knowledge about effects of uncertainty on process/product performance and help practitioners decide the proper adjustments of factor levels in order to enhance performance of electrical discharge machining and sputtering process.

A R T I C L E I N F O

Keywords:
Modeling and optimization;
Fuzzy goal programming;
Pentagon regression modelling;
Pentagon fuzzy numbers;
Wire electro-discharge machining (WEDM);
Surface roughness (SR);
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1. Introduction

In practice, inherent variations in a manufacturing process and measurement system are unavoidable. When conducting designed experiments, such variations may result in obtaining erroneous combination of optimal process factor settings, and thereby may not lead to produce product/process improvement as expected [1]. Most literature studies ignored the effect on uncertainty and provided a certain combination of optimal factor settings [2, 3]. Hocine *et al.* [4] extended the conventional fuzzy goal programming model to solve a wide range of uncertainties decision-making problems. The model was validated by optimizing the renewable portfolio for electricity generation in Italy. Komsiyah *et al.* [5] analysed production planning problem in a furniture Company with different operational constraint, including production time, quantity of raw materials, and warehouse capacity. The fuzzy goal programming was applied to minimize the production cost and raw material cost, and maximize the profit. Mirzaee et el. [6] emphasized the problem of supplier selection, which was mathematically formulated by a mixed integer linear programming model. This model was then solved by a pre-emptive fuzzy goal programming approach. Johnson and Bogle [7] presented a model-based approach to risk analysis to aid design for pharmaceutical processes which combined systematic modelling procedures with Hammersley sampling-based uncertainty analysis and sensitivity analysis used to quantify predicted performance uncertainty and to identify key uncertainty contributions. Authors demonstrated the methodology on an industrial case study where the process flowsheet was fixed and some pilot data was available.

In order to enhance process knowledge about the influence of such variations on process/product performance, an effective optimization procedure is required. The fuzzy goal programming (FGP) is found an effective technique in handling the challenge of optimizing process performance under responses' uncertainty [8-11]. Further, Al-Refaie *et al.* [12] expressed each quality characteristic by triangular fuzzy number and then used fuzzy regression combined with desirability function to optimize process performance. However, in order to obtain better evaluation of process performance under fuzziness over a wider range of optimal factor levels and guide process engineers on taking proper adjustment of key factor levels, this research treats each quality response as a pentagon fuzzy number. Then, a pentagon fuzzy regression-desirability procedure is developed to optimize process performance. The remaining of this paper is organized as follows. Section two defines materials and methods, Section three presents results and discussion, finally, section four summarizes conclusion.

2. Materials and methods

2.1 Optimization procedure

Assume a production process is studied via designed experiments. Typically, at each combination of process controllable factors product samples are collected and then observations of each quality characteristic's replicate are recorded. Then, the proposed procedure to optimal process performance for fuzzy multiple quality responses goes as follows:

Step I: Formulate the multiple regression model between the $y_{jr}(x)$; the response value of the r-th replicate of response j, and controllable process factors, x, as shown in Eq. 1.

$$y_{jr}(x) = \beta_{0r} + \sum_{f=1}^{v} \beta_{fr} x_f + \sum_{f=1}^{v} \beta_{ffr}\, x_f^2 + \sum_{g<f} \sum \beta_{fgr}\, x_f x_g + \varepsilon, \quad r = 1,2,\ldots,k \tag{1}$$

where β_{0r} is the intercept and the coefficients, β_f, β_{fg}, and β_{ff}, are crisp values, while, x_f, x_f^2, and $x_f x_g$ denote independent factor variables and ε is random error.

Utilizing the intercept and coefficients for each response replicate, obtained in Eq. 1, the fuzzy multilinear regression is developed for each fuzzy response, $y_j(\tilde{x})$, as follows:

$$\tilde{y}_j(\tilde{x}) = \tilde{\beta}_0 + \sum_{f=1}^{v} \tilde{\beta}_f \tilde{x}_f + \sum_{f=1}^{v} \tilde{\beta}_{ff}\, \tilde{x}_f^2 + \sum_{g<f} \sum \tilde{\beta}_{fg}\, \tilde{x}_f \tilde{x}_g + \varepsilon \quad \forall j, \forall f \tag{2}$$

where $\tilde{\beta}_0$, $\tilde{\beta}_{ff}$, and $\tilde{\beta}_{fg}$ are pentagonal fuzzy coefficients $(\beta^a, \beta^b, \beta^c, \beta^d, \beta^e)$ and calculated as:

$$\tilde{\beta} = \begin{cases} \beta^a = \beta^c - s \\ \beta^b = \beta^c - \lambda s \\ \beta^c = Average\,(\beta_1, \ldots, \beta_k) \\ \beta^d = \beta^c + \lambda s \\ \beta^e = \beta^c + s \end{cases} \tag{3}$$

where s is the estimated standard deviation of β values and λ is a constant between zero and one.

Step II: Represent each of the quality responses and process factors by adequate membership functions from Fig. 1, MSFs, as follows:

a) For the nominal-the-best (NTB) type quality response, the negative deviations, d_1^- and d_2^-, and positive deviations, d_1^+ and d_2^+, from the target, T, are the decision variables. The maximal negative admissible violation, D_1^- and D_2^-, and positive admissible violation, D_1^+ and D_2^+, from T are the parameters.

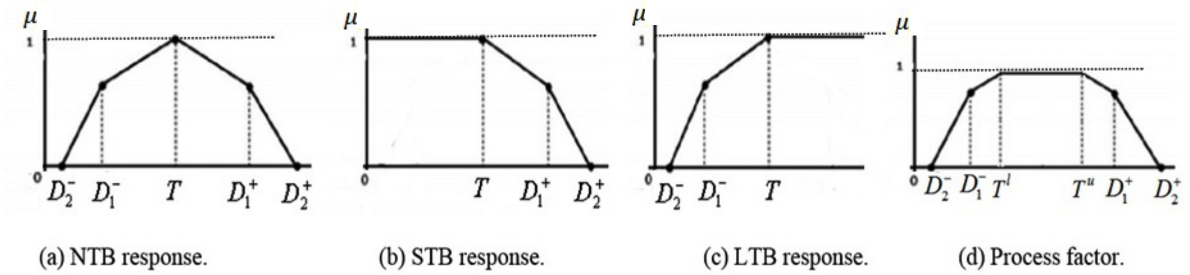

Fig. 1 The selected membership functions for quality responses

The objective function for the NTB type response is to minimize the sum of the weighted positive and negative deviations as shown in Eq. 4a.

$$Min \sum\nolimits_{\forall NTB} \frac{d_1^-}{D_1^-} + \frac{d_2^-}{D_2^-} + \frac{d_1^+}{D_1^+} + \frac{d_2^+}{D_2^+} \quad \text{[Minimize sum of deviation ratios]} \tag{4a}$$

The objective function is subject to the following constraints:

$$y + d_1^+ + d_2^+ - d_1^- - d_2^- = T \quad \text{[Response target]} \tag{4b}$$

$$\mu + \frac{d_1^-}{D_1^-} + \frac{d_2^-}{D_2^-} + \frac{d_1^+}{D_1^+} + \frac{d_2^+}{D_2^+} = 1 \quad \text{[Response membership value]} \tag{4c}$$

$$0 \le d_1^- \le D_1^-\ , 0 \le d_2^- \le D_2^- \quad \text{[Negative deviation ranges]} \tag{4d}$$

$$0 \le d_1^+ \le D_1^+\ , 0 \le d_2^+ \le D_2^+ \quad \text{[Positive deviation ranges]} \tag{4e}$$

b) For the smaller-the-better (STB) type quality response, the objective function is to minimize the sum of weighted positive deviations:

$$Min \sum\nolimits_{\forall STB} \frac{d_1^+}{D_1^+} + \frac{d_2^+}{D_2^+} \quad \text{[Minimize sum of positive deviation ratios]} \tag{5a}$$

Subject to:

$$y - d_1^+ - d_2^+ \le T \quad \text{[Response target]} \tag{5b}$$

$$\mu + \frac{d_1^+}{D_1^+} + \frac{d_2^+}{D_2^+} = 1 \quad \text{[Response membership value]} \tag{5c}$$

$$0 \le d_1^+ \le D_1^+ , 0 \le d_2^+ \le D_2^+ \quad \text{[Positive deviation ranges]} \tag{5d}$$

c) For the larger-the-better (STB) type response, the objective function is to minimize the sum of weighted negative deviations.

$$Min \sum\nolimits_{\forall LTB} \frac{d_1^-}{D_1^-} + \frac{d_2^-}{D_2^-} \quad \text{[Minimize sum of positive deviation ratios]} \tag{6a}$$

Subject to:

$$y + d_1^- + d_2^- \ge T \quad \text{[Response target]} \tag{6b}$$

$$\mu + \frac{d_1^-}{D_1^-} + \frac{d_2^-}{D_2^-} = 1 \quad \text{[Response membership value]} \tag{6c}$$

$$0 \le d_1^- \le D_1^-, 0 \le d_2^- \le D_2^- \quad \text{[Negative deviation ranges]} \tag{6d}$$

d) Each process factor, x, is represented by a trapezoidal MSF with preferable upper and lower limits, T^l and T^u, respectively. Let the maximal negative admissible violations from T^l be denoted as D_1^- and D_2^- while the positive admissible violations from T^u be denoted as D_1^+ and D_2^+. The objective function is to minimize the sum of the weighted positive and negative deviations and is formulated mathematically as shown in Eq. 7a.

$$Min \sum\nolimits_{\forall x} \frac{d_1^-}{D_1^-} + \frac{d_2^-}{D_2^-} + \frac{d_1^+}{D_1^+} + \frac{d_2^+}{D_2^+} \quad \text{[Minimize sum of positive deviation ratios]} \tag{7a}$$

Subject to:

$$x + d_1^- + d_2^+ \ge T^l \quad \text{[Factor target]} \tag{7b}$$

$$x - d_1^+ - d_2^+ \le T^u \quad \text{[Factor target]} \tag{7c}$$

$$\mu + \frac{d_1^-}{D_1^-} + \frac{d_2^-}{D_2^-} + \frac{d_1^+}{D_1^+} + \frac{d_2^+}{D_2^+} = 1 \quad \text{[Factor membership value]} \tag{7d}$$

$$0 \le d_1^- \le D_1^-, \;\; 0 \le d_2^- \le D_2^- \quad \text{[Negative deviation ranges]} \tag{7e}$$

$$0 \le d_1^+ \le D_1^+, \;\; 0 \le d_2^+ \le D_2^+ \quad \text{[Positive deviation ranges]} \tag{7f}$$

Finally, the objective function for the full optimization model minimizes the sum of ratios of positive and negative deviations subject to the set of constraints for all responses and process factors. Solving the complete optimization model for each response's replicate, the combination of optimal factor settings, x^*, can then be obtained for all response replicates.

Step III: Let $\tilde{y}_j(\tilde{x}^q)$ be the value of response j; $j = 1, \dots, Q$ obtained by substituting optimal factor levels, $\tilde{x}^q$, for quality characteristic q; $q = 1, \dots, Q$. Calculate the $\tilde{y}_j(\tilde{x}^q)$ values for all q responses utilizing Eq. (2). Let $\tilde{d}_j(\tilde{y}_j(\tilde{x}^q))$ denotes the fuzzy desirability function are defined as:

NTB response (8a)

$$\tilde{d}_j(\tilde{y}_j(\tilde{x}^q)) = \begin{cases} 0, & \tilde{y}_j(\tilde{x}^q) \le y_{\min} \\ \frac{\tilde{y}_j(\tilde{x}^q) - y_{\min}}{y_{\max} - y_{\min}}, & y_{\min} \le \tilde{y}_j(\tilde{x}^q) \le y_{\max} \\ 1, & \tilde{y}_j(\tilde{x}^q) \ge y_{\max} \end{cases} \quad \text{LTB response} \tag{8b}$$

$$\tilde{d}_j(\tilde{y}_j(\tilde{x}^q)) = \begin{cases} 1, & \tilde{y}_j(\tilde{x}^q) \le y_{\min} \\ \frac{\tilde{y}_j(\tilde{x}^q) - y_{\max}}{y_{\min} - y_{\max}}, & y_{\min} \le \tilde{y}_j(\tilde{x}^q) \le y_{\max} \\ 0, & \tilde{y}_j(\tilde{x}^q) \ge y_{\max} \end{cases} \quad \text{STB response} \tag{8c}$$

Utilizing the $\tilde{d}_j(\tilde{y}_j(\tilde{x}^q))$ functions, obtain the $\tilde{d}_j^{a,b,c,d,e}(\tilde{x}^q)$ values for each response j. The $\widetilde{U}_j$ (Eq. 9) and $\tilde{L}_j$ (Eq. 10); upper and lower deviation values, respectively, are then estimated as:

$$\widetilde{U}_j^{a,b,c,d,e} = \tilde{d}_j(y_j(\tilde{x}^j)) = \tilde{d}_{jj} = (\widetilde{U}_j^a, \widetilde{U}_j^b, \widetilde{U}_j^c, \widetilde{U}_j^d, \widetilde{U}_j^e), \forall j \tag{9}$$

$$\tilde{L}_j^{a,b,c,d,e} = Min\,\{\tilde{d}_j(y_j(\tilde{x}^{(1)})), \dots, \tilde{d}_j(y_j(\tilde{x}^Q))\} = (\tilde{L}_j^a, \tilde{L}_j^b, \tilde{L}_j^c, \tilde{L}_j^d, \tilde{L}_j^e), \forall j \tag{10}$$

Let $\widetilde{D}_j(\tilde{y}_j(\tilde{x}^q))$ denotes the deviation function of $\tilde{y}_j(\tilde{x}^q)$ and is calculated as (Eq. 11):

$$\widetilde{D}_j(\tilde{y}_j(\tilde{x}^q)) = \frac{\tilde{y}_j^e(\tilde{x}^q) - \tilde{y}_j^d(\tilde{x}^q)}{1 - \lambda}, \forall j \tag{11}$$

Then, calculate the $\widetilde{D}_j(\tilde{y}_j(\tilde{x}^q))$ values for all $\tilde{y}_j(\tilde{x}^q)$. Finally, calculate the $\tilde{P}_j$ (Eq. 12) and $\tilde{Q}_j$ (Eq. 13) as follows:

$$\tilde{P}_j^{a,b,c,d,e} = \widetilde{D}_j(\tilde{x}^j) = \widetilde{D}_{jj} = (\tilde{P}_j^a, \tilde{P}_j^b, \tilde{P}_j^c, \tilde{P}_j^d, \tilde{P}_j^e), \forall j \tag{12}$$

$$\tilde{Q}_j^{a,b,c,d,e} = Max\{\widetilde{D}_j(\tilde{y}_j(\tilde{x}^{(1)}), \dots, \widetilde{D}_j(\tilde{y}_j(\tilde{x}^G)\} = (\tilde{Q}_j^a, \tilde{Q}_j^b, \tilde{Q}_j^c, \tilde{Q}_j^d, \tilde{Q}_j^e), \forall j \tag{13}$$

Step IV: Formulate the complete fuzzy optimization models.
The optimization model consists of two multiple objectives; to maximize desirability and minimize the deviation. That is,

$$\begin{aligned} &Max\{\text{Desirability Function}\} = Max\{\tilde{d}_1(\tilde{y}_1(\tilde{x})), \dots, \tilde{d}_j(\tilde{y}_Q(\tilde{x}))\} \\ &Min\{\text{Deviation Function}\} = Min\{\widetilde{D}_1(\tilde{y}_1(\tilde{x})), \dots, \widetilde{D}_Q(\tilde{y}_Q(\tilde{x}))\} \\ &\text{s.t.} \\ &x \in \text{[Factor levels]} \end{aligned}$$

To formulate the model with a single objective function, two fuzzy functions; $\tilde{S}_j^{a,b,c,d,e}(\tilde{y}_j(\tilde{x}^j))$ and $\tilde{T}_j^{a,b,c,d,e}(\tilde{y}_j(\tilde{x}^j))$, will be introduced which are calculated respectively using Eqs. 14 and 15.

$$\tilde{S}_j^{a,b,c,d,e}(\tilde{y}_j(\tilde{x}^j)) = \begin{cases} 0, & \tilde{d}_j^{a,b,c,d,e}(\tilde{y}_j(\tilde{x}^j)) \le \tilde{L}_j^{a,b,c,d,e} \\ \dfrac{\tilde{d}_j^{a,b,c,d,e}(\tilde{y}_j(\tilde{x}^j)) - \tilde{L}_j^{a,b,c,d,e}}{\tilde{U}_j^{a,b,c,d,e} - \tilde{L}_j^{a,b,c,d,e}}, & \tilde{L}_j^{a,b,c,d,e} \le \tilde{d}_j^{a,b,c,d,e}(\tilde{y}_j(\tilde{x}^j)) \le \tilde{U}_j^{a,b,c,d,e}\ \forall j \\ 1, & \tilde{d}_j^{a,b,c,d,e}(\tilde{y}_j(\tilde{x}^j)) \ge \tilde{U}_j^{a,b,c,d,e} \end{cases} \quad (14)$$

$$\tilde{T}_j^{a,b,c,d,e}(\tilde{y}_j(\tilde{x}^j)) = \begin{cases} 1, & \tilde{D}_j^{a,b,c,d,e}(\tilde{y}_j(\tilde{x}^j)) \le \tilde{P}_j^{a,b,c,d,e} \\ \dfrac{\tilde{Q}_j^{a,b,c,d,e} - \tilde{D}_j^{a,b,c,d,e}(\tilde{y}_j(\tilde{x}^j))}{\tilde{Q}_j^{a,b,c,d,e} - \tilde{P}_j^{a,b,c,d,e}}, & \tilde{P}_j^{a,b,c,d,e} \le \tilde{D}_j^{a,b,c,d,e}(\tilde{y}_j(\tilde{x}^j)) \le \tilde{Q}_j^{a,b,c,d,e}\ \forall j \\ 0, & \tilde{D}_j^{a,b,c,d,e}(\tilde{y}_j(\tilde{x}^j)) \ge \tilde{Q}_j^{a,b,c,d,e} \end{cases} \quad (15)$$

Let

$$Min\ \tilde{S}_j(\tilde{y}_j(\tilde{x}^j)) = \tilde{S} \quad (16)$$

$$Min\ \tilde{T}_j(\tilde{y}_j(\tilde{x}^j)) = \tilde{T} \quad (17)$$

Let w_1 and w_2 represent the assigned weights for desirability and robustness, which are usually chosen by decision maker based on cost and warranty. Then, formulate the final optimization model as:

$$\begin{gathered} Max\ \ w_1 \tilde{S}^{a,b,c,d,e} + w_2 \tilde{T}^{a,b,c,d,e} \\ \text{s.t.} \\ \tilde{d}_j^{a,b,c,d,e}(x) - \tilde{S}^{a,b,c,d,e}(\tilde{U}_j^{a,b,c,d,e} - \tilde{L}_j^{a,b,c,d,e}) \ge \tilde{L}_j^{a,b,c,d,e} \quad ,\forall j \\ \tilde{D}_j^{a,b,c,d,e}(x) + \tilde{T}^{a,b,c,d,e}(\tilde{Q}_j^{a,b,c,d,e} - \tilde{P}_j^{a,b,c,d,e}) \le \tilde{Q}_j^{a,b,c,d,e} \quad ,\forall j \\ 0 \le \tilde{S}^{a,b,c,d,e} \le 1 \\ 0 \le \tilde{T}^{a,b,c,d,e} \le 1 \\ x \in [\text{Factor levels}]. \end{gathered}$$

Solve the optimization model to identify the optimal factor levels for each of the fuzzy number elements; a, b, c, d, and e.

Step V: Apply the proposed optimization procedure on real case studies, and then analyze and discuss the optimization results. Finally, compare the anticipated improvements in the fuzzy quality characteristics by the proposed optimization procedure with those using other technique(s) in previous studies.

2.2 Experimental work

Several studies [13-18] were conducted to optimize process performance for a product's multiple quality characteristics. In this research, two processes were employed to illustrate the proposed optimization procedure and presented as follows:

Case study 1: Wire electro-discharge machining

Ramakrishnan and Karunamoorthy [13] optimized performance of wire electro-discharge machining (WEDM) of material removal rate (MRR, y_1) and surface roughness (SR, y_2) using artificial neural network (ANN) models and multi-response optimization technique. The MRR and SR the larger-the-better (LTB) and the smaller-the-better (STB) type quality characteristics, respectively. Four three-level controllable process factors were examines; the pulse on time (x_1), delay time (x_2), wire feed speed (x_3), and ignition current (x_4). The Taguchi's L_9 orthogonal array was utilized for experimental design as shown in Table 1. After randomization, each experiment was conducted. Experimentation was replicated twice. Finally, the MRR (mm^2/min) and SR (mm^2/min) values were measured. Let y_{i11} and y_{i12} denote the MRR values for the first and second replicate at the i-th experiment, respectively. Similarly, let y_{i21} and y_{i22} denote the SR values for the first and second replicate at the i-th experiment, respectively. Table 1 also displays the measured MRR and SR values for all experiments.

Table 1 Results of designed experiments for WEDM [13]

Exp. i	Control factor				MRR (mm²/min)		SR(mm²/min)	
	x_1	x_2	x_3	x_4	y_{i11}	y_{i12}	y_{i21}	y_{i22}
1	0.6	4	8	8	46	46	3.2	3.1
2	0.6	6	12	12	48	47	3.3	3.2
3	0.6	8	15	16	42	41	3.3	3.3
4	0.8	4	12	16	56	55	3.8	3.7
5	0.8	6	15	8	50	49	3.4	3.5
6	0.8	8	8	12	52	53	3.2	3.3
7	1.2	4	15	12	70	71	4.2	4
8	1.2	6	8	16	74	73	3.8	3.5
9	1.2	8	12	8	46	46	3.2	3.1

The multilinear regression models for MRR (y_{11}, y_{12}) and SR replicates, (y_{21} and y_{22}) are expressed respectively as:

Regression model for response replicate	$R^2{}_{adjusted}$
$y_{11} = -2.68 + 41.88x_1 + 4.26x_2 - 1.251\,x_3 + 3.82\,x_4 - 0.458x_2^2 + 0.119x_2x_3 - 0.166\,x_4^2$	99.48 %
$y_{12} = -2.53 + 42.319x_1 + 4.21x_2 - 1.175\,x_3 + 3.76\,x_4 - 0.333x_2^2 + 0.1087\,x_2x_3 - 0.1934\,x_4^2$	99.99 %
$y_{21} = 2 + 2.207\,x_1 - 0.04x_2 + 0.0233x_3 + 0.0284\,x_4 - 0.278\,x_1^2 - 0.1364\,x_1x_2 + 0.00417\,x_2^2$	97.43 %
$y_{22} = 1.326 + 2.155\,x_1 - 0.0446x_2 + 0.0254\,x_3 + 0.0199\,x_4 - 0.2805x_1^2 - 0.169\,x_1x_2 + 0.0083\,x_2^2$	98.30 %

The fuzzy numbers of the intercept and regression coefficients are calculated and then the multiple regression, $\tilde{y}_1(\tilde{x})$, for y_1 is constructed as ($\lambda = 0.15$):

$$\begin{aligned}\tilde{y}_1(\tilde{x}) = {} & (-2.9,-2.71,-2.68,-2.65,-2.46) + (41.27,41.79,41.88,41.97,42.49)\tilde{x}_1 + (4.18,4.24,4.26,4.27,4.33)\tilde{x}_2 + \\ & (-1.26,-1.22,-1.21,-1.2,-1.15)\tilde{x}_3 + (3.73,3.8,3.82,3.83,3.9)\tilde{x}_4 + (-0.63,-0.48,-0.45,-0.43,-0.28)\tilde{x}_2^2 \\ & +(0.1,0.11,0.12,0.121,0.13)\tilde{x}_2\tilde{x}_3 + (-0.2,-0.17,-0.166,-0.16,-0.13)\tilde{x}_2\tilde{x}_4\end{aligned}$$

The multiple regression, $\tilde{y}_2(\tilde{x})$, for y_2 is constructed in a similar manner as:

$$\begin{aligned}\tilde{y}_2(\tilde{x}) = {} & (0.37,1.18,1.32,1.46,2.27) + (2.08,2.14,2.15,2.16,2.22)\tilde{x}_1 + (-0.051,-0.045,-0.044,-0.043,-0.038)\tilde{x}_2 \\ & + (0.022,0.025,0.0254,0.026,0.028)\tilde{x}_3 + (0.007,0.018,0.019,0.021,0.031)\tilde{x}_4 \\ & + (-0.284,-0.281,-0.28,-0.279,-0.276)\tilde{x}_1^2 \\ & +(-0.21,-0.17,-0.169,-0.162,-0.122)\tilde{x}_1\tilde{x}_2 + (0.002,0.007,0.008,0.009,0.014)\tilde{x}_2^2\end{aligned}$$

Utilizing the formulations in Steps I and II, the optimization model is developed and then solved to determine the combination of optimal factor settings for each replicate of the two responses. The obtained optimization results for each replicate are displayed for all response replicates in Table 2. The optimization models for $\tilde{y}_{12}$, $\tilde{y}_{21}$, and $\tilde{y}_{22}$, are formulated and then solved in a similar manner. The results of optimal factor settings are also shown in Table 2.

The results shown in Table 2 are utilized in Steps III and IV to generate the pentagon fuzzy numbers for optimal factor settings; $\tilde{x}^{(1)}$ and $\tilde{x}^{(2)}$, of $\tilde{y}_1$ and $\tilde{y}_2$, respectively. The obtained $\tilde{y}_j(\tilde{x}^g)$ values are displayed in Table 3.

The $\tilde{d}_1(\tilde{y}_1(\tilde{x}^q))$, $\tilde{d}_2(\tilde{y}_2(\tilde{x}^q))$, $\tilde{D}_1(\tilde{y}_1(\tilde{x}^q))$, and $\tilde{D}_2(\tilde{y}_2(\tilde{x}^q))$ values are then calculated for both responses and the results are shown in Table 4.

Table 2 Optimal results for WEDM

Optimal level	Response replicate			
	$\tilde{y}_{11}$	$\tilde{y}_{12}$	$\tilde{y}_{21}$	$\tilde{y}_{22}$
x_1^*	0.9341	1.0254	0.6000	1.1345
x_2^*	4.0000	4.0000	4.0000	8.0000
x_3^*	8.0000	13.6120	8.0000	8.0000
x_4^*	8.0000	8.0000	8.0000	8.0000

Table 3 The calculated $\tilde{y}_j(\tilde{x}^g)$ values for WEDM

Optimal combination	$\tilde{y}_1(\tilde{x}^q)$	$\tilde{y}_2(\tilde{x}^q)$
$\tilde{x}^{(1)}$	(52.46, 58.8, 60, 61.2, 68.5)	(1.29, 2.65, 2.89, 3.13, 4.54)
$\tilde{x}^{(2)}$	(33.5, 54.65, 58.61, 62.7, 89.91)	(1.09, 2.26, 2.49, 2.74, 4.53)

Table 4 The calculated matrix of $\tilde{d}_j^{a,b,c,d,e}\left(\tilde{y}_j(\tilde{x}^q)\right)$ and $\widetilde{D}_j^{a,b,c,d,e}\left(\tilde{y}_j(\tilde{x}^q)\right)$ of MMR and SR

	$\tilde{d}_1(\tilde{y}_1(\tilde{x}^q))$	$\tilde{d}_2(\tilde{y}_2(\tilde{x}^q))$	$\widetilde{D}_1(\tilde{y}_1(\tilde{x}^g))$	$\widetilde{D}_2(\tilde{y}_2(\tilde{x}^g))$
$\tilde{x}^{(1)}$	(0.098, 0.352, 0.399, 0.448, 0.7402)	(0.012, 0.317, 0.371, 0.423, 0.717)	(7.746, 8.164, 8.238, 8.312, 8.73)	(1.432, 1.457, 1.461, 1.466, 1.491)
$\tilde{x}^{(2)}$	(0, 0.186, 0.344, 0.508, 1)	(0.014, 0.402, 0.457, 0.508, 0.762)	(6.41, 10.78, 11.762, 12.81, 19.94)	(1.263, 1.567, 1.635, 1.708, 2.203)

The optimization models a to e were formulated for WEDM and then solved. For illustration, the Model a is formulated as:

$$
\begin{gathered}
Max \quad 0.5 \times S^a + 0.5 \times T^a \\
\text{s. j.} \\
-2.11 + 1.65x_1^a + 0.167x_2^a - 0.05x_3^a + 0.149x_4^a - 0.025x_2^a x_2^a + 0.0041x_2^a x_3^a - 0.0081x_4^a x_4^a - 0.0985S^a \geq 0 \\
0.504 - 0.484x_1^a + 0.0082x_2^a - 0.0062x_3^a - 0.0069x_4^a + 0.0602x_1^a x_1^a + 0.026x_1^a x_2^a - 0.0031x_2^a x_2^a - 0.0026S^a \geq 0.01208 \\
0.219 + 0.6144x_1^a + 0.0707x_2^a + 0.0537x_3^a + 0.084x_4^a + 0.1765x_2^a x_2^a + 0.015x_2^a x_3^a + 0.0374x_4^a x_4^a + 0T^a \leq 7.7462 \\
0.953 + 0.0735x_1^a + 0.0065x_2^a + 0.003x_3^a + 0.012x_4^a + 0.0035x_1^a x_1^a + 0.0468x_1^a x_2^a + 0.0058x_2^a x_2^a + 0.1691T^a \leq 1.4323 \\
0 \leq S^a \leq 1 \\
0 \leq T^a \leq 1
\end{gathered}
$$

The optimization models b to e are formulated and then solved in a similar manner.

Case study 2: Sputtering process of gallium-doped zinc oxide GZO films

Chen *et al.* [14] optimized five process controllable factors; R.F. power (x_1), sputtering pressure (x_2), deposition time (x_3), substrate temperature (x_4), and post-annealing temperature (x_5) of GZO films deposited on polyethylene terephthalate substrates by magnetron sputtering using the Taguchi method. Three important quality responses were considered including: deposition rate (DR, y_1) electrical resistively (ER, y_2) and optical transmittance (OT, y_3), which are LTB, STB, and LTB type quality characteristics, respectively. The Taguchi's L_{18} orthogonal array shown in Table 5 was employed to provide experimental layout, where each experiment was repeated twice. Let y_{i1r}, y_{i2r}, and y_{i3r} denote the r-th replicate ($r = 1$ or 2) of DR, ER, and OT responses at experiment i; $i = 1, \ldots, 18$, respectively. Table 5 also displays the experimental data of y_{i1r}, y_{i2r}, and y_{i3r} for the sputtering process.

Following the proposed procedure, the optimization models were constructed to determine the combination of optimal factor settings that optimizes each response replicate. Table 6 lists the obtained optimal factor settings for each replicate for all responses.

Utilizing the results in Table 6, the optimization model a to e were formulated for the sputtering process and then the combination of optimal fuzzy process factors were determined.

Table 5 Experimental data for sputtering process [14]

Exp. i	Factor					DR		ER		OT	
	x_1	x_2	x_3	x_4	x_5	y_{i11}	y_{i12}	y_{i21}	y_{i22}	y_{i31}	y_{i32}
1	50	0.13	30	25	0	4.5	4.7	14.9	15.3	88.4	88.4
2	50	0.67	60	50	100	5.6	5.6	9.8	9.7	87.7	87.7
3	50	1.33	90	100	200	5.0	4.9	7.9	7.8	88.1	88.1
4	100	0.13	30	50	100	9.6	9.3	5.4	5.6	89.2	89.3
5	100	0.67	60	100	200	11.1	11.3	4.6	4.3	87.1	87.0
6	100	1.33	90	25	0	10.0	10.0	6.5	6.6	84.7	84.7
7	200	0.13	60	25	200	19.9	20.2	1.6	1.7	86.6	86.6
8	200	0.67	90	50	0	21.6	21.6	1.9	2.0	82.3	82.4
9	200	1.33	30	100	100	20.9	20.9	1.8	1.6	85.6	85.3
10	50	0.13	90	100	100	4.8	4.6	7.3	7.0	87.6	87.6
11	50	0.67	30	25	200	5.0	4.9	6.9	7.1	89.1	89.1
12	50	1.33	60	50	0	4.9	4.8	7.8	7.7	87.4	87.4
13	100	0.13	60	100	0	9.7	9.7	6.1	5.9	87.0	87.0
14	100	0.67	90	25	100	11.1	11.6	6.0	5.8	83.7	83.7
15	100	1.33	30	50	200	10.7	10.8	5.5	5.7	88.4	88.3
16	200	0.13	90	50	200	19.5	19.4	1.0	1.1	83.1	83.1
17	200	0.67	30	100	0	22.1	22.0	1.2	1.3	85.7	85.7
18	200	1.33	60	25	100	20.5	20.5	1.4	1.3	83.9	83.7

Table 6 Optimization results for sputtering process for each response replicate

Factors	y_{11}	y_{12}	y_{21}	y_{22}	y_{31}	y_{32}
x_1^*	92.60	94.03	200.00	200.00	50.00	50.00
x_2^*	1.33	1.33	1.33	1.33	1.31	0.13
x_3^*	30.00	30.00	30.00	30.00	30.00	30.00
x_4^*	25.00	25.00	25.00	25.00	25.50	25.00
x_5^*	200.00	1.24	1.22	4.71	1.24	1.24

3. Results and discussion

In Step V, two case studies were utilized to illustrate the proposed procedure. The optimal results for each case study are discussed and compared with those obtained by previously used techniques in the following subsections.

3.1 Optimal wire electrical-discharge machining

Using Lingo 11 package, the combinations of the fuzzy optimal factor settings for each of models a, b, c, d and e of the Wire electrical-discharge machining were obtained and are then listed in Table 7. For example, the obtained combination of optimal factor settings (x_1^*, x_2^* , x_3^* , x_4^*) by solving model a is (0.9019, 4.00, 8.00, 8.331).

Finally, the $\tilde{y}_1$ and $\tilde{y}_2$ values of MRR and SR, respectively, are calculated at all combinations of optimal factor settings and found to be (51.05, 56.96, 59.22, 61.21, 70.02) and (1.31, 2.48, 2.72, 2.89, 4.16) mm^2/min, respectively. The results reveal the effect of process variability on MRR (LTB type) and SR (STB type). For illustration, when the combination of optimal process factor settings by model a is selected, the MRR and SR are 51.05 and 1.31, respectively. However, when the optimal combination of factor settings by model e is chosen, the MRR and SR are 70.02 and 4.16, respectively. Clearly, model a provides the largest improvement in SR, whereas model e provides the largest improvement in MRR. Consequently, process engineering can select and then control the proper combination of optimal settings from models a to e that satisfies product requirements.

Table 7 Fuzzy optimal factor settings for WEDM

Optimal level	Model a	Model b	Model c	Model d	Model e
$\tilde{x}_1^*$	0.9019	0.8809	0.9045	0.9067	0.8810
$\tilde{x}_2^*$	4.0000	4.1870	4.2220	4.6920	4.9460
$\tilde{x}_3^*$	8.0000	8.0000	8.0000	8.0000	8.0000
$\tilde{x}_4^*$	8.3310	8.0000	8.0000	8.0000	8.8750

3.2 Optimal sputtering process

Solving models a to e of the sputtering process, the combination of optimal factor settings (x_1^*, x_2^*, x_3^*, x_4^*) were found determined and displayed in Table 8.

Then, the $\tilde{y}_1$, $\tilde{y}_2$, and $\tilde{y}_3$ values for DR (nm/min), ER (10^{-4} Ω cm), and OT (%), respectively, were calculated at the optimal factor settings of models a, b, c, d and e and found to be (11.895, 12.282, 14.886, 18.071, 21.946), (2.848, 3.942, 4.098, 4.784, 5.19) and (88.218, 88.276, 88.395, 88.626, 89.174), respectively. Obviously, the combinations of optimal factor settings vary due to the observed fuzziness in the DR, ER, and OT. For example, the DR value at optimal setting of model a is 11.895, while it is 21.946 using optimal setting of model e. Note that the optimal setting of factor x_4, x_4^*, changes from 62.852 to 25. Similarly, optimal setting of factor x_1, x_1^*, changes from 127.605 to 195.153. Although the optimal settings of model e results in improving the DR (LTB type), but it worsens the ER (STB type). Consequently, process engineers should decide and control the settings of these two factors to avoid the negative impacts of their variations on quality characteristics.

Table 8 Fuzzy optimal factor settings for sputtering process

Factor	Model a	Model b	Model c	Model d	Model e
x_1^*	127.605	129.091	143.363	161.715	195.153
x_2^*	0.130	0.130	0.130	0.130	0.130
x_3^*	30.00	30.000	30.000	30.000	30.000
x_4^*	62.852	63.852	69.967	60.160	25.000
x_5^*	0.000	0.000	0.000	0.000	0.000

3.3 Comparison of the results

For WEDM process, the comparison between the optimization results by ANN [13] and these obtained by the proposed procedure is made in Table 9, where it is noticed that at the initial (ANN) process factor settings of WEDM process, the values of MRR (mm^2/min) and SR (mm^2/min) were 64 (71) and 3.48 (3.198), respectively. However, these approaches fail to consider the effects of process variations/fuzziness on quality responses. Using the proposed procedure, the MRR and SR values of (51.05, 56.96, 59.22, 61.21, 70.02) and (1.31, 2.48, 2.72, 2.89, 4.16), respectively. This result clearly reveals the existence of fuzziness between the observed measurements of each response replicates, which is ignored when optimizing process settings by ANN, thereby misleading process engineers. Moreover, the proposed procedure provides better understanding on how to choose the optimal factor settings that satisfy both quality responses. In other words, if the SR (mm^2/min) is more important than the MRR, the minimal value that can be reached by ANN technique is 3.198. However, utilizing the optimal settings of model a from the proposed approach results in reducing the values of SR and MRR to 1.31 and 51.05, respectively. Further, the ANN technique requires defining initial parameter settings of learning and prediction stages which are usually unknown, which the uncertainty in the decision making process about optimal factor settings.

Table 10 displays the comparison between the DR, ER, and OT values at the initial and the optimal factor settings of the sputtering process using the Grey-Taguchi [14] and the proposed approach. It is found that the DR (nm/min), ER (10^{-4} Ω cm), and OT (%) values at initial (Grey-Taguchi) factor settings were 21.033 (20.922), 11.9 (8.627) and 86.627 % (90.00 %). While, their corresponding values by using the proposed procedure are (11.895, 12.282, 14.886, 18.071, 21.946), (2.848, 3.942, 4.098, 4.784, 5.19), and (88.218, 88.276, 88.395, 88.626, 89.174), respectively. Obviously, settings process factors of sputtering process at those obtained by model e of the proposed approach leads to achieve significant improvement; that is, DR, ER, and OT values of 21.946, 5.19, and 89.174, respectively, in the three quality responses when compared to those at the initial and the Grey-Taguchi method. Further, the Grey-Taguchi method is a non-parametric approach that cannot guarantee optimal factor settings and failed to consider the effect of fuzziness on the three quality characteristics.

Table 9 MRR and SR at the combination of optimal fuzzy factor settings of WEDM

Response	Initial setting	ANN [13]	Proposed models
MRR (mm^2/min)	64	71	(51.05, 56.96, 59.22, 61.21, 70.02)
SR (mm^2/min)	3.48	3.198	(1.31, 2.48, 2.72, 2.9, 4.16)

Table 10 MRR and SR at the combination of optimal fuzzy factor settings of sputtering process

Response	Initial setting	Grey-Taguchi [14]	Proposed models
DR (nm/min, LTB)	21.033	20.922	(11.895, 12.282, 14.886, 18.071, 21.946)
ER (10^{-4} Ω cm, STB)	11.9	8.627	(2.848, 3.942, 4.098, 4.784, 5.19)
OT (%, LTB)	86.148	90	(88.218, 88.276, 88.395, 88.626, 89.174)

In summary, the proposed procedure has the following advantages:

- effectiveness in dealing with fuzziness through the use of pentagon fuzzy regression models;
- depiction of the relationship between quality response and process factors, which helps process engineers in identifying the critical process factors and determining the required adjustments in process levels to increase the anticipated improvements in desirable quality responses;

- guaranteeing optimality of factor levels, and
- conveying valuable information to process engineers in understanding the impact of fuzziness on process performance and then guiding them to take proper improvement actions.

4. Conclusion

This research proposed an efficient procedure for optimizing process performance for fuzzy multiple quality responses utilizing pentagon regression modelling. Initially, the optimal factor settings were determined for each replicate of a quality response. Then, the optimal factor settings for the replicates of each quality response were combined to construct a pentagon fuzzy regression model for this quality response. The combinations of fuzzy responses, desirability, and deviations values were constructed and utilized in formulating an optimization model, which was then solved for each element of the pentagon fuzzy number to decide optimal factor settings for multiple quality responses concurrently. Two case studies were employed for illustration, where the results of both cases revealed that the proposed procedure efficiently dealt with fuzziness problem in quality responses. In conclusion, the developed optimization procedure provided valuable assistance to process engineering when optimizing process performance for fuzzy multiple quality responses.

Conflicts of interest

The authors declare no conflict of interest.

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