Modelling and simulation of hot direct extrusion process for optimal product characteristics: Single and multi-response optimization approach

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A B S T R A C T

The study of eccentricity minimization in cylindrical products helps to reduce mechanical vibrations and wear of related mechanical parts such as bearings, columns, and gears which positively affects in maintenance costs savings and increasing production quality reliability. The main purpose of this paper is to investigate the effect of the eccentricity between the billet material and the die parts on the quality of the final product in the direct extrusion process. The input parameters to produce a cylindrical product shape are optimized in MINITAB based on Taguchi method and ANOVA. The selected material of the billet is the aluminium alloy AA2024, and the die material is Steel H13. The inputs parameters are the temperature, the die angle, the ram speed, and the presumed eccentricity. The finite element model of the process is simulated in DFORM-3D for providing the extrusion information such as the pressure, the effective stress and strain, the final product eccentricity, and the roundness error. The study is carried out on two cases of the presumed eccentricity in addition to a case of zero eccentricity. The single and multi-response optimizations are executed to obtain the optimum parameters for the minimum product eccentricity and roundness error.

Keywords:
Metal forming; Hot direct extrusion; Eccentricity; Roundness; Modelling; Simulation; Optimization; Single response; Multi-response; DFORM-3D; MINITAB

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1. Introduction

Extrusion process is used to create different and complex cross-sectional areas with a good surface finish. It is utilized in multiple shapes production such as strips, bars, tubes, hollow and solid profiles, and it could achieve the energy and material savings [1]. Aluminium alloys (Al-alloy) are the most popular materials that provide a lot of valuable properties such as light metal weight, high process ability, rust resistance, eco-friendly and high strength with good ductility and toughness [2]. Al-extrusion is a widespread metal forming process for producing long and durable parts. Hot Al-extrusion process depends on forcing a preheated billet into a steel die with a specific design. Proper selection of the extrusion initial parameters such as billet temper-
ature, ram pressure, die angle and ram pressure is the main aid to obtain a right extruded product. However, during hot Al-extrusion process, various defects may happen and are responsible for reducing the quality of the final product and rising the scrap percentage and the product cost. These defects such as internal cracking, Pipe Formation, subsurface defects, Funnel formation, centre burst and the eccentricity whereas it is a common defect in the extrusion process [3]. Studying and optimizing the Al-extrusion parameters may help in improving the product quality, minimizing the defects, and decreasing the cost.

In the past, extrusion parameters were selected according to the experience and consequently, the researchers tried to investigate the process parameters to obtain the optimum setting of the initial conditions. Yanran et al. [4] used the Finite-Element (FE) method to analyse the deformation stage of extrusion process according to the principle of minimum deformation force for optimizing the angle of the die. Jang et al. [5] aimed to study the numerical analysis of radial extrusion process combined with direct extrusion to investigate the forming properties of an AA 2024 alloy in terms of material flow into the direct can and radial flange sections. Jurković et al. [6] determined the optimal direct extrusion parameters to minimize the extrusion load using Taguchi approach. Many researchers studied the effects of Al-extrusion parameters (billet temperature, ram speed, die angle, etc.) that simulated through different finite element analysis solvers on the output responses. Zhang et al. [7] simulated the Al-extrusion process using HYPERXTRUDE to optimize the symmetry of the metal flow and the extrusion force using Taguchi and ANOVA. Jajimoggala [8] studied the effect of the extrusion parameters of AL6061 on the extrusion load and the effective stress using DOE and the analysis of variance. Bressan et al. [9] modelled the hot direct extrusion process of AL6060 through FVM and compared the results with the experimental results. Mai et al. [10] determined the ram speed and billet temperature in bars of Al-alloy to obtain the optimum of the surface roughness and the extrusion pressure. Francy et al. [11] dealt with the input parameters of the extrusion process such as half die angle, coefficient of friction, logarithmic strain and ram velocity using DEFORM-3D software and Taguchi approach to minimize the power consumption. Fernández et al. [12] studied the influence of the extrusion parameters on the manufacturing of Bimetallic cylinders combining of Ti-Alloy sleeve and Mg-Alloy core using DOE by Taguchi Method. Medvedev et al. [13] developed an approach to increase productivity rate of the Al-extrusion by optimizing of extrusion dies design coupled with subsequent optimizing of extrusion parameters such as extrusion ratio, ram speed, lubrication using FM simulation. Few researchers have dealt with the defects of the aluminium extrusion process such as centre burst, surface cracking, eccentricity, and product roundness. Parghazeh and Haghighat [14] investigated the prediction of the central bursting defects in the extrusion process through Conical-Die. Ngernbamrung et al. [15] determined the extrudability of Aluminium alloy is by tearing appearance under extrusion parameters (temperature and speed). Yang et al. [16] used the velocity-transformed central-flow model for analysing the centre-shifted of punch of the backward-extrusion for eccentric tubes from circular billets. The work hardening is considered, and the extrusion pressure is computed.

Most of the pre-existing works carried on developing the optimized design with some variations in the input process parameters under different optimization techniques. There have been no reports on the attempts in the numerical studying of the effects of main input parameters including the misalignment between the billet and the extrusion on the eccentricity and the roundness of an extruded aluminium tubes.

In this work, a combination of Finite Element Modelling (FEM) and Design of Experiments (DOE) is employed to study the effects of main extrusion process inputs on the eccentricity of a cylindrical product and its roundness. As a FEM for Hot-Direct-Extrusion process of AA2024 by DEFORM-3D software is applied to simulate the input parameters such as the billet temperature, the ram speed, the die angle, and the pre-eccentricity between the billet and the die. The outputs of the process such as the extrusion pressure, the stress and strain distribution, the eccentricity of the produced cylinder and its roundness are investigated. Taguchi Method is employed for DOE and an orthogonal array L9 is constructed by using a statistical software MINITAB 18. The multi-response of the outputs is discussed as the objectives are to minimize the extrusion pressure, the product eccentricity and the roundness error and obtain the effective stress and strain.
Also, the ANOVA is adopted to verify and identify the sequence of importance of input parameters on the final product quality.

2. Materials and method

Optimization of production parameters is a desirable research field in the recent eras [17]. To perform the modelling and simulation in this study for the experimental design of the certain cases of the hot extrusion process, the Taguchi-technique [18] using the statistical program MINITAB 18 is applied, the process elements (billet and die) are modelled in SolidWorks platform and the Finite Element Analysis (FEA) for the process in each experiment is simulated in DEFORM-3D software. Finally, the ANOVA is adopted to identify and verify the importance of extrusion parameters that affected on the eccentricity and the roundness error of the final product.

2.1 Materials selection

In this work, AA2024 alloy is selected as the billet-material. AA2024 has a high strength, lightweight, a good fatigue and corrosion resistance and it is ranked as one of an optimal selected material for the aircraft and vehicle applications. The billet has a dimension with 64 mm Dia. and 100 mm long. While H13 tool steel that offers High hardenability, excellent wear resistance and high toughness is selected as the die-material. Tables 1, 2 and Table 3 illustrate the chemical composition and the mechanical properties of the billet and the die materials, respectively. Fig. 1 represents the SolidWorks 2018 modelling of the billet, the container, the ram, and the die with its proposed dimensions.

2.2 Methodology

For studying the effect of presumed eccentricity $E_{pre}$ in the hot extrusion process of AA2024, four input parameters namely, the billet temperature $T$, ram speed $Rs$, die angle $\alpha$ and the presumed eccentricity are considered in an optimization problem whereas the coveted responses will be the pressure $p$, stress $\sigma$, strain $\varepsilon$, the extruded product eccentricity $E_c$ and its roundness $Ro$. The levels of parameters are shown in Table 4. A L9 Taguchi orthogonal array for the selected parameters is constructed in MINITAB 18 and it is illustrated in Table 5.
To study the effect of the presumed eccentricity on the quality of the final product, two cases of presumed eccentricity are adopted, and the results are compared to a zero case ($E_{pre} = 0$). The supposed two cases are (1) eccentricity between the ram and billet (the billet, container and die have the same enter line) (2) eccentricity between the container and the billet (the ram, container and die have the same centreline) as shown in Fig. 2. So, Finite Element Models for each experiment in the designed cases are constructed to study the optimum parameters for minimum pressure, stress, strain, eccentricity in the extrude and the roundness error. The FEA of the modelled process is carried through in DEFORM-3D software whereas the material properties are selected from the software database. The Boundary Conditions (BCs) and the Mesh Procedures (MP) for the simulated problem are introduced in Table 6. The outputs (pressure, stress, strain, the product eccentricity, and its roundness) for each experiment are exported to MINITAB to optimize the input parameters for the multi-responses in each case of eccentricity. Fig. 3 illustrates the simulation of extrusion steps in DEFORMTM-3D. The flowchart in Fig. 4 displays the procedures of the applied methodology and the steps sequence that are performed throughout the research.

Table 4 Hot extrusion process input parameters and its levels

<table>
<thead>
<tr>
<th>Process parameter</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
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<tr>
<td>$T$ ($^\circ$C)</td>
<td>300</td>
<td>350</td>
<td>400</td>
</tr>
<tr>
<td>$Rs$ (mm/s)</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$\alpha$ ($^\circ$)</td>
<td>12</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>$E_{pre}$ (mm)</td>
<td>0.3</td>
<td>0.5</td>
<td>0.7</td>
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</tbody>
</table>

Table 5 Taguchi orthogonal array L9

<table>
<thead>
<tr>
<th>No. of experiments</th>
<th>$T$ ($^\circ$C)</th>
<th>$Rs$ (mm/s)</th>
<th>$\alpha$ ($^\circ$)</th>
<th>$E_{pre}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1</td>
<td>12</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>300</td>
<td>2</td>
<td>16</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>300</td>
<td>3</td>
<td>18</td>
<td>0.7</td>
</tr>
<tr>
<td>4</td>
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<td>3</td>
<td>12</td>
<td>0.3</td>
</tr>
<tr>
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<td>350</td>
<td>1</td>
<td>16</td>
<td>0.5</td>
</tr>
<tr>
<td>6</td>
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<td>2</td>
<td>18</td>
<td>0.7</td>
</tr>
<tr>
<td>7</td>
<td>400</td>
<td>2</td>
<td>12</td>
<td>0.3</td>
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<td>8</td>
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<td>3</td>
<td>16</td>
<td>0.5</td>
</tr>
<tr>
<td>9</td>
<td>400</td>
<td>1</td>
<td>18</td>
<td>0.7</td>
</tr>
</tbody>
</table>

(a) Case 1: $E_{pre}$ between ram and billet (b) Case 2: $E_{pre}$ between container and billet

Fig. 2 Presumed eccentricity ($E_{pre}$) cases in the simulated extrusion process
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![Fig. 3 Schematic demonstration of the extrusion simulation at different steps during the process](image)

**Table 6 Boundary conditions of the simulated problem**

<table>
<thead>
<tr>
<th>Meshing element type</th>
<th>Tetrahedral</th>
<th>Starting step number</th>
<th>-1</th>
<th>Step increment</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh elements (elements)</td>
<td>8000</td>
<td>No. of simulation steps</td>
<td>500, 250, 1000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient of friction</td>
<td>0.4</td>
<td>Room temperature</td>
<td>20 °C</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.3 Product eccentricity calculation

The axis eccentricity of any cylindrical part may have much influence on the measurement results and the quality of its performance. Because of this came the importance of detecting the eccentricity of the final extruded product in this work. So, the eccentricity of the extruded product is calculated by taking an image of each model from the DEFORM-3D platform and exporting it into SolidWorks platform. The tracking setting is used to identify the model and its dimensions. Then, the billet axis is determined, as well as the axis of the extruded product. The axial eccentricity is computed by detecting the horizontal distance between the billet-centre and the product-centre as shown in Fig. 5.

**Fig. 5 Extruded Product Eccentricity**

![Fig. 4 The flowchart of the procedures of the applied methodology and the steps sequence](image)

**Fig. 6 Roundness Error using MZC method**
2.4 Roundness error calculation

Roundness means to how tightly an object’s shape resembles that of an accurately ideal circle. It is generally measured by rotational techniques by calculating radial deviations from a rotating fixed axis that becomes the main reference for all measurements. Roundness error can measure by four methods: Least Squares Circles, Maximum Inscribed Circle, Minimum Circumscribed Circle and Minimum Zone circle [22]. In this work, a Minimum Zone circle (MZC) is used to measure the roundness error by utilizing two circles as references. One circle (C1) is depicted outside the rounded product profile just to contain the whole of it and the second circle (C2) is drawn inside so that it only addresses the profile of the product. The roundness error (Ro) here is the difference between the radius of the two circles outside and inside as shown in Fig. 6.

3. Results and discussion

As earlier mention, the input parameters in the simulated extrusion process problem include (T, Rs, α and Epre) and the output responses are (P, σ, ε, Ec and Ro). In this work, the single and multi-response optimization using Taguchi approach are applied for catching the optimum parameters to minimize the normal pressure, the product eccentricity, and the roundness-error in the suggested cases of eccentricity. ANOVA analysis is also applied to study the effect of inputs on the required responses.

As known, the statistical optimization has three types for optimizing data, they are (1) Nominal-the-best “N-type” (2) Larger-the better “L-type” and (3) Smaller-the-better “S-type”. The objectives of “S-type & L-type” attempts are to identify the level of the optimal parameters to reach the smallest or greatest value of quality-characteristics. So, the smaller-the-better “S-type” is planned to use in this optimization problem.

3.1 Ideal case of eccentricity (No-presumed eccentricity)

In this step of the research, the problem has no-presumed eccentricity (Epre = 0) while the billet and die parts are coaxial. After completion the simulation process in DEFORM-3D of nine experiments (L9) of input parameters and the implementation of the statistical optimization in Minitab platform, it found that the optimum parameters are 400 °C as the temperature, 2 mm/s as the ram speed, and 16° as the die angle while the outputs are 59.9 MPa as the normal pressure, 22.7 MPa as the stress and 10.8 as the strain. It was noted that the values of the product eccentricity Ec and the roundness error Ro were constant in all experiments. Their values are 0.02 mm as the product eccentricity and 0.03 mm as the roundness. And this is logical while there is no presumed eccentricity between the die parts and the billet. Fig. 7 illustrates the main effects of means of the process parameters and Fig. 8 represents the simulation of the process and the values of the optimum outputs (pressure, stress and strain).

![Fig. 7 Main effects plot for means of process parameters in Ideal Case of Eccentricity](image-url)
3.2 Presumed eccentricity cases

The proposed eccentricity cases in this research are:

Case (1): The presumed eccentricity $E_{pre}$ is between the billet and the ram while the billet, the container and the die have the same centreline.

Case (2): The presumed eccentricity $E_{pre}$ is between the billet and the container while the ram, the container and the die have the same centreline.

3.3 Single response optimization and ANOVA technique

In this step, a single response optimization using MINITAB 18 is performed for both suggested eccentricity cases to realize the effect of the presumed eccentricity on the responses for each experiment. Then, ANOVA technique is applied to assess the competences of the input process parameters while extruding the AA2024.

In the case (1), and by observing the results of the optimization problem, it was found that the increasing in $E_{pre}$ has an active impact on all outputs as the responses ($P, Ec$) are increased while $\sigma, \varepsilon$, and $Ro$ decreases. In the case (2), it was observed that the increase in $E_{pre}$ was accompanied by a decrease in $P, \sigma$, and $\varepsilon$, and an increase in $Ec$ and $Ro$.

In the case (1), and by studying the one-way ANOVA results for each parameter, it was noticed that the most effective parameter on the final product eccentricity $Ec$ is the ram speed $Rs$ with the contribution percentage about 37.81% then the die angle $\alpha$ with the contribution percentage about 26.09%. The results also showed that the most influential parameter on the roundness error $Ro$ is the temperature $T$ with the contribution percentage about 51.48% then the ram speed $Rs$ with the contribution percentage about 38.59%.

In the case (2), it was noted that the most effective parameter on $Ec$ is the $Rs$ with the contribution percentage about 44.63% then the presumed eccentricity $E_{pre}$ with the contribution percentage about 41.51%. For the roundness error $Ro$ response, the most influential parameter is $T$ with the contribution percentage about 40.82% then the die angle $\alpha$ with the contribution percentage about 27.79%.

3.4 Multi response optimization

Under the situations of the single pattern of process parameters that may be optimal for certain specific characteristic however the same settings may have negative consequences for other quality aspects, the multi-characteristics-response-optimization may be the key. Since the multi-response in Taguchi method is a common technique that can deal with multi-data at the same time, it will be used in this work, and it will be followed by ANOVA technique to get the contribution percentage of the input parameters as it reveals the influence of each parameter on the responses.

In this statistical optimization problem, and for the nine proposed experiments (L9), the Signal to Noise ratio (S/N) plots are studied to determine the optimal parameters values for minimizing the objectives.
In the case (1), by using the responses data that is represented in Table 7, and by applying the multi response technique, it was found that the experiment No. 3 has the optimum process parameters that lead to obtaining the desired objectives of optimizing the responses \( P, Ec, \sigma, \varepsilon, \) and \( Ro \). The parameters are the temperature \( T = 300 \, ^\circ C \), the ram speed \( Rs = 3 \, mm/s \), presumed eccentricity \( E_{pre} = 0.7 \, mm \), and the die angle \( \alpha = 18^\circ \). The optimal values of output responses are the pressure \( P = 376 \, MPa \), the effective stress \( \sigma = 85.6 \, MPa \), the effective strain \( \varepsilon = 6.75 \), the product eccentricity \( Ec = 0.46 \, mm \), and the roundness \( Ro = 2.61 \, mm \). Fig. 9 shows the main effects of means for the input process parameters and Fig. 10 represents the DEFORM-3D simulation of the extrusion process of the experiment No. 3 for the optimum responses \( P, \sigma, \varepsilon \).

**Table 7** Input parameters and output responses data of L9 Taguchi array for case (1)

<table>
<thead>
<tr>
<th>No.</th>
<th>( T ) ((^\circ C ))</th>
<th>( Rs ) (mm/s)</th>
<th>( E_{pre} ) (mm)</th>
<th>( \alpha ) ((^\circ ))</th>
<th>( P ) (MPa)</th>
<th>( \sigma ) (MPa)</th>
<th>( \varepsilon )</th>
<th>( Ec ) (mm)</th>
<th>( Ro ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>300</td>
<td>1</td>
<td>0.3</td>
<td>12</td>
<td>937</td>
<td>89.9</td>
<td>7.43</td>
<td>7.71</td>
<td>3.09</td>
</tr>
<tr>
<td>2</td>
<td>300</td>
<td>2</td>
<td>0.5</td>
<td>16</td>
<td>757</td>
<td>122.0</td>
<td>6.78</td>
<td>0.76</td>
<td>1.84</td>
</tr>
<tr>
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<td>18</td>
<td>376</td>
<td>85.6</td>
<td>6.75</td>
<td>0.46</td>
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</tr>
<tr>
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<td>0.5</td>
<td>18</td>
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<td>7.75</td>
<td>8.77</td>
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<td>5</td>
<td>350</td>
<td>2</td>
<td>0.7</td>
<td>12</td>
<td>621</td>
<td>88.0</td>
<td>8.19</td>
<td>3.55</td>
<td>4.49</td>
</tr>
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<td>6</td>
<td>350</td>
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<td>586</td>
<td>96.0</td>
<td>6.45</td>
<td>2.51</td>
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<td>0.7</td>
<td>16</td>
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<td>6.66</td>
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<td>2.38</td>
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<td>12</td>
<td>499</td>
<td>72.6</td>
<td>6.06</td>
<td>8.35</td>
<td>2.96</td>
</tr>
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</table>

**Fig. 9** Main effects plot for means of process parameters in Case (1)

**Fig. 10** Simulation of the Extrusion process of the optimal responses in Case (1) using multi-response optimization

In the case (2), by using the responses data that is represented in Table 8, and by applying the multi- response optimization, it was noticed that the experiment No. 4 has the optimum process parameters that get the objective. The optimum responses are found at \( T = 350 \, ^\circ C, \, Rs = 1 \, mm/s, \, E_{pre} = 0.5 \, mm, \) and \( \alpha = 18^\circ \). The optimal values of output responses are the pressure \( p = 575 \, MPa \), the effective stress \( \sigma = 90.6 \, MPa \), the effective strain \( \varepsilon = 8.74 \), the product eccentricity \( Ec = 0.92 \, mm \), and the roundness \( Ro = 6.42 \, mm \). Fig. 11 shows the main effects of means for the input pro-
cess parameters of case (2), and Fig. 12 represents the DEFORM-3D simulation of the extrusion process of the optimum experiment No. 4 for the optimum responses \((P, \sigma, \varepsilon)\) and that obtains the minimum product eccentricity \(E_c\).

**Table 8** Input parameters and output responses data of L9 Taguchi array for case (2)

<table>
<thead>
<tr>
<th>No.</th>
<th>(T) (°C)</th>
<th>(Rs) (mm/s)</th>
<th>(E_{pre}) (mm)</th>
<th>(\alpha) (°)</th>
<th>(P) (MPa)</th>
<th>(\sigma) (MPa)</th>
<th>(\varepsilon)</th>
<th>(E_c) (mm)</th>
<th>(Ro) (mm)</th>
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<td>1</td>
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<td>70.9</td>
<td>10.20</td>
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<td>9.76</td>
<td>23.94</td>
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<td>14.80</td>
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</table>

**Fig. 11** Main effects plot for means of process parameters in case (2)

**Fig. 12** Simulation of the Extrusion process of the optimal responses in case (2) using multi-response optimization

### 3.5 Study the effect of process parameters on the responses

It is important to study the cause-effect relationships among inputs to understand the multivariate nature of the simulated extrusion process. Frequently in the extrusion process, the temperature, the ram speed, and the die angle are the vital and basic process parameters that have a great influence on the product quality and the amount of the energy consumption. Additionally, the extruded product and the process productivity often depend on the die parts accuracy and the coaxially between these parts and the billet material. So, the study of the eccentricity effect must be inserted in the crib of the effective extrusion parameters. In this section the impact of input parameters on the responses that lead to high productivity and performance is discussed.

In the case (1): The optimum experiment (Exp. No. 3) shows that the highest value of Ram Speed \((Rs = 3\ \text{mm/s})\) has a significant effect in reducing the pressure response which means savings in the energy consumption. Consequently, the extrusion rate increases with increase in the value of the ram speed. This happens due to the work hardening of the billet material during
the process. The selected value of ram speed leads to an appropriate effective stress and strain for the deformation occurrence. The higher the ram speed also obtains the minimum the product eccentricity and an acceptable value of roundness error while comparing with its values in the other experiments.

Die Angle plays a central role in the hot extrusion. In Exp. No. 3, Die angle recorded the maximum ($\alpha = 18^\circ$). Concurrently, the higher the die angle produces more metal flow. It is expected that while a die angle increases, the extrusion pressure will increase, but here it is combined with other parameters and a multi response optimization is executed, and the result is to obtain the lowest value of the pressure. The maximum value of the die angle also helps to obtain the minimum product eccentricity and an acceptable roundness error.

It is known that the high billet temperatures will reduce the extrusion pressure and the producer must decrease the ram speed to avoid the press and die parts deterioration. In this case of research, the lowest value of Temperature ($T = 300^\circ$C) is selected in the optimum group of input parameters. It successfully leads to obtain the lowest pressure and product eccentricity and a suitable effective stress and strain to deform the billet material.

The Presumed Eccentricity achieves its highest value ($E_{pre} = 0.7$ mm) in the optimum experiment. This parameter is combined with other optimum inputs to obtain the minimum pressure, product eccentricity, an acceptable roundness and a suitable stress and strain to produce the plastic deformation. So, this means that it has slight effect on the output responses while comparing with other parameters in the optimum experiment.

In the case (2): By studying the values of input parameters that represent the optimum experiment (Exp. No. 4), it is found that the Ram Speed achieves the lowest level ($R_s = 1$ mm/s) which will occur a larger torque. This is exhibited in the corresponding value of pressure response as it reaches the maximum whenever compared to the other values of pressure. The effective stress is also reached the highest which is desirable especially in the start of the forming region. Concurrently, the effective strain increases drastically, which helps to cause more billet deformation. As while the product eccentricity may achieve the lowest value with the selected optimum parameters the roundness error yields a relatively large value. The increase in the Temperature ($T = 350^\circ$C) and the achievement of the die angle to its highest level ($\alpha = 18^\circ$) helped in obtaining these responses value. The effect of the Presumed eccentricity appears here effectively on the quality of the final product and its ability to negatively affect the outputs if its value increases. This is evident by comparing the value of the product eccentricity and roundness error at the optimum experiment with its other values.

### 3.6 Comparison between ideal and eccentricity cases

The main target of this research is to study the presence of the presumed eccentricity whether between the die parts or with the billet and its effect on the extrusion product quality (product eccentricity and roundness error). And based on the idea that there must be an initial eccentricity, even if it is very small, between the extrusion process parts, the basis of the comparison between cases is to obtain the optimum value of product eccentricity. By observing the optimum experiment in the ideal and both eccentricity cases, it is found that the global minimum product eccentricity is ($E_c = 0.46$ mm). It occurs while the eccentricity is between the billet and the ram, case (1). Table 9 illustrates the responses values for the three cases of eccentricity at the optimum parameters of case (1), as $T = 300^\circ$C, $R_s = 3$ mm/s, $E_{pre} = 0.7$ mm and $\alpha = 18^\circ$. Table 10 shows the increase and decrease in the responses where the optimal parameters are used while comparing each two cases separately. Also, by noticing the results of comparison in the Table 10 and, it is found that the run time of simulated process under a selected optimum parameter of the case (1) is suitable and acceptable for achieving the simulation.

<table>
<thead>
<tr>
<th>Eccentricity Case</th>
<th>$P$ (MPa)</th>
<th>$\sigma$ (MPa)</th>
<th>$\varepsilon$</th>
<th>$E_c$ (mm)</th>
<th>$R_o$ (mm)</th>
<th>Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal Case ($E_{pre} = 0$)</td>
<td>106</td>
<td>29.7</td>
<td>7.34</td>
<td>0.02</td>
<td>0.03</td>
<td>50</td>
</tr>
<tr>
<td>Case (1)</td>
<td>376</td>
<td>85.6</td>
<td>6.75</td>
<td>0.46</td>
<td>2.61</td>
<td>25</td>
</tr>
<tr>
<td>Case (2)</td>
<td>291</td>
<td>76.8</td>
<td>6.55</td>
<td>47.59</td>
<td>7.18</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 9 Responses Data and Run Time consuming of DEFORM-3D simulation at the selected input parameters ($T = 300^\circ$, $R_s = 3$, $E_{pre} = 0.7$, $\alpha = 18^\circ$) for all Eccentricity Cases
3.7 ANOVA analysis of extrusion process parameters

ANOVA is a statistical technique for detecting the process parameters that notably affected the product quality. In this section, the ANOVA results of product eccentricity $E_c$ and roundness error $R_o$ responses are analysed for the optimum parameters in both cases of eccentricity. The Degree of Freedom (DF), the Sum of Squares (SS), the Mean Square (MS), F-value, P-value and the Contribution percentage are calculated and tabulated.

In the case (1), by noticing the results of ANOVA, it found that the most effective parameter on the product eccentricity $E_c$ is the presumed eccentricity $E_{pre}$ with 37.8 % contribution percentage. The ram speed $R_s$ with the contribution of 26.09 % is ranked the second most influential parameter and it is followed by the die angle $\alpha$ with 11.27 %. The least affecting parameter is the temperature $T$ with the contribution percentage 9.84 %.

Moreover, the most effective parameter on the roundness $R_o$ is the temperature $T$ with 51.48 % contribution percentage. The presumed eccentricity $E_{pre}$ with the contribution of 5.13 % is ranked the second most influential parameter and it is followed by the ram speed $R_s$ with 4.77 %. The least affecting parameter is the die angle $\alpha$ with the contribution percentage 0.63 %.

In the case (2), by observing the ANOVA results, it found that the most effective parameter on $E_c$ is the $R_s$ with 44.62 % contribution percentage. The $E_{pre}$ with the contribution of 41.52 % is the second most influential parameter and it is followed by the $T$ with 5.33 %. The least affecting parameter is the $\alpha$ with the contribution percentage 0.024 %.

Additionally, the most effective parameter on the roundness $R_o$ is the temperature $T$ with 40.82 % contribution percentage. The presumed eccentricity $E_{pre}$ with the contribution of 22.55 % is the second most influential parameter and it is followed by the ram speed $R_s$ with 8.83 % then the $\alpha$ is come with the minimum contribution percentage 0.056 %.

4. Conclusion

This research addresses the input parameters in a hot direct extrusion process of AA2024 to explore the minimization of the final product eccentricity and its roundness error.

The statistical and numerical study is successfully executed by using MINITAB 18 and DFORM-3D software. The single and the multi-response optimizations based on Taguchi method and the Analysis of Variance are applied effectively to obtain the optimum parameters for minimum objectives in both suggested cases of eccentricity in addition to the case of zero eccentricity.

In the first case of eccentricity (the eccentricity between the billet and the ram), the multi-response results displayed that the optimal input parameters are $T = 300$ °C, $\alpha = 18^\circ$, $E_{pre} = 0.7$ mm and $R_s = 3$ mm/s to obtain the minimum final product eccentricity $E_c$ that equal to 0.46 mm and while the roundness $R_o$ is equal to 2.61 mm.

In the second case of eccentricity (the eccentricity between the billet and the container), the multi response results showed that the optimal input parameters are $T = 350$ °C, $\alpha = 18^\circ$, $E_{pre} = 0.5$ mm and $R_s = 1$ mm/s to obtain the minimum final product eccentricity $E_c$ that equal to 0.92 mm and while the roundness $R_o$ is equal to 6.42 mm.

In the case (1), the results of the ANOVA analysis exhibited that the most effective input parameter on $E_c$ is $E_{pre}$ with 37.8 % contribution percentage, then $R_s$ with 26.09 %, then $\alpha$ with 11.27 % and $T$ with 9.84 %. The most effective parameter on the roundness $R_o$ is $T$ with 51.48 % contribution percentage, then $E_{pre}$ with 5.13 %, $R_s$ with 4.77 %, and $\alpha$ with 0.024 %.

In the case (2), the results of the ANOVA analysis demonstrated that the most effective parameter on $E_c$ is $R_s$ with 44.62 % contribution percentage, then $E_{pre}$ with 41.52 %, $T$ with 5.33 %, and $\alpha$ with 0.63 %. The most effective parameter on the roundness $R_o$ is $T$ with 40.82 % contribution percentage, then $E_{pre}$ with 22.55 %, $R_s$ with 8.83 % and $\alpha$ with 0.056 %.
References


