Impact of agile, condition-based maintenance strategy on cost efficiency of production systems

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ABSTRACT

Maintenance plays an increasingly important role in the life of production companies, as professional maintenance is an important prerequisite for the reliable operation of resources. A well-chosen maintenance strategy can make a major contribution to increased efficiency of production processes. The main goal of this research is to propose a novel optimization approach to define optimal maintenance strategy that ensures the efficient operation of the production process while reducing maintenance costs. The developed optimization method is based on Howard’s policy iteration and describes the objective of the planning as a Markov decision process. The novelty and the scientific contribution of the presented study is the application of Howard’s policy iteration methodology in a Markov decision process for agile, condition-based maintenance strategy optimization. As the results of the numerical analysis of the scenarios shows, the implementation of an optimized maintenance strategy based on the proposed approach can significantly increase the maintenance efficiency of the production process. The main reason for this is that the level and type of maintenance is always implemented depending on the current state of the system components, which reduces both the maintenance cost and the losses due to production downtime.

ARTICLE INFO

Keywords:
Agile maintenance strategy;
Productivity;
Process control;
Markov decision process;
Maintenance strategy;
Optimization;
Smart manufacturing

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Article history:
Received 9 May 2023
Revised 17 October 2023
Accepted 13 November 2023

1. Introduction

The global maintenance market is expected to grow from 42.66 billion USD in 2022 to 72.46 billion USD by 2029, and this grow means 7.9% Compound Annual Growth Rate [1]. This fact shows the importance of maintenance in manufacturing. Across industry, a wide range of maintenance strategies can be used to support the availability of technological and logistics resources. These strategies can be classified in many ways. The maintenance strategies can be classified as preventive or corrective types. Preventive maintenance strategies are based on the idea, that maintenance operations are performed before failure occurs, while in the case of corrective maintenance, the maintenance operations are performed after the failure has occurred. However, Telek concluded in maintenance logistics research [2], that the maintenance appears as an independent service element of the production process, but in my opinion, maintenance strategy and maintenance operations must integrated into the whole business process, including purchasing, production, distribution and reverse processes. Agility can be a very important benefit of a well-chosen maintenance strategy, as it allows to react to detected failures in a timely manner through a well-
chosen maintenance operation. A maintenance strategy can be considered well-chosen if it ensures efficient operation of the machines in a cost-effective way, so agile maintenance makes it possible to respond to changes in the condition level of the manufacturing plant.

A significant link between smart manufacturing and intelligent maintenance has been created by the fourth industrial revolution, which transformed conventional manufacturing systems into cyber-physical systems, creating real-time decision algorithms that can greatly increase the utilisation of production and logistics capacity in manufacturing systems, increasing flexibility and availability, while also greatly improving process sustainability. The connection between the smart manufacturing paradigm and the intelligent maintenance is based on digital twin technologies, which makes it possible to forecast future status of physical systems and make real-time decision regarding the maintenance strategy [3, 4].

As the literature review section shows, the existing research works are focusing on a wide range of optimization problems regarding maintenance, but only a few of them discuss the agile, condition-based maintenance. Based on this fact, the scope of this work is to propose a novel optimization approach to find a cost-efficient strategy for an agile, condition-based maintenance.

This paper is organised as follows. Section 2 presents a literature review focusing on the topic of maintenance policy optimization. Section 3 proposes a novel mathematical model, which makes it possible to define the cost-efficient strategy for agile, corrective maintenance. The section describes the transformation of the conventional P-F curve into a discrete P-F curve, which makes it possible to discretize the lead time from the possible detection point to the functional failure. The model can be described as a Markov decision process. Section 4 discusses the results of the numerical analysis of a scenario, which validates the mathematical model and the optimisation algorithm. Conclusions, future research directions and managerial impacts are discussed in Section 5.

2. Literature review

Within the frame of this section, I summarise the main results of maintenance strategy optimization related research results. I focus on the state-of-the-art technologies and give an overview of the most recent achievements in the field of maintenance strategy optimisation, in order to identify the bottlenecks that can be used to validate the research of agile, condition-based maintenance strategy optimization.

Li et al [3] in a multi-objective maintenance optimization concluded, that in the case of uncertain environment, it is possible, that the chosen maintenance strategy and performed maintenance operation is inappropriate, therefore integrated decision-making methodologies can be used to improve the conventional decision-making models to probabilistic uncertainty models. As Shi et al. [4] in a research work focusing on preventive maintenance strategy found, it is important to focus on the lifecycle safety and availability of the maintained system, which applies especially to the preventive maintenance strategies, where the decomposition of lifecycle failure states and lifecycle failure probability plays also an important role in the modelling of the optimised maintenance strategy. The integration of inspection and maintenance is a suitable, but challenging improvement direction of maintenance strategies. Guo and Liang [5] concluded in a study describing the optimization of maintenance strategies as predictive Markov decisions, that inspection and maintenance strategies must be flexible, because in the early phase of the lifecycle of the inspected and maintained system, predictive inspections are not needed as often as in later phase of the lifecycle of the system, which can lead to wastes of human and technological resources. The lifecycle related problems are also discussed by Hernández et al. [6], and their approach shows, that the maintenance of networked assets with progressively deteriorating condition levels can also be optimized considering the dynamics of data traffic.

Zhang et al. [7] found in a study regarding emergency maintenance, that the optimization of maintenance strategies is particularly complex when the scheduling of maintenance operations needs to be integrated with the scheduling of the operation of the system being maintained, a task that can become particularly complex for a hyper-connected complex system such as a high-speed railway lines. In this case, the rolling horizon framework is a suitable tool to perform real-time implementation of decisions and maintenance operations. Pinciroli et al. [8] discusses a same
topic, focusing on the integrated optimization of operation and maintenance of renewable energy systems. The reliability-centered maintenance (RCM) is suggested by Paoprasert et al. [9] to improve key performance indicators of a HDD production system. As the study concluded, RCM is suitable to increase availability of machines and reliability of the manufacturing system. However, Industry 4.0 focuses on technology, but a survey on enabling technologies [10] shows, that the upcoming industrial revolution will be directed to the operators, which means, that the role of human resources in maintenance systems will continue to grow, despite increasing technological support, which will lead to new challenges, in particular for the training of human resources. Case studies validated by Mappas et al. [11], that automated maintenance operations can significantly increase the efficiency of maintenance operations. Based on these studies, we can conclude, that maintenance strategy optimization is an extensively researched topic, including a wide range of models, solution algorithms and application fields.

The used models and methods include the followings: Monte Carlo method [3], stochastic simulation [4], Forward algorithm [5], Baum-Welch algorithm [5], Lagrangian relaxation [7], mixed-integer nonlinear optimization [7], artificial intelligence [10], deep reinforcement learning [8] and Failure-Mode-and-Effect-Analysis (FMEA) [12], Fuzzy-TOPSIS [13], simulated annealing [14] and other heuristics [15]. The applications and case studies includes a wide range of industries: wind farms [3], high-speed railway [7], HDD manufacturing [9], automotive [12], injection moulding [16], offshore floating systems [17] and nuclear power plant [18] and they analyse different types of maintenance solutions including preventive [4], predictive [5, 19], emergency [7], condition-based [12] and collaborative maintenance [20], and measurement of maintenance excellence from technical and financial point of view [21].

The consequences of the literature review are the followings:

• The articles that addressed the optimization of maintenance strategies are focusing on different maintenance types, but only a few of them discusses the agile, condition-based maintenance.

• A wide range of research articles discuss the optimization of maintenance strategies using the conventional P-F curve [22], but the transformation of this continuous P-F curve into a discretised P-F curve to describe transition probabilities between different condition levels of machines and plant is not discussed as a potential tool to integrate the cost efficiency of both manufacturing and maintenance. Therefore, this research topic still needs more attention and research.

• Mathematical models and solution algorithms are important tools for the optimization of maintenance strategies, which can lead to increased quality [23]. According to that, the main goal of this research is to propose a novel mathematical model and solution algorithm to support the optimization of agile condition-based maintenance.

3. Materials and methods

Developing an optimal strategy for maintenance processes can be defined as an assignment problem, where maintenance operations of different types, depth and cost are assigned to the technological and logistics resources, in order to ensure the efficient, smooth operation of the production process and improve the availability of machines and plant. The following assumptions can be used in this assignment task. We can define different conditions of the technological and logistics system, which can be monitored accurately in real-time either by a digital twin solution or by a conventional sensor-based monitoring of technological and logistics resources as a digital shadow of the real-world system:

\[ C = (c_1, c_2, \ldots, c_i, \ldots, c_\alpha) \]  

where \( c_i \) is the condition level \( i \) of the system and state \( i \) of the system and \( \alpha \) defines the potential conditional levels depending on the condition levels of technological and logistics resources. Condition monitoring makes it possible to collect information regarding the condition of the technological and logistics resources including temperature, pressure, vibration, abrasion, noise. The
condition level of the system significantly influences the availability of the machines and plant, because low condition level can lead to downtime or increased reject rate (lower product quality).

The transition between these condition levels can occur for two reasons. One is when the condition of the system decreases during continuous operation, causing the system’s condition level to decrease. The other is when the condition of the machines and the plant improves because of a maintenance or condition improvement operation. We can also define a set of potential maintenance operations (or maintenance levels) which can significantly influence the transition between two potential condition levels.

\[ M^q = (m_1^q, m_2^q, \ldots, m_j^q, \ldots, m_\beta^q) \]  

where \( m_j^q \) is the potential maintenance operation \( j \) for maintenance strategy \( q \), \( \beta \) is the upper limit of potential maintenance operations. We can define the upper limit of the maintenance operations as a dynamic parameter, because depending on the new, unknown condition levels, new maintenance operations can be defined and set up. It means, that \( \alpha = \alpha(t) \) and \( \alpha = \alpha(t) \).

The above-mentioned transition probabilities can statistically describe the probability between two predefined condition levels. For example, if a drilling machine is working properly (condition level 1) but the temperature of the drilling tool exceeds 175° C (condition level 2) than it can lead to decreased product quality (condition level 3) and it can also lead to failure in product (condition level 4) and machine (condition level 5). The transition probabilities

The transition probabilities define the basis for the selection of the optimal maintenance operation, as different maintenance operations lead to different condition levels.

\[ \forall \gamma: \sum_{i=1}^{\alpha} t_{i\gamma} = 1 \]  

where \( T = [t_{iy}] \) is the transition probability matrix defining the transition probability between condition level \( i \) and condition level \( \gamma \).

In conventional condition-based maintenance models, the P-F curve describes that as a failure starts manifesting, the machine or plant deteriorates to the point at which it can possibly be detected (point P). If the failure is not detected, it continues until a functional failure occurs (point F) [23]. Using transition probabilities of potential condition levels of machines and plant, it is possible to transform this conventional P-F curve into a discretised P-F curve, as Fig. 1 shows.

We can assign cost to both to these condition levels, reflecting machine and system availability, productivity, and product quality and to the maintenance operations.

It is important to note, that the elements of the transition matrix are highly influenced by maintenance operations. If maintenance operation \( \delta \) is performed in the case of condition level \( i \) of the system, then the transition probability from condition level \( i \) to condition level \( j \) of the system is not necessarily the same as the transition between condition level \( i \) to condition level \( j \) of the system. The reason for this is, that the maintenance operation results a condition level improvement from condition level \( i \) to a new condition level \( k \), and the probability of a transition from condition level \( i \) to condition level \( j \) is therefore depends on the transition between condition level \( k \) and condition level \( j \).

![Fig. 1 Discretized P-F curve describing transition probabilities between condition levels](image-url)
\[ p(c_j|c_i, m_\delta) \in T \land p(c_j|c_i, m_\delta) \leq t_{ij} \lor p(c_j|c_i, m_\delta) \geq t_{ij} \quad (4) \]

When planning maintenance processes, we can use different objective functions to find the optimal solution. In my previous study [24], I have shown the energy efficiency-based maintenance policy optimization. In this model, the discounted profit based on the maintenance cost and cost of lost production is the objective function. The optimization problem is a Markov decision problem; therefore, it is also an infinite horizon probabilistic dynamic programming problem, and the objective function is the optimization of the discounted profit as follows:

\[ DP^0(c_i) = pe_{c_i,m_\delta(c_i)} + \varepsilon \cdot \sum_{\delta=1}^{\delta_{\text{max}}} p(c_j|c_i, m_\delta(c_i)) DP^0(c_j) \quad (5) \]

where: \( DP^0(c_j) \) is the expected discounted profit depending on the condition level of the manufacturing system, \( pe_{c_i,m_\delta(c_i)} \) is the expected profit depending on the chosen maintenance operation \( m_\delta \) and condition level \( c_i \), \( \varepsilon \) is the discounting factor, which can significantly influence the value of the objective function, because higher discounting factor lead to higher discounted profit.

The expected profit can be defined in many ways. In this approach, the expected profit is defined depending on the following parameters: expected income resulted by MRP (Materials Requirement Planning), lost value caused by the downtime and cost of maintenance operations:

\[ H_M^0(c_i) = \max_{m_\delta \in M(c_i)} \left( pe_{c_i,m_\delta(c_i)} + \varepsilon \cdot \sum_{\delta=1}^{\delta_{\text{max}}} p(c_j|c_i, m_\delta(c_i)) DP^0(c_j) \right) \quad (6) \]

where \( H_M^0(c_i) \) is the Howard's parameter in the case of condition level \( i \) of the manufacturing system. Based on Eqs. 5 and 6 we can compare the Howard's parameter and the discounted profit value. If the Howard's parameter is equal to the discounted profit, then \( M \) is the optimal maintenance strategy. Otherwise, the maintenance operations assigned to each condition level must be changed and then both parameters must be recomputed.

4. Results and discussion

Within the frame of the scenario analysis, a U-shaped manufacturing system is analysed including 10 machines. 10 different conditions levels of the manufacturing system are defined: \( C = (c_1, \cdots, c_{10}) \), where \( c_1 \) represents the best condition level, as Table 1 shows.

<table>
<thead>
<tr>
<th>( T = {t_{ij}} )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_3 )</th>
<th>( c_4 )</th>
<th>( c_5 )</th>
<th>( c_6 )</th>
<th>( c_7 )</th>
<th>( c_8 )</th>
<th>( c_9 )</th>
<th>( c_{10} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_1 )</td>
<td>0.6</td>
<td>0.25</td>
<td>0.1</td>
<td>0.05</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>0</td>
<td>0.5</td>
<td>0.3</td>
<td>0.1</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( c_3 )</td>
<td>0</td>
<td>0</td>
<td>0.82</td>
<td>0.11</td>
<td>0.05</td>
<td>0.02</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( c_4 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
<td>0.3</td>
<td>0.15</td>
<td>0.07</td>
<td>0.05</td>
<td>0.03</td>
<td>0</td>
</tr>
<tr>
<td>( c_5 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.7</td>
<td>0.2</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( c_6 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>0.4</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( c_7 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.7</td>
<td>0.2</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>( c_8 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.6</td>
<td>0.4</td>
<td>0</td>
</tr>
<tr>
<td>( c_9 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.8</td>
<td>0.2</td>
</tr>
<tr>
<td>( c_{10} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

There are 10 potential maintenance operations in this scenario, which can be performed depending on the current condition level of the manufacturing system. The OMS (online monitoring system) makes it possible to collect data regarding condition level of the machines and plant and defines the expected transition possibilities between condition.

Maintenance operations are assigned to each condition level of the manufacturing system. The probability that the production system will move from one condition level to another can be determined by computing the condition level resulted from the maintenance operation and after that we can calculate the transition probability. As an example, in the case of transition probability \( t_{35} \), we can calculate the potential values as:

\[ p(c_5|c_3, m_0) = p(c_5|c_4, m_3) = \cdots = p(c_9|c_0, m_6) = p(c_9|c_{10}, m_7) = t_{35} = 0.05 \quad (7) \]

and this calculation of transition probabilities can be generalized as follows:
\[ p \forall \rho - \sigma = \gamma \land t_{i\gamma} > 0 \cdot p(c_{i\gamma} | c_{\rho}, m_{\sigma}) = t_{i\gamma} \]  \hspace{1cm} (8)

Let the expected income resulted by the material requirement planning be \( c(MRP) = 30000 \) €. The lost value of the manufacturing process, depending on the condition level of the manufacturing system can be also defined as follows as shown in Table 2:

\[
l_{v_{c_{i}}} = ilv^{\varphi_{c_{i}}} \]  \hspace{1cm} (9)

where \( ilv \) is the initial lost value, which is in this scenario 12000 €, \( \varphi \) is the specific parameter influencing the lost value depending on the condition level of the manufacturing system.

We can define in the same way the maintenance cost depending on the condition level of the manufacturing system and the performed maintenance operation as follows:

\[
mc_{c_{i}, m_{\delta}(c_{i})} = imc^{\omega \delta} \]  \hspace{1cm} (10)

where \( imc \) is the initial maintenance cost, which is in this scenario 2500 €, \( \omega \) is the specific parameter influencing the maintenance cost depending on the condition level of the manufacturing system and the performed maintenance operation. The computed values of the scenario analysis are shown in Table 3.

As Table 3 shows, the maintenance operation have increased cost depending of the complexity of them, because complex maintenance operations can lead to a more significant condition level improvement of the machines and plant.

Based on Eqs. 6, 9, and 10 we can compute the expected profit of the scenario depending on the current condition level of the machines and plant and the assigned maintenance operation, as shown in Table 4. Let define the initial maintenance strategy by the assignment of maintenance operations to condition levels as given:

\[
A^{0} = [a_{0}^{0}] = [m_{0}, m_{1}, m_{2}, m_{3}, m_{4}, m_{5}, m_{6}, m_{7}, m_{8}, m_{9}] \]  \hspace{1cm} (11)

where \( a_{0}^{0} = m_{\pi} \), and is the maintenance operation \( \pi \) is assigned to condition level \( c_{i} \) in the initial phase of the optimization.

<table>
<thead>
<tr>
<th>( c_{i} )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ilv_{c_{i}} )</td>
<td>18000</td>
<td>17185</td>
<td>16307</td>
<td>15363</td>
<td>14347</td>
<td>13252</td>
<td>12072</td>
<td>10800</td>
<td>9427</td>
<td>7946</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( m_{1} )</th>
<th>2500</th>
<th>4734</th>
<th>6641</th>
<th>9443</th>
<th>13618</th>
<th>16307</th>
<th>19929</th>
<th>29611</th>
<th>68601</th>
</tr>
</thead>
</table>

| \( mc_{c_{i}, m_{\delta}(c_{i})} \) | 50601 | 13304 | 729 | 729 | 729 | 729 | 729 | 729 | 729 |

| \( p_{e}^{c_{i}, m_{\delta}(c_{i})} \) | 50601 | 13304 | 729 | 729 | 729 | 729 | 729 | 729 | 729 |

Once the input parameters for the scenario have been defined, the discounted profit of the initial maintenance strategy can be computed based on Eq. 5 solving the following value definition equations:

\[
\text{Table 2 Lost value of the manufacturing system depending on the condition level in (€)}
\]

\[
\text{Table 3 Maintenance cost depending on the condition level and maintenance operation in (€)}
\]

\[
\text{Table 4 Expected profit of the scenario in (€)}
\]
\[ DP^0(c_1) = pe_{c_1,m_0} + \varepsilon \cdot \sum_{\theta=1}^{4} t_{1\theta} \cdot DP^0(c_{\theta}) \]  
\[ DP^0(c_2) = pe_{c_2,m_1} + \varepsilon \cdot \sum_{\theta=1}^{4} t_{1\theta} \cdot DP^0(c_{\theta}) \]  
\[ DP^0(c_3) = pe_{c_3,m_1} + \varepsilon \cdot \sum_{\theta=2}^{5} t_{2\theta} \cdot DP^0(c_{\theta}) \]  
\[ DP^0(c_4) = pe_{c_4,m_2} + \varepsilon \cdot \sum_{\theta=2}^{5} t_{2\theta} \cdot DP^0(c_{\theta}) \]  
\[ DP^0(c_5) = pe_{c_5,m_1} + \varepsilon \cdot \sum_{\theta=4}^{5} t_{4\theta} \cdot DP^0(c_{\theta}) \]  
\[ DP^0(c_6) = pe_{c_6,m_3} + \varepsilon \cdot \sum_{\theta=3}^{5} t_{3\theta} \cdot DP^0(c_{\theta}) \]  
\[ DP^0(c_7) = pe_{c_7,m_2} + \varepsilon \cdot \sum_{\theta=5}^{7} t_{5\theta} \cdot DP^0(c_{\theta}) \]  
\[ DP^0(c_8) = pe_{c_8,m_3} + \varepsilon \cdot \sum_{\theta=5}^{7} t_{5\theta} \cdot DP^0(c_{\theta}) \]  
\[ DP^0(c_9) = pe_{c_9,m_4} + \varepsilon \cdot \sum_{\theta=5}^{7} t_{5\theta} \cdot DP^0(c_{\theta}) \]  
\[ DP^0(c_{10}) = pe_{c_{10},m_2} + \varepsilon \cdot \sum_{\theta=8}^{9} t_{8\theta} \cdot DP^0(c_{\theta}) \]

The solution of the above-mentioned value definition equations resulted the discounted profit for the initial maintenance strategy describing the assignment of maintenance strategies to condition levels of the machines and plant shown in Table 5.

As Table 5 shows, the condition level of the machines and plant has a significant impact on the discounted value, because lower condition levels lead to lower discounted value.

\[ \textbf{Table 5} \text{ Discounted profit for the initial maintenance strategy in (€)} \]

| \( DP^0(c_1) \) | \( c_1 \) | \( c_2 \) | \( c_3 \) | \( c_4 \) | \( c_5 \) | \( c_6 \) | \( c_7 \) | \( c_8 \) | \( c_9 \) | \( c_{10} \) |
|----------------|
| 324528 | 322028 | 316994 | 314760 | 306318 | 309936 | 300776 | 293474 | 296067 | 285852 |

The next phase of the optimization is to check the validity of the initial maintenance strategy. Based on Eq. 6, it is possible to calculate the Howard’s parameter for each condition level of the machines and plant, and then modify the initial maintenance strategy based on the maximum value of the Howard’s parameter. In the case of \( c_1 \) condition level, the Howard’s parameter is the same as the discounted value, therefore no maintenance strategy modification is required in the second iteration phase.

\[ H^0_M(c_1) = DP^0(c_1) \rightarrow a^1_{c_1} = a^0_{c_1} = m_0 \]  

In the case of \( c_2 \) condition level, we can calculate the Howard’s parameter based on Eq. 6, as follows:

\[ H^0_M(c_2) = \max \begin{cases} m_0 \rightarrow pe_{c_2,m_0} + \varepsilon \cdot \sum_{\theta=2}^{5} t_{2\theta} \cdot DP^0(c_{\theta}) \\ m_1 \rightarrow pe_{c_2,m_1} + \varepsilon \cdot \sum_{\theta=1}^{4} t_{1\theta} \cdot DP^0(c_{\theta}) \end{cases} \]

The comparison of the Howard’s parameter and the discounted value shows, that no maintenance strategy change is required in the case of condition level \( c_2 \).

\[ H^0_M(c_2) = DP^0(c_2) \rightarrow a^2_{c_2} = a^0_{c_2} = m_1 \]

In the case of \( c_3 \) condition level, we can calculate the Howard’s parameter in the same way:

\[ H^0_M(c_3) = \max \begin{cases} m_0 \rightarrow pe_{c_3,m_0} + \varepsilon \cdot \sum_{\theta=3}^{5} t_{3\theta} \cdot DP^0(c_{\theta}) \\ m_1 \rightarrow pe_{c_3,m_1} + \varepsilon \cdot \sum_{\theta=2}^{5} t_{2\theta} \cdot DP^0(c_{\theta}) \\ m_2 \rightarrow pe_{c_3,m_2} + \varepsilon \cdot \sum_{\theta=1}^{4} t_{1\theta} \cdot DP^0(c_{\theta}) \end{cases} \]

The comparison of the Howard's parameter and the discounted value shows, that we can change maintenance operation \( m_1 \) to maintenance operation \( m_2 \) assigned to condition level \( c_3 \).

\[ H^0_M(c_3) > DP^0(c_3) \rightarrow a^1_{c_3} = a^0_{c_3} \rightarrow a^1_{c_3} = m_2 \]

This maintenance operation change resulted a 2801 € additional discounted profit in the case of condition level \( c_3 \). We can calculate the new maintenance operations assigned to each condition level leading to increased discounted profit in the same way. As Table 6 shows, the iterative method...
odology after the first iteration phase lead to the change of 8 assignment of maintenance operations to condition levels, and the value of the total additional discounted value can be calculated as follows:

\[
\forall z > 0: TADP^z = \sum_{i=1}^{a_z} ADP^z(c_i) = \sum_{i=1}^{a_z} D P^z(c_i) - D P^z-0(c_i) = 39446 \, \€
\]  

(27)

where \( TADP^z \) is the total additional discounted value after iteration phase \( z \), \( ADP^z(c_i) \) is the additional discounted value after iteration phase \( z \) in the case of condition level \( i \).

The above-described iterative calculation process must be continued as long as it is possible to increase the total discounted value by changing the maintenance strategy. The final result of the maintenance strategy optimisation is shown in Table 7.

**Table 6** The increased discounted value per condition level in (€) and the assignment of maintenance strategies and condition levels after the first iteration

<table>
<thead>
<tr>
<th>Condition Level</th>
<th>( a_1 )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_3 )</th>
<th>( c_4 )</th>
<th>( c_5 )</th>
<th>( c_6 )</th>
<th>( c_7 )</th>
<th>( c_8 )</th>
<th>( c_9 )</th>
<th>( c_{10} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assignment</td>
<td>( a_1 )</td>
<td>( c_1 )</td>
<td>( c_2 )</td>
<td>( c_3 )</td>
<td>( c_4 )</td>
<td>( c_5 )</td>
<td>( c_6 )</td>
<td>( c_7 )</td>
<td>( c_8 )</td>
<td>( c_9 )</td>
<td>( c_{10} )</td>
</tr>
<tr>
<td>( ADP^1(c_i) )</td>
<td>0</td>
<td>0</td>
<td>2801</td>
<td>3128</td>
<td>8768</td>
<td>975</td>
<td>6358</td>
<td>9484</td>
<td>581</td>
<td>7352</td>
<td></td>
</tr>
</tbody>
</table>

**Table 7** The increased discounted value per condition level in (€) and the assignment of maintenance strategies and condition levels after the first iteration

<table>
<thead>
<tr>
<th>Condition Level</th>
<th>( a_1 )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_3 )</th>
<th>( c_4 )</th>
<th>( c_5 )</th>
<th>( c_6 )</th>
<th>( c_7 )</th>
<th>( c_8 )</th>
<th>( c_9 )</th>
<th>( c_{10} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assignment</td>
<td>( a_1 )</td>
<td>( c_1 )</td>
<td>( c_2 )</td>
<td>( c_3 )</td>
<td>( c_4 )</td>
<td>( c_5 )</td>
<td>( c_6 )</td>
<td>( c_7 )</td>
<td>( c_8 )</td>
<td>( c_9 )</td>
<td>( c_{10} )</td>
</tr>
<tr>
<td>( ADP^2(c_i) )</td>
<td>8294</td>
<td>8294</td>
<td>11095</td>
<td>11421</td>
<td>17061</td>
<td>10823</td>
<td>17180</td>
<td>20307</td>
<td>14258</td>
<td>20298</td>
<td></td>
</tr>
</tbody>
</table>

Based on the above discussed methodology, it can be concluded that the optimization of the maintenance strategy and the modification of the assignment of maintenance operations to machines can significantly contribute to the increase of the efficiency of the production system, since an overall increase of 130030 € in discounted value was achieved.

For the optimization method presented above, it is important to perform a sensitivity analysis of the objective function for some parameters. By analysing the impact of the maintenance cost and the discount rate on the discounted value, it can be concluded that an increase in the maintenance cost decreases the discounted value obtained by the maintenance strategy. This finding seems trivial, but the impact of the maintenance cost on the discounted value is not trivial, since a given increase in the maintenance cost does not change the discounted value resulting from the strategy to the same extent, since the productivity resulting from the condition level of the machines can be modelled as a Markov process with transition probabilities, as illustrated in Fig. 2.

**Fig. 2** Impact of maintenance cost and discount rate on discounted value of agile maintenance strategy

Fig. 2 also shows how a decrease in the discount rate has a decreasing effect on the discounted value, a relationship that can also be explained by the transition probabilities between the condition levels of the machines. Based on this line of thinking, it can be seen that the maintenance costs
influenced by the maintenance operations, the revenues associated with the condition levels of machines and plant and other system parameters have a significant impact on the discounted value that can be achieved by implementing an agile maintenance strategy.

5. Conclusion

Optimising maintenance processes is an increasingly important goal of production companies. This is because, in order to meet the dynamically changing customer demands in a cost-effective way, the machines in the production system must be available at all times in a condition to produce a product of the right quality. In this paper, the author presents a maintenance strategy optimization methodology that is suitable for modelling the transitions between the condition levels of the manufacturing system as Markov chains and is suitable for efficient application of Markov decision process to select the optimal maintenance strategy. The presented iteration-based methodology is suitable for determining the optimal maintenance strategy. The essence of this methodology is that for each condition level the optimal maintenance operation can be determined, as a result of which the discounted value associated with that condition level can be increased, i.e. the lost value caused by downtime or rejected products can be reduced.

A new methodology has been developed in this research work and its applicability has been validated by the numerical analysis of a case study. After validation of the methodology, the following conclusions can be drawn:

- Optimisation of the maintenance strategy can significantly improve the efficiency of production systems, thereby enhancing product quality. The discounted value as a metric can be well used to measure this improvement.
- The presented iterative method is not only suitable for the optimization of small-scale tasks but is also well suited for multi-machine manufacturing systems, since the presented methodology does not include complex computational procedures that would be computationally expensive, i.e. the presented optimization task does not belong to the NP-hard optimization problems.
- The maintenance strategy is optimized in iterative steps. In each iteration phase, the discounted value is gradually increased.

The presented new methodology is important not only for the academy, but its practical applicability can also be significant since it can greatly contribute to the enhancement of the competitiveness of production companies by increasing the efficiency and availability of the manufacturing systems.

The research presented also has implications for managerial decisions, as the optimisation of the maintenance strategy can significantly influence the optimization of technological, logistics and human resources, and decisions can be taken on the outsourcing of certain processes (for example, the decision to outsource maintenance processes can be justified). The application of machine learning solutions can be defined as a potential future research direction. Cooperation and networking make it possible to improve the efficiency of maintenance operations [27], therefore the second potential future research direction is the development of novel optimization methods for networking companies.

References
