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When core sorting and quality grading is beneficial to remanufacturers: Insights from analytical models

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A B S T R A C T

In this paper, we study the core acquisition and remanufacturing problem in which the remanufactured products are produced from acquired cores with uncertain quality condition, and are used to satisfy customer demand. Decision-making models are developed to examine the potential value of core sorting and quality grading in the remanufacturing system: a single-period model with deterministic demand, and a single-period model with stochastic demand (i.e., a newsvendor-type model). In each model, both the sorting strategy and the non-sorting strategy are discussed and compared. Our theoretical and numerical results show that: (1) In the deterministic demand case, core sorting is cost-effective only when the unit sorting cost is below a threshold value and the unit acquisition cost falls into a specific interval. Furthermore, in the case with two quality grades the adoption of sorting strategy with respect to the expected fraction of high-quality cores may be non-monotone: an initial increase in the expected fraction of high-quality cores may motivate a switch to core sorting, however, further increase in the expected fraction may motivate a reverse switch; (2) Similarly, in the stochastic demand case, the sorting strategy also becomes unattractive when the unit sorting cost is sufficiently high. In addition, the value of core sorting will be better off under more fluctuating demand for remanufactured products if the sorting strategy is the dominant strategy. Otherwise, it will be worse off.

A R T I C L E I N F O

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Remanufacturing;
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1. Introduction

Remanufacturing, which refers to the industrial process of restoring used products or parts to like-new condition, has become to be a more and more important part of the circular economy (Mihai *et al.*, 2018), [1]. It also has been applied to various products such as automotive parts, electrical and electronics products, machinery, information and communication equipment, ink and toner cartridges, medical devices and furniture (Yoon and Joung, 2019; Davidavičienė *et al.*, 2019, Yoon and Joung 2021), [2-4]. Many companies, including Daimler, Volvo, Lenovo, Huawei, Hewlett-Packard, Xerox, and IKEA, have engaged in the business of acquiring and remanufacturing used products (referred to as cores) from end-users. For example, the Volvo Group has reported that the total sales of remanufactured components amounted to SEK 10 billion in 2018 with a yearly average increasing rate of 10 %. Through an old-for-new program, Huawei has collected more than 140,000 used cell phones in 2018, and recovered and reused 82.3 % materials of these recycled cell phones. In a “Buy-back service” project launched in 2017, IKEA Japan recycled 1,900 second-hand furniture products in the first six months, and sold 85 % of them to customers after repair and refurbishing.

Since the quality condition of acquired cores can be so highly variable that may complicate the remanufacturing process (Guide, 2000; He, 2018), [5, 6], cores are usually sorted and graded into different categories based on their quality conditions by remanufacturers. For example, ReCellular, a cellphone remanufacturer, classified the returned phones into six quality grades based on their functional and physical criteria in which the remanufacturing cost of the lowest quality grade is eight times more than that of the highest quality grade (Guide *et al.*, 2003), [7]. Similarly, a returned Personal Computer may also fall into different conditions: some may be brand new with their package box has never been opened; some may be used for a few times; some may need repair; and some may only be salvaged for parts or materials (Blackburn *et al.*, 2004), [8]. It was reported that Pitney-Bowes, an equipment remanufacturer, categorized the returned mechanical products into three quality grades (i.e., remanufacturing, reassembling or recycling) according to different recovery options (Ferguson *et al.*, 2009), [9]. In the reverse logistics network studied by Sedehzadeh and Seifbarghy (2021), [10], returned products are classified into two categories according to their health security: usable products and recoverable products. The sorting and grading operation resolves the quality uncertainties of acquired cores in advance, and thus allows remanufacturing these cores in a greedy and economical sequence.

In practice, core sorting and quality grading are typically implemented through visual inspection by sophisticated workers and usage information monitoring by information systems with tracking and tracing technology such as RFID. As the tasks of sorting and grading are primarily labour-intensive, it is usually costly to make such quality assessment. Furthermore, this prevailing operation helps firms to mitigate uncertainty in quality condition of cores, but requires acquiring more returned products than the demand for remanufactured products. As a result, the additional acquisition cost as well as the sorting cost offsets the remanufacturing cost savings obtained by processing a higher quality unit rather than a lower quality one. The trade-off between these two effects of core sorting and quality grading makes it less attractive in remanufacturing. Managers in remanufacturing firms are aware of the remanufacturing cost difference between different quality grades, but are unsure of the exact value of core quality information and the addition effort required to collecting and utilizing such information (Ferguson *et al.*, 2009), [9]. Some researchers also found that under certain conditions remanufacturers can hardly benefit from core sorting and quality grading (see, e.g., Li *et al.*, 2016; Yanıkoğlu and Denizel, 2021), [11, 12].

The research on core acquisition management, which refers to the management of the core acquisition process by dealing with the uncertainties with respect to return timing, quantity and core quality in reverse logistics, has intensified in recent years. An excellent review can be found in Wei *et al.* (2015), [13]. Our paper is most relevant to the research work that addressed the complications of quality uncertainty of cores in remanufacturing, thus we mainly review the literature of this research stream below. Guide *et al.* (2003), [7], first developed a single-period model to determine the optimal core acquisition prices and remanufactured products' selling price. They assumed that the collectors grade cores and sell them to the remanufacturer in different quality classes at different prices, and within a certain quality class all the cores have the same associated remanufacturing cost. Galbreth and Blackburn (2006), [14], considered the case where the remanufacturer acquires unsorted used products from the third party collectors. The quality condition of cores is highly variable, and the remanufacturer must sort each core into different quality grades. They assumed that the cumulative distribution of cores condition is known exactly. In a single-period model, they derived optimal core acquisition and sorting policies for both deterministic demand case and stochastic demand case. Their sorting policy is defined by the value of remanufacturing cost: cores with remanufacturing cost above a threshold value are scrapped, and those with cost below the threshold value are remanufactured. After that, Galbreth and Blackburn (2010), [15], studied core acquisition decisions when the quality condition of each acquired core is uncertain. Their work is extended to the case where the quality condition of each acquired core follows general distributions by Yang *et al.* (2015), [16]. By incorporating quantity discount and carbon tax scheme into the core acquisition models discussed in Galbreth and Blackburn (2006, 2010), [14, 15], Yang *et al*. (2016), [17], analysed the optimal core acquisition policies under the cases with quality variability and with both quality variability and condition uncertainty, respectively. Teunter and Flapper (2011), [18], assumed that there are multiple types of acquired cores, and that the type of a core follows a multinomial distribution. They derived optimal core acquisition and remanufacturing policies for both deterministic and stochastic demand. Mutha *et al*. (2016), [19], developed a two-period model to study core acquisition decisions when used products can be acquired either in bulk with uncertain quality levels, or in sorted grades with known quality levels, and the remanufacturer can acquire remanufacture cores before or after demand is realized. Lv *et al*. (2017), [20], discussed the two-period and multi-period manufacturing/remanufacturing models where the collection rate of used products can be influenced by a fixed investment. Mircea *et al*. (2023), [21] studied a repeated game in remanufacturing where used products are collected by the online/offline recycler and the manufacturer, and derived the optimal recycling prices for each collector. In a newsvendor-type model setting, Li *et al*. (2016), [11], analyzed the optimal decisions on core acquisition and remanufacturing quantities under both the remanufacturing-to-stock (RMTS) mode and the remanufacturing-to-order (RMTO) mode. They found that sorting may never be adopted in the RMTS mode regardless of the sorting cost. Ferguson *et al*. (2009), [9], and Yanıkoğlu and Denizel (2021), [12], studied the value of quality grading in the multi-period setting. The numerical results in Ferguson *et al*. (2009), [9], indicated that the ratio of return rates to demand rates, the number of quality grades, the distribution of the quality of returns and the cost difference between quality grades are the main drivers of the value of quality grading. Based on Ferguson *et al*. (2009)'s work [9], Yanıkoğlu and Denizel (2021), [12], study the core acquisition and remanufacturing problem under the case where the unit remanufacturing cost and unit resource requirement are uncertain. They found that when considering the unit grading cost, it may cause a significant deterioration in the value of grading, even make the grading totally useless.

This work is motivated by academic research (see, e.g., Ferrer and Ketzenberg, 2004, [22]; Ketzenberg *et al*., 2006, [23]; Ferguson *et al*., 2009, [9]; Li *et al*., 2016, [11]), and industrial practice (see, e.g., ReCellular, Pitney-Bowes, PneuLaurent, Caterpillar). We study the core acquisition and remanufacturing problem under a single-period setting in which the remanufactured products used to satisfy customer demand are produced from acquired cores with uncertain quality. Both the deterministic demand case and the stochastic demand case are discussed to examine the potential value of core sorting and quality grading in remanufacturing. In each case, two strategies are discussed and compared: (1) the sorting strategy, i.e., sorting and remanufacturing cores in a greedy sequence, and (2) the non-sorting strategy, i.e., remanufacturing cores in the natural sequence. During these different model settings, we aim to address the following questions: (1) Is core sorting and quality grading a cost-effective operation for remanufacturing firms? (2) Under what conditions does the remanufacturer benefit from core sorting and quality grading? (3) What are the impacts of demand and cost parameters on the value of core sorting and quality grading? (4) Whether these impacts change under different core acquisition and remanufacturing decision-making environments?

More specifically, we start with a single period model with deterministic demand, for which we derive the optimal core acquisition quantity for both the sorting and non-sorting strategy. By comparing with these two strategies, we find that core sorting and quality grading is cost-effective only when the unit sorting cost is sufficiently small and the unit core acquisition cost falls into a specific interval. Further analysis in a special case with two quality grades show that, an initial increase in the expected fraction of high-quality cores may motivate a switch to core sorting and quality grading, however, further increase may motivate a reverse switch. We then continue our study in a newsvendor-type model and characterize the optimal acquisition, sorting and remanufacturing policies. Similar to the deterministic demand case, core sorting and quality grading is beneficial to the remanufacturer only when the sorting cost is sufficiently low. Another observation is that when the sorting strategy is superior to the non-sorting strategy, it is more valuable under a higher level of demand variation.

The rest of this paper is organized as follows. In Section 2, we describe the basic problem setting and introduce the notations and assumptions used throughout the paper. In Section 3 and Section 4, we present a single-period core acquisition and remanufacturing model with deterministic and stochastic demand, respectively. Finally, we conclude the paper in Section 5.

2. Problem description

Consider a remanufacturing firm who acquires cores and sells remanufactured products to the market. The cores are acquired in bulk at unit acquisition cost c_r and their quality conditions are uncertain. Without loss of generality, assume that the acquired cores can be categorized into N nominal quality grades based on their quality levels, with grade 1 being the highest quality grade and grade N being the lowest. In general, the higher quality grades are cheaper to remanufacture, that is, $c_1 < c_2 <\ldots< c_N$, where c_i denotes the unit remanufacturing cost of grade i, $i = 1,2,\ldots,N$. The fraction of acquired cores belonging to grade i, θ_i, is a random variable with the domain [0, 1]. The expectation of θ_i is denoted by μ_i. The fractions $\theta_1, \theta_2,\ldots, \theta_N$ are jointly distributed with p.d.f. $g_{\theta_1,\theta_2,\ldots,\theta_N}(\cdot)$ and c.d.f. $G_{\theta_1,\theta_2,\ldots,\theta_N}(\cdot)$ and their summation is 1.

After a quantity of cores, Q units, are acquired, the firm has two alternative choices: (1) adopting a sorting procedure to resolve the quality uncertainties by incurring a unit sorting cost c_g, (2) doing nothing. If cores are sorted and graded, the fractions of core quality distribution are revealed before remanufacturing, otherwise they are known to the firm only until all the cores are remanufactured.

The market demand for remanufactured products studied in the single-period setting has two alternative forms: (a) deterministic demand, (b) stochastic demand. For the deterministic case, market demand D is known and no shortage is allowed. For the stochastic case, market demand D is a random variable with probability density function $f(D)$ and cumulative distribution function $F(D)$. In addition, we assume that D and $(\theta_1, \theta_2, \ldots, \theta_N)$ are independent. At the beginning of each period, the remanufacturing firm decides remanufacturing quantities from each grade of core before demand is realized. Then demand realization is observed. A unit shortage penalty cost, b_r, is charged for the unsatisfied demand and a unit holding cost, h_r, is charged for the leftover remanufactured product. At the end of the period, the leftover cores are disposed at unit cost, h_u. In general, the sequence of events is as follows: acquiring bulk cores, sorting and grading (or doing nothing), remanufacturing, demand realization. The decisions to be made by the remanufacturing firm include the core acquisition quantity and the remanufacturing quantity of each quality grade. The objective is to maximize the total expected profit with respect to core acquisition, remanufacturing, and demand fulfilment.

In summary, the following notions and assumptions are employed in this paper:

N	number of nominal quality grades of cores
D	market demand for remanufactured products
Q	acquisition quantity of cores
p	unit selling price for a remanufactured product
c_r	unit acquisition cost
c_i	unit remanufacturing cost of grade i, $i = 1,2,\ldots,N$
c_g	unit sorting cost
h_u	unit disposal cost
b_r	unit shortage penalty cost for unsatisfied demand
h_r	unit holding cost for the remanufactured product
θ_i	fraction of acquired cores belonging to grade i, $i = 1,2,\ldots,N$
μ_i	expected value of θ_i, $i = 1,2,\ldots,N$
c_μ	expected unit remanufacturing cost, where $c_\mu = E\left(\sum_{i=1}^{N}\theta_i c_i\right) = \sum_{i=1}^{N}\mu_i c_i$
$g_{\theta_1,\theta_2,\ldots,\theta_N}(\cdot)$	joint probability density function of $\theta_1, \theta_2, \ldots, \theta_N$
$G_{\theta_1,\theta_2,\ldots,\theta_N}(\cdot)$	joint cumulative distribution function of $\theta_1, \theta_2, \ldots, \theta_N$
$f(D)$	probability density function of D

$F(D)$	cumulative distribution function of D
q_i	quantity of cores of grade i that are remanufactured, $i = 1,2,\ldots,N$
π	total expected profit of the remanufacturing firm

3. The single-period, deterministic demand case

We begin our analysis with the single-period, deterministic demand case. Both the sorting scenario and the non-sorting scenario are discussed. For ease of exposition, we put superscript D and subscripts s and ns on some notations to represent these two scenarios, respectively.

3.1 The non-sorting scenario

When the manufacturer adopts the non-sorting operation, it is optimal to acquire the exact demanded quantity of cores. Let $Q_{ns}^{D^*}$ and $\pi_{ns}^{D^*}$ be the optimal acquisition quantity and the optimal expected profit under the non-sorting scenario, respectively. Then we have $Q_{ns}^{D^*} = D$ and $\pi_{ns}^{D^*} = (p - c_r - c_\mu)D$, where $c_\mu = \sum_{i=1}^{N} \mu_i c_i$ is the expected unit remanufacturing cost.

3.2 The sorting scenario

Let Q_s^D be the acquisition quantity of cores under the sorting scenario in the deterministic demand case. We define the acquisition ratio as $\alpha = D/Q_s^D$, where α can be interpreted as the fraction of acquired cores that must be remanufactured to satisfy the demand. It is reasonable that the remanufacturer will acquire more cores than demanded under the sorting scenario (Galbreth and Blackburn, 2006), [14]. Thus, we have $0 < \alpha \leq 1$.

The optimal remanufacturing policy is straightforward that follows a greedy rule: first processing cores of grade 1, and if the remanufacture-up-to level is still not reached, then processing cores of grade 2, and so on, stop until the remanufacture-up-to level is reached or all the cores are remanufactured.

Let $R_0 = 0$ and

$$R_i = min\left\{D, \sum_{j=1}^{i} \theta_j Q_s^D\right\}, i = 1,2,\ldots,N \quad (1)$$

Then the quantity of cores of grade i remanufactured under the sorting scenario, q_i^D, is

$$q_i^D = R_i - R_{i-1}, i = 1,2,\ldots,N \quad (2)$$

The expected profit of the remanufacturer, π_s^D, is

$$\pi_s^D = p \times D - (c_r + c_g)Q_s^D - E\left(\sum_{i=1}^{N} c_i q_i^D\right) - h_u(Q_s^D - D) \quad (3)$$

which can be rewritten as a function of α, where

$$\begin{aligned}\pi_s^D(\alpha) = (p - c_N + h_u) \times D - (c_r + c_g + h_u)\frac{D}{\alpha} \\ + D \times \sum_{i=1}^{N-1} (c_{i+1} - c_i)\left(1 - \int_0^\alpha (1 - \tfrac{x}{\alpha}) g_{\sum_{j=1}^{i}\theta_j}(x)dx\right)\end{aligned} \quad (4)$$

where $g_{\sum_{j=1}^{i}\theta_j}(x)$ and $G_{\sum_{j=1}^{i}\theta_j}(x)$ are the p.d.f. and c.d.f. of x, where $x = \sum_{j=1}^{i}\theta_j$ for any $i \in \{1,2,\ldots,N\}$.

Theorem 1. *If $c_r + c_g + h_u \geq c_N - c_\mu$, $\pi_s^D(\alpha)$ is monotonically increasing in α, and $\alpha^* = 1$. Otherwise, $\pi_s^D(\alpha)$ is quasi-concave in α, and α^* is given by*

$$\sum_{i=1}^{N-1}\left[(c_{i+1} - c_i)\int_0^{\alpha^*} x g_{\sum_{j=1}^{i}\theta_j}(x)dx\right] = c_r + c_g + h_u \quad (5)$$

Proof: Take the first derivate of $\pi_s^D(\alpha)$ with respect to α, we have

$$\pi_s'^D(\alpha) = (c_r + c_g + h_u)\frac{D}{\alpha^2} - \frac{D}{\alpha^2}\sum_{i=1}^{N-1}\left[(c_{i+1} - c_i)\int_0^{\alpha} x g_{\sum_{j=1}^{i}\theta_j}(x)dx\right]$$

It is easy to verify that the term $\sum_{i=1}^{N-1}\left[(c_{i+1} - c_i)\int_0^{\alpha} x g_{\sum_{j=1}^{i}\theta_j}(x)dx\right]$ is increasing in α, and $\pi_s'^D(\alpha)$ is decreasing in α. Note that

(i) when $\alpha \to 0$, $\sum_{i=1}^{N-1}\left[(c_{i+1} - c_i)\int_0^{\alpha} x g_{\sum_{j=1}^{i}\theta_j}(x)dx\right] \to 0$,

(ii) when $\alpha = 1$, $\sum_{i=1}^{N-1}\left[(c_{i+1} - c_i)\int_0^{\alpha} x g_{\sum_{j=1}^{i}\theta_j}(x)dx\right] = \sum_{i=1}^{N-1}\left[(c_{i+1} - c_i)\sum_{j=1}^{i}\mu_j\right] = c_N - c_\mu$.

If $c_r + c_g + h_u \geq c_N - c_\mu$, then we always have $\pi_s'^D(\alpha) \geq 0$ for any $\alpha \in (0,1]$. This implies that $\pi_s^D(\alpha)$ is increasing in α, and attains its maximum at $\alpha = 1$. If $c_r + c_g + h_u < c_N - c_\mu$, with the increase of α, $\pi_s'^D(\alpha)$ decreases and changes sign at most once from positive to negative. Denote α^* as the changing point where $\pi_s'^D(\alpha^*) = 0$. Thus, $\pi_s'^D(\alpha) > 0$ for any $\alpha \in (0, \alpha^*)$ and $\pi_s'^D(\alpha) < 0$ for any $\alpha \in (\alpha^*, 1]$. This implies that $\pi_s^D(\alpha)$ is increasing in α on the interval $(0, \alpha^*)$ and decreasing in α on the interval $(\alpha^*, 1]$. Therefore, $\pi_s^D(\alpha)$ is quasi-concave in α, and attains its maximum at $\alpha = \alpha^*$.□

Theorem 1 suggests that when the unit core processing (i.e., acquisition and sorting and disposal) cost is sufficiently high, the acquired cores are fully remanufactured. Otherwise, they are partially remanufactured. However, it is never optimal to adopt the sorting operation if all the acquired cores are remanufactured (i.e., $\alpha^* = 1$).

3.3 Comparative analysis

We examine the value of core sorting by comparing the expected profit of the remanufacturer under both the sorting scenario and the non-sorting scenario. Proposition 1 characterizes the situations when core sorting is beneficial to the remanufacturer.

Proposition 1. *There exist c_r^{max}, c_r^{min} and c_g^{max} such that (i) $\pi_s^D(\alpha^*) > \pi_{ns}^{D^*}$, if $c_r^{min} < c_r < c_r^{max}$ and $c_g < c_g^{max}$; (ii) $\pi_s^D(\alpha^*) \leq \pi_{ns}^{D^*}$, otherwise.*

Proof. By Proposition 1, when $c_r + c_g + h_u \geq c_N - c_\mu$, $\alpha^* = 1$, i.e., all the acquired cores are remanufactured. Then it is never optimal for the remanufacturer to choose the sorting strategy since additional sorting cost is required but no remanufacturing cost savings are obtained from core sorting. Thus, under this case we have $\pi_s^D(\alpha^*) < \pi_{ns}^{D^*}$.

When $c_r + c_g + h_u < c_N - c_\mu$,

$$\pi_s^D(\alpha^*) - \pi_{ns}^{D*} = (c_r + c_\mu - c_1 + h_u)D - D\sum_{i=1}^{N-1}\left[(c_{i+1} - c_i)G_{\sum_{j=1}^{i}\theta_j}(\alpha^*)\right]$$

Thus, if $c_r > c_1 + \sum_{i=1}^{N-1}\left[(c_{i+1} - c_i)G_{\sum_{j=1}^{i}\theta_j}(\alpha^*)\right] - c_\mu - h_u$, then $\pi_s^D(\alpha^*) > \pi_{ns}^{D*}$; otherwise, $\pi_s^D(\alpha^*) \leq \pi_{ns}^{D^*}$.

Let $c_r^{max} = c_N - c_\mu - c_g - h_u$, and $c_r^{min} = c_1 + \sum_{i=1}^{N-1}\left[(c_{i+1} - c_i)G_{\sum_{j=1}^{i}\theta_j}(\alpha^*)\right] - c_\mu - h_u$. Note that under this case the necessary condition for $\pi_s^D(\alpha^*) > \pi_{ns}^{D^*}$ is $c_r^{max} > c_r^{min}$, i.e., $c_g < \sum_{i=1}^{N-1}\left\{(c_{i+1} - c_i)[1 - G_{\sum_{j=1}^{i}\theta_j}(\alpha^*)]\right\} = c_g^{max}$.

Proposition 1 implies that core sorting in remanufacturing is cost-effective only when the unit sorting cost is smaller than a threshold and the unit core acquisition cost falls into a specific interval.

Furthermore, we examine the two-grade case to gain more managerial insights. It is found that the adoption of core sorting with respect to the expected fraction of high-quality cores, μ_1, may be non-monotone: an initial increase in μ_1 may motivate a switch to core sorting, however, further increase in μ_1 may motivate a reverse switch.

Proposition 2. *In the case with two quality grades, there exist μ_1^{max}, μ_1^{min} and c_g^{max} such that (i) $\pi_s^D(\alpha^*) > \pi_{ns}^{D^*}$, if $\mu_1^{min} < \mu_1 < \mu_1^{max}$ and $c_g < c_g^{max}$; (ii) $\pi_s^D(\alpha^*) \leq \pi_{ns}^{D^*}$, otherwise.*

Proof. According to the proof of Proposition 1, when $N = 2$, from $c_r^{min} < c_r < c_r^{max}$ we have

$$\mu_1^{min} = \frac{c_r + c_g + h_u}{c_2 - c_1} < \mu_1 < 1 - G_{\theta_1}(\alpha^*) + \frac{c_r + h_u}{c_2 - c_1} = \mu_1^{max}$$

Furthermore, from $\mu_1^{max} > \mu_1^{min}$ we have $c_g < (c_2 - c_1)[1 - G_{\theta_1}(\alpha^*)] = c_g^{max}$.

The results of Proposition 2 are illustrated in Fig. 1, where the parameter settings in the associated numerical example are: $D = 50, p = 100, c_r = 2, c_g = 2, h_u = 1, c_1 = 5, c_2 = 30$. The fraction of acquired cores with high-quality level, θ_1, follows a Beta distribution, i.e., $\theta_1 \sim \mathrm{B}(a, b)$. We consider different values of a and b in the numerical study, where $(a, b) \in \{a = 0.5i, b = 10 - 0.5i, i = 1, \ldots, 19\}$, thus the expectation of θ_1, μ_1, changes its value from 0.05 to 0.95 in steps of 0.05. The impacts of the expected fraction of high-quality cores, μ_1, on the expected profit of the remanufacturer are shown in Fig. 1. The explanation is as follows. When the fraction of high-quality cores is excessively high, there is no need to sorting cores as most of the acquired cores are in good quality condition. While when the fraction of high-quality cores is excessively low, core sorting could not help to improve the poor quality condition of acquired cores and thus is useless.

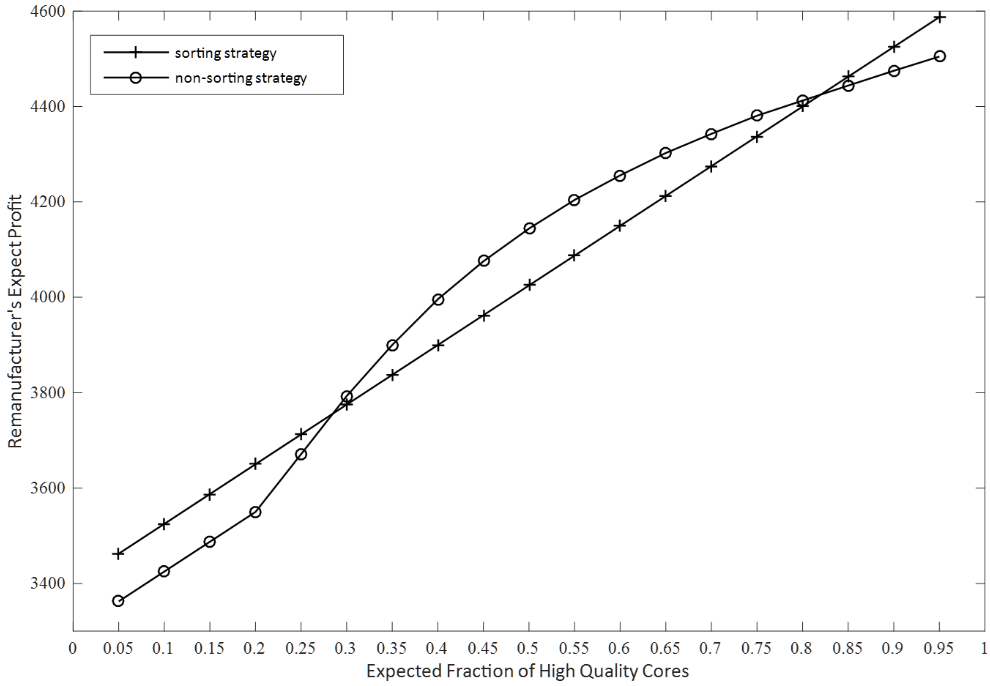

Fig. 1 Remanufacturer's expected profit under sorting and non-sorting strategies at different levels of expected fraction of high-quality cores

4. The single-period, stochastic demand case

Then we extend our analysis to the single-period, stochastic demand case. Similarly, we use superscript R and subscripts s and ns to represent this case under sorting and non-sorting scenarios.

4.1 The non-sorting scenario

In the non-sorting scenario, without the quality information of cores in advance, the firm will process all the acquired cores to satisfy market demand. The decision-making problem is a newsvendor-type problem. The expected profit of the firm is

$$\pi_{ns}^R = E[pmin(Q_{ns}^R, D) - b_r(D - Q_{ns}^R)^+ - h_r(Q_{ns}^R - D)^+] - (c_\mu + c_r)Q_{ns}^R \tag{6}$$

The optimal acquisition quantity, $Q_{ns}^{R^*}$, is

$$Q_{ns}^{R^*} = F^{-1}\left(1 - \frac{c_r + c_\mu + h_r}{p + b_r + h_r}\right) \tag{7}$$

4.2 The sorting scenario

Let Q_s^R be the acquisition quantity of the cores, and q_i^R the quantity of cores of grade i remanufactured. The expected profit of the firm under the sorting scenario, is

$$\pi_s^R = E[pmin(Q_s^R, D) - \sum_{i=1}^{N} c_i q_i^R - b_r(D - \sum_{i=1}^{N} q_i^R)^+ - h_r(\sum_{i=1}^{N} q_i^R - D)^+ - h_u(Q_s^R - \sum_{i=1}^{N} q_i^R)] \quad (8)$$
$$-(c_r + c_g)Q_s^R$$
$$s.t.\quad 0 \le q_i^R \le \theta_i Q_s^R, i = 1,2,\ldots,N$$

Similarly, let $R_0 = 0$ and R_i be the aggregate remanufactured quantity from grade 1 to grade i cores, $i = 1,2,\ldots,N$. Then following the greedy rule, $q_i^R = R_i - R_{i-1}$. The expected profit (8) can be rewritten as

$$\pi_s^R = E[pmin(Q_s^R, D) - \sum_{i=1}^{N} c_i(R_i - R_{i-1}) - b_r(D - R_N)^+ - h_r(R_N - D)^+ - h_u(Q_s^R - R_N)] \quad (9)$$
$$-(c_r + c_g)Q_s^R$$
$$s.t.\quad 0 \le R_i \le \textstyle\sum_{k=1}^{i} \theta_k Q_s^R, i = 1,2,\ldots,N$$

The remanufacture-up-to levels to the unconstrained optimization problem (9) are characterized in Proposition 3, which are also newsvendor-type solutions.

Proposition 3. *The remanufacture-up-to level from grade* 1 *to grade i core,* R_i^*, *is*

$$R_i^* = F^{-1}\left(1 - \frac{c_i + h_r - h_u}{p + b_r + h_r}\right), i = 1,2,\ldots,N \quad (10)$$

where $R_1^* > R_2^* > \ldots > R_N^*$.

Proof. If only cores of grade 1 to grade i ($i = 1,2,\ldots,N$) are remanufactured, then the remanufacturer's expected profit (9) can be rewritten as

$$\pi_s^R = E[pmin(R_i, D) - \sum_{l=1}^{i} c_l(R_l - R_{l-1}) - b_r(D - R_i)^+ - h_r(R_i - D)^+ - h_u(Q_s^R - R_i)] - (c_r + c_g)Q_s^R$$

Taking the first and second derivative of π_s^R with respect to R_i respectively, we have

$$\frac{\partial \pi_s^R}{\partial R_i} = (p + b_r + h_u - c_i) - (p + b_r + h_r)F(R_i)$$

and

$$\frac{\partial^2 \pi_s^R}{\partial {R_i}^2} = -(p + b_r + h_r)f(R_i) < 0$$

Thus, π_s^R is concave in R_i and the optimal remanufacture-up-to level is given by the first-order conditions. Moreover, since $c_1 < c_2 < \ldots < c_N$, it is easy to verify that $R_1^* > R_2^* > \ldots > R_N^*$.□

Let y_i^* be the optimal aggregate remanufactured quantity from grade 1 to grade i cores after remanufacturing, then the optimal remanufacturing quantity of cores of grade i is $q_i^{R^*} = y_i^* - y_{i-1}^*$, where $y_0^* = 0$. Based on the remanufacture-up-to levels derived in Proposition 3, the optimal remanufacturing policy under the sorting strategy with stochastic demand is characterized in Theorem 2.

Theorem 2. *After core sorting, the fractions* $\theta_1, \theta_2, \ldots, \theta_N$ *are realized, then for a given core acquisition quantity* Q_s^R, *the optimal remanufacturing policy,* $\{y_1^*, \ldots, y_i^*, \ldots, y_N^*\}$, *is as follows:*

(1) if $R_1^* \le \theta_1 Q_s^R$, $y_N^* = \ldots = y_1^* = R_1^*$;
(2) for $i = 2,\ldots,N$,
if $\sum_{k=1}^{i-1} \theta_k Q_s^R < R_i^* \le \sum_{k=1}^{i} \theta_k Q_s^R$, $y_N^* = \ldots = y_i^* = R_i^*$, $y_l^* = \sum_{k=1}^{l} \theta_k Q_s^R$, $l = 1,2,\ldots,i-1$;
if $R_i^* \le \sum_{k=1}^{i-1} \theta_k Q_s^R < R_{i-1}^*$, $y_N^* = \ldots = y_i^* = y_{i-1}^*$, $y_l^* = \sum_{k=1}^{l} \theta_k Q_s^R$, $l = 1,2,\ldots,i-1$;
(3) if $Q_s^R < R_N^*$, $y_i^* = \sum_{k=1}^{i} \theta_k Q_s^R$, $i = 1,2,\ldots,N$.

Proof. According to Proposition 3, when cores of grade 1 to grade i ($i = 1,2,\ldots,N$) are remanufactured, the optimal remanufacture-up-to level is R_i^*, which is given by Eq. (10). However, for a given core acquisition quantity Q_s^R and the realized fractions $\theta_1, \theta_2, \ldots, \theta_N$, the available quantity of grade 1 to grade i cores is $\sum_{k=1}^{i} \theta_k Q_s^R$. Therefore, the optimal aggregate remanufactured quantity from grade 1 to grade i cores after remanufacturing, y_i^*, depends on the relationship between R_i^* and $\sum_{k=1}^{i} \theta_k Q_s^R$.

When $R_i^* > \sum_{k=1}^{i} \theta_k Q_s^R$, we have $R_1^* > \ldots > R_{i-1}^* > R_i^* > \sum_{k=1}^{i} \theta_k Q_s^R > \sum_{k=1}^{i-1} \theta_k Q_s^R > \ldots > \theta_1 Q_s^R$, then it is optimal to make grade 1 to grade i cores fully remanufactured. Otherwise, it is optimal to remanufacture up to R_i^*, however, if $R_i^* < \sum_{k=1}^{i-1} \theta_k Q_s^R$, then grade i cores will never be remanufactured.

Theorem 2 implies that in the optimal remanufacturing policy there exists a certain quality grade so that the acquired cores from grades that are lower than this grade are fully remanufactured, and the acquired cores from grades that are higher than this grade are disposed of, while the acquired cores of this grade are partially remanufactured.

Some properties of π_s^R are derived in Proposition 4, which can facilitate us to derive the optimal core acquisition quantity.

Proposition 4. $\pi_s^R(Q_s^R)$ *has the following properties*:
(i) $\pi_s^R(Q_s^R)$ *is piece-wise concave and continuous with respect to* Q_s^R;
(ii) $\pi_s^R(Q_s^R)$ *has a unique global optimum*.

Proof. The proof follows along the lines of proof in Brown and Lee (2003), [24]. In Eq. (8), since $\{Q_s^R : Q_s^R \geq 0\}$ is a convex set, $\{q_1^R, \ldots, q_N^R : 0 \leq q_i^R \leq \theta_i Q_s^R, i = 1,2,\ldots,N\}$ is a nonempty set, and $-\pi_s^R$ is a convex function on the nonempty convex set, then following Proposition A.4 in Porteus (2002), [25], $\min_{0 \leq q_i^R \leq \theta_i Q_s^R} -\pi_s^R(Q_s^R)$ is convex in Q_s^R, i.e., $\max_{0 \leq q_i^R \leq \theta_i Q_s^R} \pi_s^R(Q_s^R)$ is concave in Q_s^R. Hence, $\pi_s^R(Q_s^R)$ has a unique global optimum. Moreover, based on Theorem 2 it is easy to show that $\pi_s^R(Q_s^R)$ is also a piece-wise continuous function with respect to Q_s^R.□

Fig. 2 illustrates the results of Proposition 4. The parameter settings of the numerical example are: $N = 2$, $D \sim N(100,20)$, $p = 100$, $c_r = 2$, $c_g = 2$, $h_r = 2$, $b_r = 5$, $h_u = 1$, $c_1 = 6$, $c_2 = 30$, $\theta_1 \sim B(8,2)$. It can be seen from Fig. 2 that $\pi_s^R(Q_s^R)$ is divided into three pieces by $Q_s^R = R_1^*$ and $Q_s^R = R_2^*$, and each piece is a concave function.

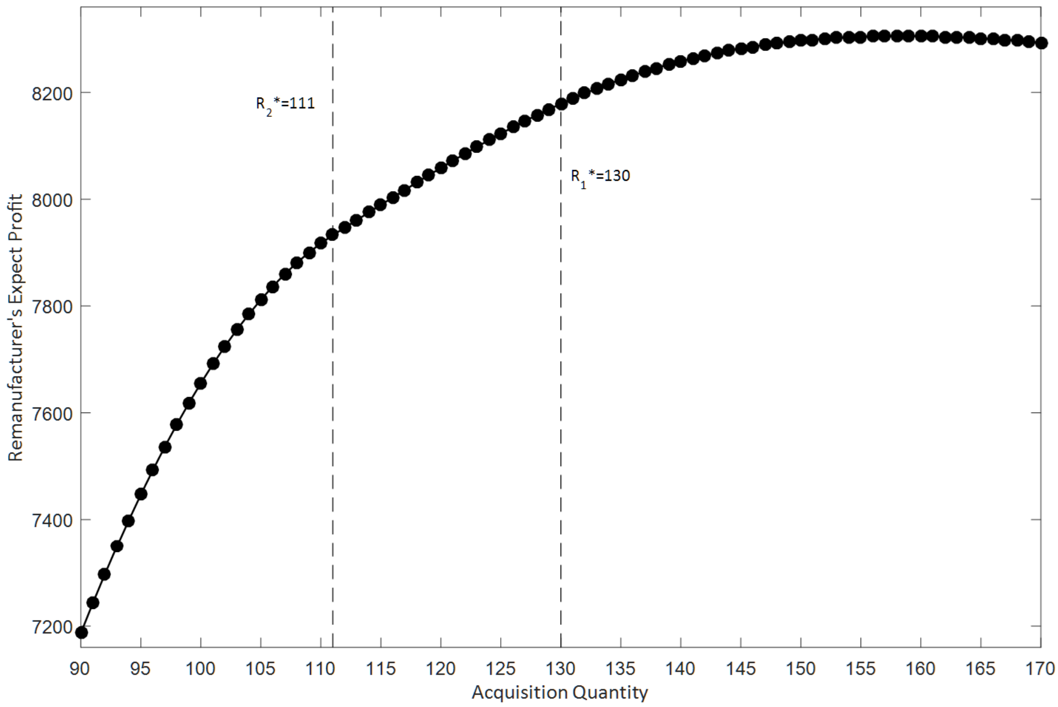

Fig. 2 Remanufacturer's expected profit with different core acquisition quantities

4.3 Comparative analysis

We also examine the value of core sorting under the stochastic demand case. By comparison, it is also found that core sorting in remanufacturing is cost-effective only when the unit sorting cost is below a threshold value and the unit core processing (i.e., acquisition, sorting and disposal) cost is sufficiently low.

Proposition 5. *There exists* c_g^{max} *such that* (i) $\pi_s^R(Q_s^{R^*}) > \pi_{ns}^R(Q_{ns}^{R^*})$, *if* $c_r + c_g + h_u < c_N - c_\mu$ *and* $c_g < c_g^{max}$; (ii) $\pi_s^R(Q_s^{R^*}) < \pi_{ns}^R(Q_{ns}^{R^*})$, *otherwise.*

Proof. It follows Theorem 2 that when $Q_s^R \leq R_N^*$, $y_i^* = \sum_{k=1}^{i} \theta_k Q_s^R$, $i = 1,2,\ldots,N$. Then, on the interval $[0, R_N^*]$ we have

$$\pi_s^R = E[pmin(Q_s^R, D) - b_r(D - Q_s^R)^+ - h_r(Q_s^R - D)^+] - (c_\mu + c_r + c_g)Q_s^R$$

Taking the first derivative of π_s^R with respect to Q_s^R, we have

$$\pi_s'^R = (p + b_r - c_r - c_g - c_\mu) - (p + b_r + h_r)F(Q_s^R)$$

It is easy to verify that $\pi_s'^R$ is decreasing in Q_s^R. Moreover, when $Q_s^R = 0, \pi_s'^R(Q_s^R = 0) > 0$; when $Q_s^R = R_N^*, \pi_s'^R(Q_s^R = R_N^*) = c_N - c_\mu - (c_r + c_g + h_u)$.

When $c_r + c_g + h_u \geq c_N - c_\mu, \pi_s'^R(Q_s^R = R_N^*) \leq 0$, thus the optimal core acquisition quantity $Q_s^{R^*}$ is on the interval $[0, R_N^*]$. This implies that all the acquired cores should be remanufactured even with the sorting strategy. As mentioned above, under this condition core sorting has no value to the remanufacturer.

When $c_r + c_g + h_u < c_N - c_\mu, \pi_s'^R(Q_s^R = R_N^*) > 0$, thus π_s^R is increasing in Q_s^R on the interval $[0, R_N^*]$ since $\pi_s'^R$ is always positive on that interval. This suggests that $Q_s^{R^*}$ is larger than R_N^*, i.e., at least the acquired cores of grade N are partially remanufactured. Note that if $c_g = 0$, then for any given quantity of cores, the sorting strategy is not inferior to the non-sorting strategy due to the potential remanufacturing cost savings. Thus, we have $\pi_s^R(Q_s^{R^*}) > \pi_s^R(Q_{ns}^{R^*}) \geq \pi_{ns}^R(Q_{ns}^{R^*})$, where the first inequality is valid due to the optimality of $Q_s^{R^*}$ for π_s^R. With the increase of c_g, $\pi_s^R(Q_s^{R^*})$ decreases but $\pi_{ns}^R(Q_{ns}^{R^*})$ remains the same. Therefore, there exists a value of c_g on the interval $(0, c_N - c_\mu - c_r - h_u)$, denoted as c_g^{max}, so that when $0 < c_g < c_g^{max}$, $\pi_s^R(Q_s^{R^*}) > \pi_{ns}^R(Q_{ns}^{R^*})$.□

Proposition 5 implies that when the sorting cost is too high, core sorting is useless to the remanufacturer. It also can be seen from Table 1 that when $c_g \leq 2$, the remanufacturer's expected profit under the sorting strategy is higher, while when $c_g \geq 2.5$, the non-sorting strategy makes the remanufacturer more profitable.

We further examine the impacts of demand uncertainty on the value of core sorting. Let Δ_g be the increase in remanufacturer's expected profit by using the sorting strategy relative to the case where the remanufacturer adopts the non-sorting strategy, where $\Delta_g = \frac{\pi_g - \pi_m}{\pi_m} \times 100\%$. Based on the same demand and cost parameter settings with that of Fig. 2, we vary the values of a and b (distribution parameters of θ_1) where $(a, b) \in \{a = 0.5i, b = 10 - 0.5i, i = 1, \ldots, 19\}$, and consider three different levels of demand variation: $\delta_D \in \{10, 20, 30\}$.

Fig. 3 illustrates the increase in remanufacturer's expected profit by adopting the sorting strategy under different levels of demand variation. The observation is similar to the deterministic demand case that the sorting strategy is valuable to remanufacturer only when the expected fraction of high-quality cores is within a suitable interval. Another observation is that when the sorting strategy is superior to the non-sorting strategy, it is more valuable under a higher level of demand variation. This is because a higher level of demand variation may increase the possibility that a high-quality core is remanufactured rather than being disposed of, which facilitates the adoption of sorting strategy.

Table 1 Remanufacturer's expected profit under sorting and non-sorting strategies at different levels of sorting cost

c_g	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
π_s^R	8633.3	8549.0	8466.4	8385.2	8305.4	8226.7	8149.1	8072.5	7996.8	7921.9	7847.9
π_{ns}^R	8238.8	8238.8	8238.8	8238.8	8238.8	8238.8	8238.8	8238.8	8238.8	8238.8	8238.8

Note: Other parameters take the same value as in the numerical example of Fig. 2.

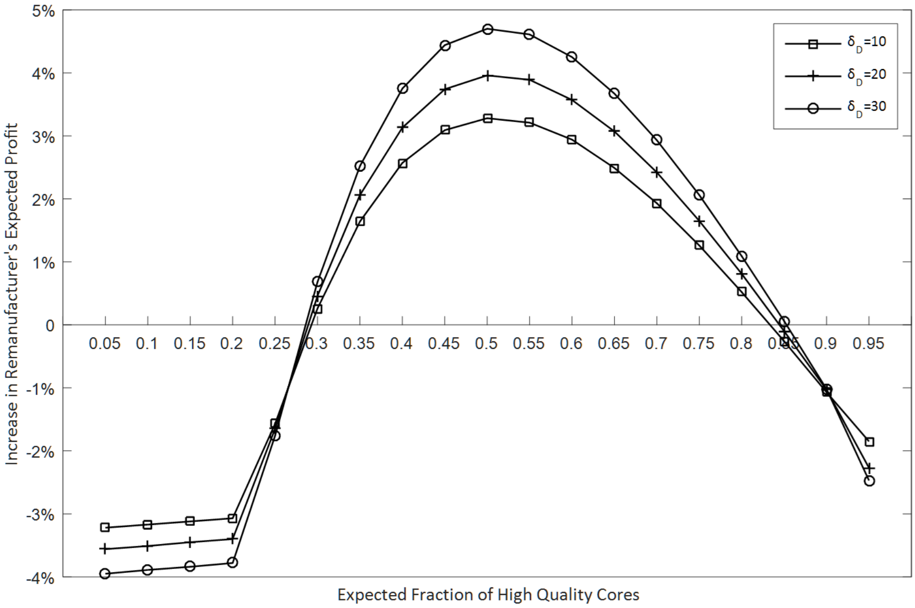

Fig. 3 Increase in Remanufacturer's expected profit under sorting and non-sorting strategies at different levels of demand variation

5. Conclusion

In this paper, we study the core acquisition and remanufacturing problem under a single-period setting in which the remanufactured products are produced from acquired cores with uncertain quality. Both the deterministic demand case and the stochastic demand case are discussed to examine the potential value of core sorting and quality grading in remanufacturing. For deterministic demand, it is found that core sorting is cost-effective only when the sorting cost is below a threshold value and the acquisition cost falls into a specific interval. Furthermore, in the case with two quality grades the sorting strategy is valuable to remanufacturer only when the expected fraction of high-quality cores is within a suitable interval. For stochastic demand, it is observed that the sorting strategy also becomes unattractive when the sorting cost is sufficiently high. In addition, the value of core sorting will be larger with more variable demand for remanufactured products if the sorting strategy is the dominant strategy. Future research may include considering the core acquisition and remanufacturing problem in a multi-period setting, and/or incorporating acquisition pricing decisions into the models.

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