A multi-criteria decision-making in turning process using the MAIRCA, EAMR, MARCOS and TOPSIS methods: A comparative study

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ABSTRACT

Multi-criteria decision-making is important, and it affects the efficiency of a mechanical processing process as well as an operation in general. It is understood as determining the best alternative among many alternatives. In this study, the results of a multi-criteria decision-making study are presented. In which, sixteen experiments on turning process were carried out. The input parameters of the experiments are the cutting speed, the feed speed, and the depth of cut. After conducting the experiments, the surface roughness and the material removal rate (MRR) were determined. To determine which experiment guarantees the minimum surface roughness and maximum MRR simultaneously, four multi-criteria decision-making methods including the MAIRCA, the EAMR, the MARCOS, and the TOPSIS were used. Two methods the Entropy and the MEREC were used to determine the weights for the criteria. The combination of four multi-criteria making decision methods with two determination methods of the weights has created eight ranking solutions for the experiments, which is the novelty of this study. An amazing result was obtained that all eight solutions all determined the same best experiment. From the obtained results, a recommendation was proposed that the multi-criteria making decision methods and the weighting methods using in this study can also be used for multi-criteria making decision in other cases, other processes.

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1. Introduction

Multi-criteria decision-making methods are used in many fields. These methods help to compare alternatives and find the best one [1]. For a mechanical machining process as well as a turning process, multi-criteria decision-making is very important. It can be said that because among
many input parameters to evaluate the turning process, sometimes there are parameters that the objective function sets for them are often opposite. For example, when high-speed turning to improve machining productivity, the tool wear rate is also large, causing the decrease of the tool life [2]. To increase the MRR, it is necessary to increase the feed rate and the depth of cut, but this increases the surface roughness [3]. Besides, to reduce the tool wear rate, it is necessary to increase the flow and the concentration of the coolant to reduce the cutting heat. However, doing that will not only increase the manufacturing cost but also affect the environment. In addition, increasing the machining productivity will often increase the cutting tool vibration, and lead to the reduction of the tool life and increase the surface roughness [4], etc. For the above reasons, many studies on multi-criteria decision-making for turning process have been carried out.

The TOPSIS is the most used method for multi-criteria decision-making in many different fields [5, 6]. This method has also been used for multi-criteria decision-making for turning processes in many studies. These studies usually focus on selecting optimal input process parameters to ensure multiple criteria at the same time such as: Ensuring the minimum surface roughness and the maximum MRR when processing Glass fiber reinforced polyester materials (GFRP) [7]; Ensuring all of six parameters of the surface roughness (including $R_{q}$, $R_{a}$, $R_{z}$, $R_{ku}$, $R_{z}$, $R_{sm}$) have the same minimum value when turning GFRP materials [8]; Ensure the minimum surface roughness ($R_{a}$ and $R_{c}$) and the maximum MRR when turning EN19 steel [9]; Simultaneously ensuring the minimum surface roughness and tool wear rate, and the maximum MRR when turning 1030 steel [10]; Simultaneously ensuring the minimum surface roughness, the cutting force, the tool wear and the cutting heat, and the maximum MRR when turning pure Titanium [11]. Ensuring the minimum surface roughness, the cutting force and the tool wear when turning CP-Ti grade II material [12]; Simultaneously ensuring the minimum surface roughness, the cutting force, the tool wear and the cutting temperature when turning Ti-6Al-4V alloy [13]; Simultaneously ensuring the minimum surface roughness, and the maximum MRR when turning AISI D2 steel [14]; Simultaneously ensuring the minimum surface roughness, and the maximum MRR when turning Al 6351 alloy [15]; Simultaneously ensuring the minimum surface roughness, the minimum roundness deviation and the minimum tool wear when turning 9XC steel [16], etc. Recently, the TOPSIS method and six other methods including the SAW, the WASPAS, the VIKOR, the MOORA, the COPRAS, and the PIV, have been used in multi-criteria decision-making when turning 150Cr14 steel and the best option was received for all of methods [17].

In the last few years, scientists have also proposed new decision-making methods. Three of those methods are MAIRCA, EAMR, and MARCOS methods.

The MAIRCA method was first introduced in 2018 [18]. The outstanding advantage of this method other than other methods is that the objectives can be in both qualitative and quantitative types. There have been several studies which applied this method to multi-criteria decision-making. For example, determining the most effective time (year) in mergers and acquisitions of companies in Turkey during the period 2015-2019 [19]; determining the best performing airline out of eleven emerging airlines from Turkey, Mexico, China, Indonesia and Brazil [20]; selecting a partner for a food company in Turkey [21]; preventing the Covid-19 epidemic to the sustainable development of OECD countries [22].

The EAMR method was discovered in 2016 [23]. This method has been used for a number of studies such as: selecting partners to hire for logistics [24]; selecting contract types of health care services [25]; deciding the order quantity for each supplier to ensure environmental criteria [26].

The MARCOS method was first used in 2019 [27]. This method has been applied in several studies such as: in the selection of intermediate modes of transport between countries in the Danube region [28]; for minimizing risks in the transportation [29], in selection of lifting equipment for services in warehouses [30]; for the selection of human resources for transportation companies [31], or for the cost selection in the construction [32].

Although the three methods MAIRCA, EAMR and MARCOS have been used in some studies as described above, so far there has been no research on the application of any of these methods for multi-criteria decision-making for the turning process. The combination of three methods
A multi-criteria decision-making in turning process using the MAIRCA, EAMR, MARCOS and TOPSIS methods: A …

(MAIRCA, EAMR and MARCOS) with TOPSIS method is the basis for assessing the accuracy of the results obtained. This is the first reason for doing this study.

When performing multi-criteria decisions, an important task is to determine the weights for the criteria. This has a great influence on the ranking order of the alternatives [33]. If it is done by the decision maker, the accuracy achieved is not high because it depends on the knowledge as well as the subjective thoughts of that person. If it is determined by consulting the experts, its accuracy will depend on the experience of the experts as well as the way the questionnaires are presented, which is also very time consuming and high cost [34]. To overcome these limitations, it is necessary to determine the weights for the criteria based on mathematical models. In this way, the weight of the criteria is determined independent of the subjectivity of the decision maker. The Entropy is known as a method of determining weights with high accuracy, which has been used in many cases. When it is necessary to compare multi-criteria decision-making methods, the Entropy method is also recommended to use to determine the weights for the criteria [17].

MEREC is a weighting method which introduced in 2021 [35]. This method has been used to determine the weights for criteria such as: decision-making to determine the location of logistics distribution centers [36]; decision-making for documental classification [37]; etc. However, up to now this method has not been used to determine the weights for criteria in turning processes. The simultaneous use of two methods (the MEREC and the Entropy) to determine weights is the basis for evaluating stability when determining the best solution of multi-criteria decision-making methods. This is the second reason for doing this study.

Surface roughness and MRR are two commonly used parameters to evaluate turning processes. The reason is that the surface roughness has a great influence on the workability and durability of the products through the wear resistance, the chemical corrosion resistance, and the accuracy of the joint (for tight joints) [3], while MRR is an important factor to evaluate the cutting productivity [38]. Besides, determining the values of these parameters is also simpler than other that of parameters, such as the cutting force, the cutting temperature, or the vibration in the cutting process. This is the reason why this study will also use the surface roughness and MRR as two indicators to evaluate turning process.

This study presents the results of experimental research on turning process with two parameters to evaluate the turning process, namely surface roughness and MRR. In addition, the Entropy and the MEREC are two methods used to determine the weights for the criteria (surface roughness and MRR). Also, four methods including the MAIRCA, the EAMR, the MARCOS, and the TOPSIS will be used to make multi-criteria decision for turning process. The purpose of multi-criteria decision-making is to ensure simultaneous minimum surface roughness and maximum MRR.

2. Used methods of multi-criteria decision-making

2.1 The MAIRCA method

The steps to implement multi-criteria decision-making according to the MAIRCA method are as follows [18].

**Step 1:** Building the initial matrix according to the following equation:

\[ X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \] (1)

where \( m \) is the number of options; \( n \) is the number of criteria; \( x_{mn} \) is the value of the \( n \) criterion in \( m \).
Step 2: Determining the priority for an indicator. When the decision maker is neutral, the role of the indicators is the same (no priority is given to any). Then the priority for the criteria is the same and is calculated as follows:

\[ P_{Aj} = \frac{1}{m}, j = 1, 2, \ldots, n \]  

(2)

Step 3: Calculating the quantities \( t_{p_{ij}} \) according to the equation:

\[ t_{p_{ij}} = P_{Aj} \cdot w_j, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n \]

where \( w_j \) is the weight of the \( j \)-th criterion.

Step 4: Calculating the quantities \( t_{r_{ij}} \) according to the equations:

\[ t_{r_{ij}} = t_{p_{ij}} \cdot \left( \frac{x_{ij} - x_i^+}{x_i^+ - x_i^-} \right) \text{ if } j \text{ is the criterion the bigger the better} 
\]

(4)

\[ t_{r_{ij}} = t_{p_{ij}} \cdot \left( \frac{x_{ij} - x_i^-}{x_i^+ - x_i^-} \right) \text{ if } j \text{ is the criterion as small as better} 
\]

(5)

Step 5: Calculating the quantities \( g_{ij} \) according to the equation:

\[ g_{ij} = t_{p_{ij}} - t_{r_{ij}} \]

Step 6: Summing the \( g_i \) values according to the equation:

\[ Q_i = \sum_{i=1}^{m} g_{ij} \]

(7)

Ranking the options according to the principle that the one with the smallest \( Q_i \) is the better.

2.2 The EAMR method

The steps according to the EAMR method are summarized as follows [23].

Step 1: Building a decision matrix:

\[ X_d = \begin{bmatrix} x_{11}^d & \cdots & x_{1n}^d \\ x_{21}^d & \cdots & x_{2n}^d \\ \vdots & \ddots & \vdots \\ x_{m1}^d & \cdots & x_{mn}^d \end{bmatrix} \]

(8)

where \( 1 \leq d \leq k, k \) is the number of decision makers; \( d \) is the index representing the decision maker \( d \).

Step 2: Calculating the mean value of each alternative for each criterion according to the equation:

\[ \bar{x}_{ij} = \frac{1}{k} \left( x_{ij}^1 + x_{ij}^2 + \cdots + x_{ij}^k \right) \]

(9)

It should be noted that \( k \) is the index of the \( k \) decision maker, not the exponent.

Step 3: Determining the weights for the criteria. At this step, each decision maker can choose a different weighting method.

Step 4: Calculating the average weighted value for each criterion according to the equation:

\[ \bar{w}_j = \frac{1}{k} \left( w_j^1 + w_j^2 + \cdots w_j^k \right) \]

(10)

Step 5: Calculating \( n_j \) values according to the equation:
\[ n_{ij} = \frac{x_{ij}}{e_j} \]  

(11) 

in which, \( e_j \) is determined by the equation: 

\[ e_j = \max_{i \in \{1, \ldots, m\}} (\bar{x}_{ij}) \]  

(12) 

**Step 6:** Calculating the normalized weight values according to the equation: 

\[ v_{ij} = n_{ij} \cdot \bar{w}_j \]  

(13) 

**Step 7:** Calculating the normalized score for the criteria: 

\[ G_i^+ = v_{11}^+ + v_{12}^+ + \cdots + v_{1m}^+ \]  

if \( j \) is the criterion the bigger the better 

\[ G_i^- = v_{11}^- + v_{12}^- + \cdots + v_{1m}^- \]  

if \( j \) is the criterion as small as better 

(14) 

(15) 

**Step 8:** The rank of value (RV) is found based on \( G_i^+ \) and \( G_i^- \). 

**Step 9:** Calculating the evaluation score for the options according to the equation: 

\[ S_i = \frac{RV(G_i^+)}{RV(G_i^-)} \]  

(16) 

The solution with the largest \( S_i \) will be the best one, which is the ranking principle of the EAMR method. 

### 2.3 The MARCOS method 

The steps to implement multi-criteria decision-making according to the MARCOS method are as follows [27]. 

**Step 1:** Similar to step 1 of the MAIRCA method. 

**Step 2:** Constructing an initial matrix that expands by adding an ideal solution (AI) and the opposite solution to the ideal solution (AAI) : 

\[ X = \begin{bmatrix} AAI & \begin{bmatrix} x_{a1} & \cdots & x_{aan} \\ x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots \\ x_{m1} & \cdots & x_{mn} \\ AI & \begin{bmatrix} x_{a1} & \cdots & x_{ain} \end{bmatrix} \end{bmatrix} \\ A_1 & \begin{bmatrix} x_{11} & x_{1n} \end{bmatrix} \\ A_2 & \begin{bmatrix} x_{21} & x_{2n} \end{bmatrix} \\ \vdots & \vdots & \vdots \\ A_m & \begin{bmatrix} x_{m1} & x_{mn} \end{bmatrix} \end{bmatrix} \]  

(17) 

where: 

\[ AAI = \min (x_{ij}); i = 1, 2, \ldots, m; j = 1, 2, \ldots, n \]  

if \( j \) is the criterion the bigger the better. 

\[ AAI = \max (x_{ij}); i = 1, 2, \ldots, m; j = 1, 2, \ldots, n \]  

if \( j \) is the criterion as small as better. 

\[ AI = \max (x_{ij}); i = 1, 2, \ldots, m; j = 1, 2, \ldots, n \]  

if \( j \) is the criterion the bigger the better. 

\[ AI = \min (x_{ij}); i = 1, 2, \ldots, m; j = 1, 2, \ldots, n \]  

if \( j \) is the criterion as small as better. 

**Step 3:** Calculating the normalized values according to the following equations: 

\[ u_{ij} = \frac{x_{ai}}{x_{ij}} \]  

if \( j \) is the criterion as small as better 

\[ u_{ij} = \frac{x_{ij}}{x_{ai}} \]  

if \( j \) is the criterion the bigger the better 

(18) 

(19) 

**Step 4:** Calculating the weighted normalized values using the equation: 

\[ c_{ij} = u_{ij} \cdot w_j \]  

(20) 

where \( w_j \) is the weight of the criterion \( j \). 

**Step 5:** Calculating coefficients \( K_i^+ \) and \( K_i^- \) by the following equations:
\[ K_i^- = \frac{S_i}{S_{AAi}} \]  
\[ K_i^+ = \frac{S_i}{S_{Ai}} \]

where \( S_i, S_{AAi} \) and \( S_{Ai} \) are the sum of the values of \( c_{ij}, x_{aa} \) and \( x_{ai} \), respectively; with \( i = 1, 2, \ldots, m \).

**Step 6:** Calculating functions \( f(K_i^-) \) and \( f(K_i^+) \) by:

\[
 f(K_i^-) = \frac{K_i^+}{K_i^+ + K_i^-} 
\]
\[
 f(K_i^+) = \frac{K_i^-}{K_i^+ + K_i^-} 
\]

**Step 7:** Calculating function \( f(K_i) \) according to the following equation and rank the alternatives:

\[
 f(K_i) = \frac{K_i^+ + K_i^-}{1 + \frac{1 - f(K_i^+)}{f(K_i^+)} + \frac{1 - f(K_i^-)}{f(K_i^-)}} 
\]

Ranking the solutions according to the best solution is the one with the largest value of the function \( f(K_i) \).

### 2.4 The TOPSIS method

The steps performed in the TOPSIS method are described as follows \([39, 40]\).

**Step 1:** Similar to step 1 of the MAIRCA method.

**Step 2:** Calculating the normalized values of \( k_{ij} \) according to the equation:

\[
 k_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \]

**Step 3:** Calculating the weighted normalized values using the equation:

\[
 l_{ij} = w_j \times k_{ij} \]

**Step 4:** Determine the best solution \( A^+ \) and the worst solution \( A^- \) for the criteria according to the equations:

\[
 A^+ = \{l_{1^+}, l_{2^+}, \ldots, l_{j^+}, \ldots, l_{n^+}\} 
\]
\[
 A^- = \{l_{1^-}, l_{2^-}, \ldots, l_{j^-}, \ldots, l_{n^-}\} 
\]

wherein \( l_{j^+} \) and \( l_{j^-} \) are the best and worst values of the \( j \) criterion, respectively.

**Step 5:** Calculating values \( S_i^+ \) and \( S_i^- \) by the following equations:

\[
 S_i^+ = \sqrt{\sum_{j=1}^{n} (l_{ij} - l_{j^+})^2} \quad i = 1, 2, \ldots, m 
\]
\[
 S_i^- = \sqrt{\sum_{j=1}^{n} (l_{ij} - l_{j^-})^2} \quad i = 1, 2, \ldots, m 
\]
Step 6: Calculating values $C_i^*$ by:

$$C_i^* = \frac{s_i^-}{s_i^+ + s_i^-} \quad i = 1, 2, ..., m; 0 \leq C_i^* \leq 1$$  \hspace{1cm} (32)

Step 7: Rank the alternatives according to the principle that the one with the largest $C_i^*$ is the best one.

3. Used methods of determining the weight

3.1 The Entropy method

Determining the weights of the indicators by the Entropy method is performed according to the following steps [41].

Step 1: Determining the normalized values for the indicators:

$$p_{ij} = \frac{x_{ij}}{m + \sum_{i=1}^{m} x_{ij}^2}$$  \hspace{1cm} (33)

Step 2: Calculating the value of the Entropy measure for each indicator:

$$me_j = -\sum_{i=1}^{m} [p_{ij} \times \ln(p_{ij})] - \left(1 - \sum_{i=1}^{m} p_{ij}\right) \times \ln \left(1 - \sum_{i=1}^{m} p_{ij}\right)$$  \hspace{1cm} (34)

Step 3: Calculating the weight for each indicator:

$$w_j = \frac{1 - me_j}{\sum_{j=1}^{m} \left(1 - me_j\right)}$$  \hspace{1cm} (35)

3.2 The MEREC method

The steps to determine the weights according to the MEREC method are as follows: [35]:

Step 1: Similar to step 1 of the MAIRCA method.

Step 2: Calculating the normalized values using the following equations:

$$h_{ij} = \frac{\min x_{ij}}{x_{ij}} \quad \text{if } j \text{ is the criterion the bigger the better}$$  \hspace{1cm} (36)

$$h_{ij} = \frac{x_{ij}}{\max x_{ij}} \quad \text{if } j \text{ is the criterion as small as better}$$  \hspace{1cm} (37)

Step 3: Calculating the overall efficiency of the alternatives by the following equation:

$$S_i = \ln \left[1 + \left(\frac{1}{n} \sum_{j}^{n} \ln(h_{ij})\right)\right]$$  \hspace{1cm} (38)

Step 4: Calculating the efficiency of the alternatives according to the equation:

$$S_{ij}' = \ln \left[1 + \left(\frac{1}{n} \sum_{k,k \neq j}^{n} \ln(h_{ij})\right)\right]$$  \hspace{1cm} (39)

Step 5: Calculating the absolute value of the deviations using the equation:

$$E_j = \sum_{i}^{m} |S_{ij}' - S_i|$$  \hspace{1cm} (40)
Step 6: Calculating the weight for the criteria according to the equation:

\[ w_j = \frac{E_j}{\sum_k E_k} \] (41)

4. Used materials and execution of turning experiment

The experimental setup is described as follows: A conventional lathe ECOCA SJ460 (Taiwan) was used for the experiment. Besides, SKS3 steel samples with a diameter of 32 mm and a length of 260 mm were selected. In addition, three input parameters including cutting speed, feed rate and depth of cut were investigated. The Taguchi method was used to design an orthogonal matrix of 16 experimental runs (Table 1). After conducting the experiment, the MRR values were calculated according to the Eq. 42:

\[ \text{MRR} = \frac{1}{60} \cdot n_w \cdot \pi \cdot d_w \cdot f_d \cdot a_p \, (\text{mm}^3/\text{s}) \] (42)

where \( n_w \) is the number of revolutions of the part per minute; \( d_w \) is the diameter of the workpiece; \( f_d \) is the feed rate, and \( a_p \) is the depth of cut (mm).

The surface roughness was also determined at each test using an SJ-201 equipment. Table 1 shows the obtained results of surface texture and MRR.

From Table 1, the minimum surface roughness is 0.455 \( \mu \text{m} \) in option A13, but the maximum value of MRR is 362.046 \( \text{mm}^3/\text{s} \) in option A7. It is therefore necessary to define an alternative where the surface roughness is considered to be the “minimum” and the MRR is considered the “maximum”. This work can only be done by using mathematical methods in decision-making, and of course a mandatory job is also to determine the weights for the criteria. These two important contents will be presented in section 5 of this paper.

<table>
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<th>Actual values</th>
<th>Responses</th>
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<td>( n_w ) (rev/min)</td>
<td>( f_d ) (mm/rev)</td>
</tr>
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5. Results and discussion

5.1 Determining the weights for criteria

Eqs. 33 to 35 were used to determine the weights for the criteria according to the Entropy method. The weights of \( Ra \) and MRR are 0.6149 and 0.3851, respectively.

Eqs. 36 to 41 were applied to determine the weights for the criteria according to the MEREC method. The results have determined that the weights of \( Ra \) and MRR are 0.7042 and 0.2958, respectively.
5.2 Multi-criteria decision-making with the use of the entropy method for determining the weights of the criteria

Applying the MAIRCA method

Eq. 1 was used to build the initial matrix, which is the last two columns in Table 1. Eq. 2 was applied to determine the priority \( P_{Aj} \) for the criteria. As the criteria are considered equal, that is, the decision maker does not give importance to one criterion over the other. Therefore, the priority for both criteria \( Ra \) and MRR is equal to \( 1/16 = 0.0625 \).

Eq. 3 was applied to determine the value of parameter \( t_{p_{ij}} \), with the weight of the criteria defined in section 5.1. The result has determined the value \( t_{p_{ij}} \) of \( Ra \) and MRR are 0.0384 and 0.0241 respectively.

Eqs. 4 and 5 was used to calculate the values of \( t_{r_{ij}} \); apply Eq. 6 to calculate \( g_{ij} \); apply Eq. 7 to calculate \( Q_i \). All these values have been included in Table 2. The results of ranking options according to the value of \( Q_i \) have also been included in this table.

<table>
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<tr>
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<th>( t_{r_{ij}} )</th>
<th>( t_{r_{ij}} )</th>
<th>( g_{ij} )</th>
<th>( Q_i )</th>
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<td>0.0276</td>
<td>0.0110</td>
<td>0.0387</td>
</tr>
</tbody>
</table>

Applying the EAMR method

Eq. 8 was applied to build decision matrix. If the number of decision makers is \( k \), then each person has a different decision matrix (possibly due to different experimental results). However, in this study, there is only one set of results shown in Table 1, i.e. \( k \) equals 1. Therefore, this step of EAMR method is similar to step 1 of MAIRCA method, which means that the main decision matrix is the last two columns in Table 1.

Eq. 9 was used to calculate the mean value of the alternatives for each criterion. For \( k \) equals 1, then \( \bar{x}_{ij} = x_{ij} \).

Eq. 10 was applied to calculate the average weight for the criteria. Since \( k \) equals 1, so \( \bar{w}_j = w_j \).

Eq. 11 was used to calculate \( n_{ij} \) values; apply Eq. 12 to calculate \( e_j \) values. Also, Eq. 13 was used to calculate \( v_{ij} \).

Eqs. 14 and 15 were applied to calculate the respective values \( G_i \).

Eq. 16 was used to calculate the \( S_i \) values.

The results of calculating these quantities are presented in Table 3. The results of ranking the alternatives according to the value of \( S_i \) have also been compiled into this table.
Applying the MARCOS method

Eq. 17 has been applied to determine the ideal solution (AI) and the opposite solution to the ideal solution (AAI). Accordingly, in the ideal solution, the values of $Ra$ and $MRR$ are 0.455 $\mu$m and 362.046 mm$^3$/min, respectively. At the opposite solution, the values of $Ra$ and $MRR$ are 2.897 $\mu$m and 36.255 mm$^3$/min, respectively.

Calculating the normalized values $u_{ij}$ according to the Eqs. 18 and 19.

The normalized value considering the weight $c_{ij}$ is calculated according to the Eq. 20. The coefficients $K^+$ and $K^-$ were calculated according to Eqs. 21 and 22.

The value of $f(K^*)$ that has been calculated by Eq. 23 is equal to 0.9025. Also, the value of $f(K_1)$ has been calculated by Eq. 24 is equal to 0.0975.

Eq. 25 has been applied to calculate the values of $f(K_1)$.

The results of calculating these quantities are presented in Table 4. The ranking results of the alternatives are also presented in this table.

| Table 3 EAMR parameters and ratings |
|-------------------|-----|-----|-----|-----|
| Trial.            | $u_{ij}$ | $c_{ij}$ | $K^+$ | $f(K)$ | Rank |
|-------------------|-----|-----|-----|-----|
| A1                | 0.7955 | 0.1001 | 0.00146 | 0.00144 | 4    |
| A2                | 0.3126 | 0.2727 | 0.00084 | 0.00083 | 5    |
| A3                | 0.1683 | 0.6357 | 0.00096 | 0.00095 | 13   |
| A4                | 0.1571 | 0.2818 | 0.00114 | 0.00113 | 10   |
| A5                | 0.8553 | 0.1890 | 0.00165 | 0.00163 | 3    |
| A6                | 0.3902 | 0.2288 | 0.00090 | 0.00090 | 14   |
| A7                | 0.1709 | 0.1000 | 0.00135 | 0.00134 | 6    |
| A8                | 0.1749 | 0.8274 | 0.00118 | 0.00116 | 9    |
| A9                | 0.8395 | 0.2837 | 0.00173 | 0.00171 | 2    |
| A10               | 0.3316 | 0.6438 | 0.00125 | 0.00123 | 8    |
| A11               | 0.1977 | 0.4503 | 0.00081 | 0.00081 | 16   |
| A12               | 0.1819 | 0.6985 | 0.00105 | 0.00104 | 11   |
| A13               | 1.0000 | 0.4471 | 0.00127 | 0.00125 | 1    |
| A14               | 0.4205 | 0.6492 | 0.00140 | 0.00139 | 5    |
| A15               | 0.2049 | 0.8514 | 0.00125 | 0.00124 | 7    |
| A16               | 0.2058 | 0.5870 | 0.00097 | 0.00096 | 12   |

Applying the TOPSIS method

Eq. 26 was used to calculate the normalized values of $k_{ij}$. The normalized values taking into account the weight $l_{ij}$ are calculated according to the Eq. 27.

The $A^+$ value of $Ra$ and $MRR$ has been determined by Eq. 28, with values of 0.0366 and 0.1597 respectively.
The $A$ value of $Ra$ that $MRR$ has also been determined by Eq. 29 is 0.2331 and 0.0160 respectively.

The values $S_i^r$ and $S_i$ have been calculated according to the Eqs. 30 and 31, respectively.

The value $C^*$ has been calculated by Eq. 32.

The results of calculating these quantities are presented in Table 5. The ranking results of the alternatives are also presented in this table.

### Table 5 TOPSIS parameters and ratings

<table>
<thead>
<tr>
<th>Trial</th>
<th>$k_{ij}$</th>
<th>$MRR$</th>
<th>$l_{ij}$</th>
<th>$MRR$</th>
<th>$S_i^r$</th>
<th>$S_i$</th>
<th>$C^*$</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.0748</td>
<td>0.0415</td>
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<td>0.0160</td>
<td>0.1440</td>
<td>0.1871</td>
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</tr>
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<td>A2</td>
<td>0.1825</td>
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<td>0.1122</td>
<td>0.0436</td>
<td>0.1386</td>
<td>0.1240</td>
<td>0.4721</td>
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<tr>
<td>A3</td>
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<td>0.2637</td>
<td>0.2176</td>
<td>0.1015</td>
<td>0.1901</td>
<td>0.0869</td>
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</tr>
<tr>
<td>A4</td>
<td>0.3791</td>
<td>0.3409</td>
<td>0.2331</td>
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<td>0.1985</td>
<td>0.1153</td>
<td>0.3673</td>
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<tr>
<td>A5</td>
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<td>0.0784</td>
<td>0.0428</td>
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<td>0.1908</td>
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<td>0.5090</td>
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<td>0.1597</td>
<td>0.1776</td>
<td>0.1450</td>
<td>0.4495</td>
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<td>0.2094</td>
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<td>0.1186</td>
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<td>0.0932</td>
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<td>0.0737</td>
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<td>0.2013</td>
<td>0.1116</td>
<td>0.1716</td>
<td>0.1007</td>
<td>0.3699</td>
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<td>0.1854</td>
<td>0.0366</td>
<td>0.0714</td>
<td>0.0883</td>
<td>0.2041</td>
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<td>0.1037</td>
<td>0.0754</td>
<td>0.1703</td>
<td>0.6932</td>
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<td>0.1441</td>
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<td>0.1559</td>
<td>0.0954</td>
<td>0.3795</td>
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</table>

### 5.3 Multi-criteria decision-making with the use of the MEREC method for determining the weights of the criteria

Doing the same as in section 5.2, the results of ranking options according to four multi-criteria decision-making methods (MAIRCA, EAMR, MARCOS and TOPSIS) when the weights are determined by the MEREC method (presented in section 5.1) are presented in Table 6. In addition, the ranking results of the alternatives when the weights are determined by the Entropy method (in Tables 2, 3, 4, 5) have also been summarized in Table 6.

### Table 6 Ranking of alternatives by two methods of determining weight

<table>
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<tr>
<th>Trial</th>
<th>Entropy weight</th>
<th>MEREC weight</th>
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<tr>
<td></td>
<td>MAIRCA</td>
<td>EAMR</td>
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<tr>
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<td>A2</td>
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<td>6</td>
</tr>
<tr>
<td>A16</td>
<td>12</td>
<td>5</td>
</tr>
</tbody>
</table>
The ranking results of the options in Table 6 show that:

- With three methods MAIRCA, MARCOS, and TOPSIS: for different weighting methods, the ranking order of options is also different [33].
- All four multi-criteria decision-making methods identify $A_{13}$ as the best option. This result is consistent when the weights of the criteria are determined by two different methods.
- The order of ranking the alternatives according to the EAMR method is completely the same when using two different weighting methods. This shows that the EAMR method has very high stability in ranking the alternatives.
- To ensure the "minimum" surface roughness and "maximum" MRR at the same time, the values of cutting speed, feed rate and cutting depth are 1050 rev/min, 0.092 mm/rev and 1.0 mm respectively.

6. Conclusion

This paper presents the results of an experimental study on the SKS3 steel turning process, with a total of 16 experiments designed according to the orthogonal matrix by the Taguchi method. Three cutting parameters were selected for the process input. Besides, surface roughness and MRR were selected as two parameters to evaluate turning process. Four methods including the MAIRCA, the EAMR, the MARCOS, and the TOPSIS were used for multi-criteria decision-making. The determination of the weights for the criteria was done by two methods Entropy and MEREC. From the results of the study, some conclusions are drawn as follows:

- For the first time, three methods including MAIRCA, EAMR, MARCOS are used to make multi-criteria decision for turning process. An excellent result has been obtained that all three methods as well as the TOPSIS method have consistently identified a best alternative.
- The MEREC method is applied for the first time in this study to determine the weights for the criteria of the turning process. The use of weights determined by the Entropy method or the MEREC method does not affect the determination of the best solution in all four cases where the different methods are used. Thus, with this study, determining the best solution when using four methods (MAIRCA, EAMR, MARCOS and TOPSIS) does not depend on the method of determining the weights.
- When using the Entropy method, for different multi-criteria decision-making methods, the same best solution can be determined [17]. In addition, when using two methods Entropy and MEREC, for different decision-making methods, the best solutions still only one option.
- To determine the best option when making a multi-criteria decision, the weighted method is Entropy and (or) MEREC should be used.
- The order of ranking the alternatives when using the EAMR method is completely the same when using two different weighting methods. This shows the use of the EAMR method to rank the alternatives for high stability. This can be explained that when applying this method, the weights of the criteria were normalized according to Eq. 13.
- The above conclusions are drawn based on the results of this study. To solidify them, there is a need for some more studies in which other weighting options are considered, in other machining processes.

References


