Optimization approaches for solving production scheduling problem: A brief overview and a case study for hybrid flow shop using genetic algorithms

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Abstract

The aim of this paper is to investigate scheduling problems in manufacturing. After a brief introduction to the modelling approach to the scheduling problem, the study focuses on the optimization approach to the scheduling problem. Firstly, the different optimization approaches are categorised and their respective advantages and disadvantages are shown. This is followed by a detailed analysis of the characteristics and applicability of each of the commonly used optimization approaches. Finally, a case study is presented. A mathematical model is developed with the objective of minimising the maximum completion time for a mixed flow shop scheduling problem, and a genetic algorithm is used to solve the problem. The validity of the model is verified through the case study, which can provide a reasonable scheduling solution for actual manufacturing. This provides a reference for the selection and use of methods for solving scheduling problems in practical production.

1. Introduction

In today’s rapidly developing technology, manufacturing plays an important role in the development of the national economy. How to reduce manufacturing costs is often the primary issue that manufacturing companies need to consider, and how to strengthen production management capabilities, improve production efficiency and reduce production and operating costs is attracting increasing attention. These problems can be solved by optimising the production scheduling arrangements [1, 2]. Production scheduling is the efficient arrangement of production processes based on certain scheduling objectives to achieve job scheduling and optimal allocation of resources within the constraints of time and other conditions [3]. The level of scheduling decisions often determines the ability of the business to achieve cost reductions and respond quickly to market demands. Scheduling optimisation on the manufacturing floor is therefore a key concern for modern manufacturing companies.
Scholars have used different algorithms to model and optimise different job shop scheduling problems such as single-machine scheduling and multi-machine scheduling. This paper firstly reviews and summarises the optimization approaches used by scholars for scheduling problems in the past, and analyses the advantages and disadvantages of different approaches and the applicable environment, which can provide reference for subsequent scholars in the selection of optimization approaches for production scheduling problems. The hybrid shop scheduling problem is more complex than the general shop scheduling problem. This is because there are parallel machines for certain processes in the process and machine selection is required for the machining of the workpiece. Therefore, this paper chooses to conduct a case study for the hybrid flow shop scheduling problem. A mathematical model is developed and solved using a genetic algorithm, and the validity of the model is then verified.

2. Literature review

The problem of workshop scheduling has been studied by a very large number of national and international scholars. Song and Yang solved the scheduling problem with a real-time scheduling algorithm based on a branch-and-bound algorithm and an artificial neural network [4]. Yang proposed a multi-objective hybrid genetic algorithm that incorporates local search into evolutionary computation to solve the scheduling problem for flow shops [5]. Zhou designed a genetic algorithm-based production scheduling optimization model for multiple processes and machines [6]. Zheng and Pan proposed an improved discrete firefly optimization algorithm combined with a local search algorithm to solve the flexible job shop scheduling problem, and conducted simulations to verify the effectiveness of the algorithm [7]. Li et al. solved the dual-objective hybrid flow shop scheduling problem based on genetic algorithms, combining with small habitat techniques [8]. Wang et al. established a model for the single-machine scheduling problem considering equipment availability constraints with the optimization objective of minimizing the total delay time, and then solved it with a genetic algorithm, which can effectively cope with the impact of equipment availability on production scheduling [9].

There are always uncertainties in a system that cause project delays [10, 11]. Regarding the uncertainty condition constraint, Chen et al. established a fuzzy multi-objective scheduling model to effectively solve the replacement flow shop scheduling problem considering processing time and fuzzy lead time [12]. You et al. established a dynamic scheduling method based on game theory, considering machine failures in flexible job shop scheduling, and developed a multi-stage complete information static game model for the scheduling problem under failures [13].

In foreign research, Chen, Y.R. et al. developed two mixed integer programming models for the single-machine scheduling problem with flexible maintenance and non-recoverable operations [14]. Grznar et al. have developed a simulation model to simulate the logistics system, which can determine the state of the plant layout and effectively suggest solutions to problems in production and supply [15]. Wang et al. studied the disassembly of a product and applied an intelligent algorithm to solve the disassembly line balancing problem [16]. Liu et al. developed a mathematical model considering constraints such as variable machining times and intermittent machining with an objective of completion time and energy consumption, using a forging shop as a study [17]. Paternina-Arboleda et al. presented a heuristic algorithm based on bottleneck stage identification for the flow shop completion time minimization problem, rescheduling the sequence of workpieces on the bottleneck stage [18]. Yang and Wang proposed a novel hybrid method of adaptive neural networks and heuristics in solving the shop floor scheduling problem [19]. Torabi et al. presented a new mixed-integer nonlinear programming design for determining machine allocation, sorting, batching, and scheduling decisions, which was solved using an enumeration method [20]. Vincent et al. took constraints such as machine capacity and time lag into account and proposed a mixed integer linear programming and constrained programming model, which was solved using a meta-heuristic algorithm [21].
3. A brief overview of methods for solving production scheduling problem

The production scheduling problem can be described as the scheduling of materials, processing time and sequences of operations with the objective of optimising manufacturing time or costs, etc., subject to various conditions. In a shop floor production system, production scheduling problems can be classified into the following types depending on the processing method: single machine scheduling problems, parallel machine scheduling problems, flow shop scheduling problems, hybrid flow shop scheduling problems, job shop scheduling problems and open shop scheduling problems [22, 23]. There are several modelling methods for solving shop floor scheduling problems, comprising three main categories, mathematical programming method, graph and network method and simulation method [24]. The core of the mathematical planning method is that the relationships between variables in a system can be expressed by corresponding mathematical relationships, the diagram and network method mainly includes activity cycle diagrams, critical path method, etc. The simulation method is based on the establishment of a system model to express the system behaviour, which is transformed into a computer simulation program. Each of the three types of methods has its own advantages and disadvantages, and the one that is more frequently used is the mathematical planning method. Once a mathematical model has been built for the shop floor scheduling problem, the problem needs to be solved using the appropriate algorithms, which can be divided into two main categories: exact methods and approximate methods.

Precision methods can be used to solve scheduling optimisation or sub-optimisation problems, with the advantage that the global optimum solution is guaranteed and is suitable for solving problems that require high precision. The precise methods commonly used include mathematical programming method and branch and bound method. Mathematical planning methods can be divided into integer programming, mixed integer programming, Lagrangian relaxation method and decomposition method. However, the disadvantage is that it can only solve small-scale problems and is slow, and its use is limited for complex problems.

![Fig. 1 Methods for solving scheduling problems](image)
The opposite is true of the approximation method, which gives a quick solution to the problem, but has the disadvantage that there is no guarantee of an optimal solution. It is suitable for large-scale problems and better meets the needs of real-world problems. Approximation methods can be further divided into three main categories: construction method, artificial intelligence method and local search algorithm. The construction methods mainly include dispatching rule method, interpolation method and bottleneck-based heuristic method, etc. Artificial intelligence algorithms mainly include constraint satisfaction, expert systems, neural networks, multi-intelligent body techniques, and meta-heuristics (e.g., genetic algorithms, ant colony algorithms, etc.) Finally local search algorithms, mainly represented by simulated annealing method and taboo search, etc. The main structure and some common algorithm features are shown in Figs. 1 and 2.
When using exact methods to solve scheduling problems, the optimal solution to the problem can be obtained. However, traditional optimisation methods such as linear programming, dynamic programming and branch-and-bound methods are slow and more suitable for solving small-scale shop floor scheduling problems, while complex shop floor scheduling problems are difficult or impossible to solve quickly. For large-scale scheduling problems, approximate methods are often a better choice. Approximation methods differ from exact methods in that they do not seek the optimal solution to a scheduling optimisation problem, but rather aim to find a more satisfactory solution in a given amount of time.

Among the construction methods, the assignment rule method contains First Come First Served, Shortest Processing Time, Earliest Due Date and so on. They are based on information such as processing times, delivery deadlines, number of parts and workshop conditions, which determine which rules are used to optimise the scheduling solution. They are characterised by their simplicity of calculation and the possibility of obtaining a satisfactory solution, but only the current state is considered. In practice, it can be used in combination with a variety of rules or with other intelligent optimisation algorithms. In addition, the construction method also contains a bottleneck-based heuristic method, which in turn includes the bottleneck shifting method and the shot-beam search method. The advantage of this method is the better quality of the solution, the disadvantage is the long computation time and the complexity of the implementation.

The most popular and widely used algorithms are intelligent optimisation algorithms, which originate from the summary and simulation of certain unique natural phenomena and laws, and involve a wide range of disciplines, and have unique advantages for solving complex scheduling optimisation problems. These algorithms can be divided into artificial intelligence and local search algorithms according to their principles. Commonly used artificial intelligence methods are constraint satisfaction, multi-intelligence techniques, expert systems, neural networks, genetic algorithms, ant colony algorithms and particle swarm optimisation. Constraint satisfaction methods can give guidance on scheduling problems to a certain extent. Multi-intelligence body techniques methods require experience and knowledge. The expert system approach has a long development cycle and is strictly domain specific. Neural network methods are less computationally efficient and more suitable for small-scale problems. Genetic algorithms have low mathematical requirements and are highly flexible in their application. Ant colony algorithms can be used to solve complex combinatorial optimisation but are slow to converge. Particle swarm optimisation methods were originally used for continuous problem optimisation.

Finally, there are local search algorithms. The main local search algorithms commonly used are simulated annealing and taboo search. The simulated annealing method is not able to obtain a good solution very quickly and is suitable for use in combination with other algorithms. The taboo search method is faster and is also suitable for use in combination with other algorithms.

As mentioned above, different types of job shops have different modes of operation, so different models can be built to solve the production scheduling problem for different job shops using different algorithms. This paper selects the mixed flow shop scheduling problem as the research object, establishes the scheduling optimization mathematical model, and solves it by genetic algorithm.

4. Problem statement and research methodology

4.1 Production scheduling model for hybrid flow shop

Hybrid flow shop scheduling, which is also called flexible flow shop scheduling, exists in a wide range of areas and is more complex than the normal flow shop [25]. Its machining structure is shown in Fig. 3. Workpieces are processed in several stages, each of which may have several optional machines, and the workpieces are processed in a fixed sequence. The hybrid flow shop scheduling problem is more closely related to actual manufacturing and is therefore the most used. In this paper, we aim to develop a mathematical model for production scheduling in a hybrid flow shop with the objective of minimising completion time.
The scheduling problem in a mixed flow shop can be characterised as follows: there are \( n \) parts to be machined \( \{N_1, N_2, ..., N_n\} \) on \( m \) machines \( \{M_1, M_2, ..., M_m\} \), where at least one machine exists for each process and parallel machines exist for at least one process, and any of the parallel machines can be selected to process the process. The purpose of scheduling is to find the sequence of workpieces in the overall production plan and to select the most suitable equipment for each process so that the scheduling solution is optimal.

Model hypothesis:

1) Only a maximum of one workpiece can be machined on one machine at any one time.
2) A workpiece can only be processed by one machine at any one time.
3) Once the machine has started working on the workpiece, it cannot be interrupted and must complete the process.
4) Each workpiece is processed strictly in its own sequence.
5) Each workpiece is processed in each process in strict accordance with the optional equipment.
6) Equal machining priority between different workpieces.

Model building:

The objective function is as follows:

\[
min f = \min \{ \max T_k \}, \quad 1 \leq k \leq m
\]  
(1)

The constraints are as follows.

\[
\sum_{p=1}^{n} x_{ip} = 1, \quad p = 1, 2, ..., n
\]  
(2)

\[
\sum_{i=1}^{n} x_{ip} = 1, \quad i = 1, 2, ..., n
\]  
(3)

\[
\sum_{j=1}^{m} M_j > m
\]  
(4)

\[
\sum_{k=1}^{M_j} O_{ijk} = 1, j = 1, 2, ..., m, k = 1, 2, ..., M_j
\]  
(5)

\[
T_{ij} - T_{i(j-1)} \geq t_{ijk} O_{ijk}, 2 \leq j \leq n
\]  
(6)
Where $T_k$ is the time of completion of the $k$-th machine. $x_{ij}$ indicates that the $j$-th workpiece is assigned to be processed at the $p$-th location. $M_j$ indicates the number of machines available for the $j$-th process. $O_{ijk}$ is a 0/1 variable that takes the value of 1 when the $j$-th process of workpiece $i$ is processed on the $j$-th machine and 0 otherwise. $T_{ij}$ indicates the completion time of the $j$-th process of workpiece $i$ and $t_{ijk}$ indicates the time spent by workpiece $i$ on the $k$-th machine for the $j$-th process.

Eq. 1 indicates the minimised maximum completion time, $k = 1, 2, .., m$. Eqs. 2 and 3 show that a machine can only be used to machine one workpiece at a time and that a workpiece can only be machined by one machine at a time. Eq. 4 shows that there are concurrent machines for at least one process. Eq. 5 indicates that only one machine can process each process of the same workpiece. Eq. 6 means that the current process is completed before moving on to the next process and cannot be interrupted.

### 4.2 Algorithm design

This paper applies a genetic algorithm to optimize the hybrid flow shop scheduling problem with the target of minimising the maximum completion time.

**Step 1: Coding design**

Considering the specificity of the hybrid flow shop model, in order to be able to describe both the processing sequence of the workpiece and the assigned machine information, this paper adopts a two-layer coding approach, containing both the process sequence and the machine sequence. The first layer is based on the workpiece process coding, and the other chromosome coding layer numbers the processing machines for each process. Each two-layer code then corresponds to a scheduling scheme based on the known set of available machines and their corresponding processing schedules.

Let $M(a, b)$ denote the set of machines that process the $b$-th process of workpiece $a$, and let $T(a, b)$ denote the set of time spent on the different machines that process the $b$-th process of workpiece $a$. Now suppose that a job shop has four machines to process three workpieces, each of which has to go through three processes, and that the corresponding $M$ and $T$ are shown in Tables 1 and 2.

In the chromosome code shown, each gene in the process sequence represents a workpiece number, where the number of times it appears in the chromosome indicates the number of processes it represents. As shown in the Fig. 4.

The sequence of execution of each workpiece process and the arrangement of machines for the scheduling scheme corresponding to this chromosome is derived as shown in the Fig. 4. The Gantt chart of the processing corresponding to this chromosome code can be obtained as shown in Fig. 5. A chromosome can be obtained using the two-layer coding method as shown in Table 3.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Optional set of machines for machining workpieces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Workpiece</td>
<td>Working procedure 1</td>
</tr>
<tr>
<td>N1</td>
<td>M4</td>
</tr>
<tr>
<td>N2</td>
<td>M2, M4</td>
</tr>
<tr>
<td>N3</td>
<td>M4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Time set for machining of workpieces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Workpiece</td>
<td>Working procedure 1</td>
</tr>
<tr>
<td>N1</td>
<td>3</td>
</tr>
<tr>
<td>N2</td>
<td>[5,3]</td>
</tr>
<tr>
<td>N3</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Chromosome coding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process sequence</td>
<td>1 2 2 3 2 3 1 1 3</td>
</tr>
<tr>
<td>Machine sequence</td>
<td>1 2 1 1 2 1 2 1</td>
</tr>
</tbody>
</table>
Step 2: Select design

In genetic algorithms, the higher the fitness level, the higher the probability of being selected. This algorithm design is based on ranked fitness selection. The probability of selection of individuals is determined by ranking individuals in order of fitness. Firstly, the individuals in the population are ranked linearly according to their fitness, and then the values of fitness in the population are selected proportionally using a 'roulette wheel' approach.

Step 3: Crossover and variation

Crossover is the process of replacing and recombining parts of the structure of two parents to create a new individual, and the search power of genetic algorithms can be greatly improved by crossover. The crossover operation starts by selecting two chromosomes at random from the population, crossing the chromosomes at random locations, and then adjusting the crossed chromosomes to match the number of expressions.
Variation is used to give the genetic algorithm the ability to search locally at random and to maintain the diversity of the population [26]. The basic element of the mutation operator is to make changes to gene values. First individuals determine whether or not to mutate based on a pre-determined mutation probability, and then mutations are made to randomly selected mutation sites for those individuals making mutations.

5. Simulation analysis: Results and discussion

5.1 Results

The production scheduling problem is a key issue in manufacturing and a proper scheduling solution can effectively improve the operational efficiency of a manufacturing company. In this case study, the objective is to minimise the completion time and to optimise the scheduling problem in a hybrid shop. There are six workpieces that need to be processed in six stages, and ten machines are responsible for processing them. Some of the workpieces have parallel machines for some of the processes and can be selected to be processed on different machines, with the specific information on the machines available in the Table 4.

The different machine options correspond to different processing time, as shown in the Table 5.

<table>
<thead>
<tr>
<th>Workpiece1</th>
<th>Workpiece2</th>
<th>Workpiece3</th>
<th>Workpiece4</th>
<th>Workpiece5</th>
<th>Workpiece6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working procedure 1</td>
<td>3</td>
<td>7</td>
<td>4</td>
<td>[2,8]</td>
<td>[3,6]</td>
</tr>
<tr>
<td>Working procedure 2</td>
<td>4</td>
<td>[2,8]</td>
<td>9</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Working procedure 3</td>
<td>5</td>
<td>[7,8]</td>
<td>6</td>
<td>[2,1]</td>
<td>10</td>
</tr>
<tr>
<td>Working procedure 4</td>
<td>3</td>
<td>2</td>
<td>[4,6]</td>
<td>10</td>
<td>[2,5]</td>
</tr>
<tr>
<td>Working procedure 5</td>
<td>[4,3]</td>
<td>3</td>
<td>[8,10]</td>
<td>7</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Workpiece</th>
<th>Workpiece 1</th>
<th>Workpiece 2</th>
<th>Workpiece 3</th>
<th>Workpiece 4</th>
<th>Workpiece 5</th>
<th>Workpiece 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working procedure 1</td>
<td>11</td>
<td>12</td>
<td>7</td>
<td>[4,2]</td>
<td>[2,5]</td>
<td>8</td>
</tr>
<tr>
<td>Working procedure 2</td>
<td>5</td>
<td>[3,4]</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>[4,4]</td>
</tr>
<tr>
<td>Working procedure 3</td>
<td>7</td>
<td>[6,5]</td>
<td>4</td>
<td>[4,2]</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>Working procedure 4</td>
<td>5</td>
<td>3</td>
<td>[3,3]</td>
<td>7</td>
<td>[7,6]</td>
<td>[4,7]</td>
</tr>
<tr>
<td>Working procedure 5</td>
<td>[6,6]</td>
<td>7</td>
<td>[5,2]</td>
<td>7</td>
<td>3</td>
<td>[8,5]</td>
</tr>
<tr>
<td>Working procedure 6</td>
<td>[3,1]</td>
<td>5</td>
<td>[7,4]</td>
<td>5</td>
<td>8</td>
<td>[6,9]</td>
</tr>
</tbody>
</table>

This paper uses Matlab software to program and solve the above production scheduling optimisation problem using a genetic algorithm, setting the initial population size to 40 and the maximum number of iterations to 50. The Gantt chart of the optimal scheduling solution obtained after solving using Matlab software is shown in the Fig. 6.
As shown in Fig. 6, the x-axis shows the processing time of the workpiece and the y-axis shows the machine number corresponding to the workpiece processing. Based on the above Gantt chart, it is clear that in this case of a hybrid flow shop, the selection of machines for the different processes of the different workpieces and the time taken to process them can be seen. Where the number in the rectangle indicates the \( j \)-th process of the \( i \)-th workpiece, it can be seen that the final time taken to complete all processes is 49 minutes. In solving the above problem using Matlab, the resulting evolutionary iteration diagram is shown in the Fig. 7.

![Variation of the optimal solution](image)

**Fig. 7** Evolutionary iteration diagram of the optimal solution

The above images show the variation of the solution and the variation of the population mean in solving the production scheduling optimisation problem for a hybrid shop floor. The results show that the algorithm used in this paper can effectively find the optimal scheduling solution for the hybrid shop floor problem in a short time.

5.2 Discussion

The simulation results show that the genetic algorithm used in this paper can effectively solve the scheduling problem for a hybrid flow shop. Fig. 6 clearly shows the machine and sequence arrangement and the corresponding processing time for scheduling each workpiece process, while Fig. 7 shows the evolutionary iterative process of the optimal solution.

Each algorithm for solving production scheduling problems often has its own advantages and disadvantages, such as global search, local search, speed of solution, accuracy of solution, scale of application, etc. It is usually necessary to choose the right algorithm according to the needs of the problem.

The genetic algorithm used in this paper first encodes the variables to be decided upon, and then operates on their encoding. The sequence is manipulated directly, simulating the process of chromosome evolution. It is based on probabilistic rules that make the search more flexible, as well as its population search feature, which starts from the population and avoids some points that do not need to be searched, allowing for better global search, but at the same time has disadvantages in terms of local search. It can be used in combination with other algorithms when necessary to solve practical problems. And more research could be done in the future on retaining good individuals while maintaining population diversity.

6. Conclusion

Based on the understanding and analysis of different types of job shop production scheduling problems, this paper provides an overview of modelling and solution methods for the optimisation of scheduling problems. The advantages, disadvantages and applicability characteristics of the different methods are analysed, and situations in which the different optimisation methods are applicable are derived. Where necessary, a reasonable combination of different solution
methods can be used to cope with complex scheduling problems. This can provide a reference for the choice of modelling and solution methods in the study of scheduling problems. Later, a case study is conducted on the hybrid flow shop scheduling problem. A production scheduling optimisation model with the objective of minimising the maximum completion time is constructed, and a genetic algorithm based on two-layer coding is applied to solve the problem, proving the validity of the model and the algorithm.

Acknowledgement

This work was financially supported by the Liaoning Planning Office of Philosophy and Social Science Project L19BXW006.

References


