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Comparing Fault Tree Analysis methods combined with Generalized Grey Relation Analysis: A new approach and case study in the automotive industry

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A B S T R A C T

The failure modes of products gradually show a diversified trend with the precision and complexity of the product structure. The combination of fault tree analysis and generalized grey relational analysis is widely used in the fault diagnosis of complex systems. In this study, we utilize a method that combines fault tree analysis and generalized grey relational analysis. This method is applied to diagnose the *Expansion Adhesive Debonding* fault of automobile doors. Then, we analyse and compare the differences in actual fault diagnosis results. The comparison involves three analysis methods: Fault Tree Analysis combined with Absolute Grey Relation Analysis (F-AGRA), Fault Tree Analysis combined with Relative Grey Relation Analysis (F-RGRA), and Fault Tree Analysis combined with Comprehensive Grey Relation Analysis (F-CGRA). Subsequently, we compare the findings with actual production results. This comparison allows us to discuss the differences between the three methods in the fault diagnosis of complex systems. We also discuss the application occasions of these methods. This study will provide a new method for fault analysis and fault diagnosis in the actual production of the automobile manufacturing industry. This method can eliminate faults effectively and accurately and improve product quality and productivity.

A R T I C L E I N F O

Keywords:
Fault tree analysis (FTA);
Generalized Grey Relation Analysis (GGRA);
Failure mode;
Fault diagnosis;
Complex system;
Fault Tree Analysis combined with Absolute Grey Relation Analysis (F-AGRA);
Fault Tree Analysis combined with Relative Grey Relation Analysis (F-RGRA);
Fault Tree Analysis combined with Comprehensive Grey Relation Analysis (C-GRA)

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1. Introduction

The diversified needs of customers and the increasingly improved automation and intelligent production have brought about product function diversity and fast-paced production. However, the diagnosis and analysis of the complicated and diversified failure modes have led to new challenges. The products in the automobile manufacturing industry exhibit more complex failure modes, more diverse fault causes, and a stronger correlation between fault causes than those in other industries. The simple fault analysis method cannot solve the complex fault problem.

Therefore, a fault analysis method for complex system analysis that can accurately, quickly, and efficiently determine the cause of product failure and take corresponding troubleshooting measures is urgently needed.

Complex systems are generally composed of various interacting components with complex relationships. This scenario also leads to system failures that typically involve multiple levels of organization and interrelated factors. As the system's complexity increases, the interactions between components and the complexity of the hierarchical structure and interrelationships between components increase. Thus, the complexity of analysis increases with the increase in system complexity.

Fault Tree Analysis (FTA) is currently recognized as an effective technique for determining and analysing failure modes of complex products or systems [1]. This kind of inverted-tree logical cause-and-effect diagram is established for studying the system's functional failure [2]. This analysis method has been widely used in the mechanical engineering [3], medicine [4], chemistry [5], transportation [6], construction [7], aviation [8] and others for qualitative and quantitative fault diagnoses. It plays an important role in system reliability analysis, safety analysis, and risk assessment [9]. However, the correlation between the basic and top events is ignored in the practical application of FTA. The components' potential and relative failure events in complex systems often exist simultaneously. Thus, the system failure and the resulting failure are usually uncertain [10]. Moreover, the correlation information between them is lacking. Thus, inaccurate results easily occur when FTA is used to analyse complex systems. Therefore, the correlation between fault events and system failures must be analysed by other effective methods combined with FTA. Grey relational analysis (GRA) is one of the most widely used methods.

Based on the sample data of various factors, GRA uses grey relational degrees to describe the strength, size, and order of the relationships between factors [11-12]. The basic idea is that if the changing trends of the two factors are consistent, they can be considered relatively related. Otherwise, the correlation between them is small. Therefore, according to the basic idea of GRA, it can be applied to comprehensive evaluation. The evaluated object and the ideal object are regarded as two systems, and the advantages and disadvantages of the evaluated object can be obtained by calculating the correlation degree of the two systems. The greater the correlation degree of the two systems is, the better the evaluated object is. Alternatively, the evaluated object and the negative ideal object can be regarded as two systems. The greater the correlation between the two systems is, the more similar the failure mode of the evaluated object is to the negative ideal solution [13].

The combination of FTA and GRA has been extensively researched in recent years. Pang calculated the relationship between basic event and top event failure rate trend in the electromagnetic machining process by combining T-S FTA and GRA. In this way, they realized the targeted repair of the fault system in the electromagnetic machining process and achieved the purpose of quality control [14]. Tien optimized the process parameters by combining fuzzy GRA and particle swarm optimization based on the Taguchi experiment, considerably improving the performance indicators of materials [15]. Applying this machine learning method to fault diagnosis enables effective handling of large-scale, high-dimensional data and the discovery of fault features and trends hidden behind extensive datasets. This method is crucial for fault diagnosis in complex systems. It can analyse and learn from data in real time. Thus, it has the potential for real-time fault diagnosis. It can also provide objective, data-driven analyses, thereby reducing subjective interventions. Wang *et al*. introduced grey theory and fuzzy theory into the FTA to analyse the wind turbine. They proposed the gearbox transmission system based on grey theory and T-S fuzzy FTA to solve uncertain failure probability [16]. Wang introduced the method of combining GRA and FTA in the analysis and diagnosis of the loosening accident in a mine hoist. He analysed the possibility of various fault modes in the FTA of excessive looseness in a mine hoist [17]. Zhang *et al*. proposed a failure mode analysis method to improve the generalized grey relational fault tree for mine car fall accidents. They used this method to determine and rank the possibility of occurrence of various mine car fall failure modes [18]. Chen *et al*. proposed the LSSC reliability diagnosis method for hazardous materials by improving the generalized grey relation fault tree. This method can quickly identify the accident factors of the key chemical logistics operation system [19].

The works above show that FTA is mainly used for deterministic fault analysis, whereas GRA is primarily used for uncertain fault analysis. Deterministic methods have low data requirements and may be fast because they focus on deterministic faults. On the contrary, uncertain fault analysis methods are comprehensive and can handle the interrelationships among multiple faults in a system. They usually have high data requirements to capture the relationships and uncertainties. This comprehensiveness can enhance problem identification accuracy but may require increased computational resources and time.

Therefore, combining FTA and GRA methods can leverage the advantages of deterministic and nondeterministic methods for fault analysis and diagnosis. This approach allows for the rapid and effective analysis of the interrelationships among multiple fault causes in system failures. Therefore, applying the combination of FTA and GRA to solve product failures encountered in industrial product manufacturing has a great generalization value. Chen *et al.* determined the weak link in the LSSC system for a hazardous chemical by combining generalized GRA with a fault tree [19]. This method can be extended to product failure analysis in industrial product manufacturing.

At present, the analysis methods combining generalized GRA with fault trees commonly include FTA combined with Absolute Grey Relation Analysis (called F-AGRA), FTA combined with Relative Grey Relation Analysis (called F-RGRA), and FTA combined with Comprehensive Grey Relation Analysis (called F-CGRA). These three methods have their respective advantages, disadvantages, and applications. However, the current study has no special research and focused discussion on the applicability and accuracy of the three methods in the failure mode of complex products. As a result, the failure analysis using the combination of generalized GRA and FTA cannot quickly and accurately choose among the three methods. Thus, the production efficiency is affected.

The present study established a fault tree by taking the typical automobile doors' *Expansion Adhesive Debonding* fault encountered in the production of G Group as the research object to make up for the shortcomings of the studies above. The automobile doors' *Expansion Adhesive Debonding* fault was also analysed based on F-AGRA, F-RGRA, and F-CGRA methods. We followed the actual problem-solving process in the production site to understand the causes and final solutions of the failure in the actual production. In this study, the theoretical analysis of the three methods was compared with that of the troubleshooting methods in the actual production. We studied and analysed the reasons for the differences between the three methods to choose the suitable fault analysis method. This study can provide an efficient and accurate idea of analysing the causes of faults for enterprise fault improvement to solve system faults quickly.

2. Expansion Adhesive Debonding fault analysis

In the disassembly and self-inspection processes for the exit door assembly of a certain model of G Group, the expansion adhesive filled between the inner and outer plates of the automobile doors did not fit the surface of the sheet metal parts. Moreover, the expansion adhesive section was neat. No attachment was found between the door plate and the door plate, and a large gap exists. According to the "Specification for Complete Destruction of Body in White Connection Quality" of the group, the expansion adhesive should have adhesion traces on all the panels on both sides of the split. Otherwise, the expansion adhesive may have insufficient filling, leading to noise caused by the vibration during driving. The noise becomes increasingly obvious with the increase in speed, seriously affecting user comfort. Therefore, the *Expansion Adhesive Debonding* fault of the automobile door is defined as a system fault, and the F-AGRA, F-RGRA, and F-CGRA methods are adopted for analysis. The results are compared with the actual practice to analyse the reasons for the differences and applicable occasions of the three methods.

2.1 Fault analysis process based on Fault Tree Analysis and Generalized Grey Relation Analysis

The Fault Tree and Grey Relation Analysis are used to analyse the problem. The specific process is as Fig. 1.

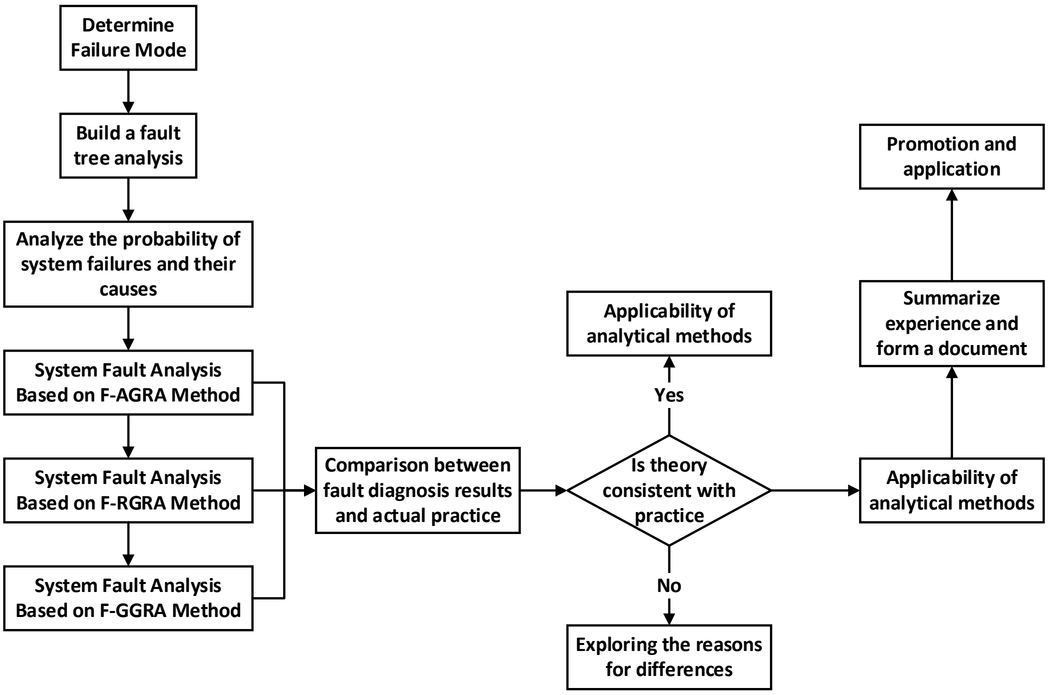

Fig. 1 Fault analysis process based on Fault Tree Analysis and Generalized Grey Relation Analysis

2.2 Constructing the Fault Tree Analysis construction

FreeFta software is used to construct the fault tree of automobile doors' *Expansion Adhesive Debonding*, as shown in Fig. 2. T represents the fault tree top event, Mi represents the intermediate event, and Ni represents the basic event. In this case, the *Expansion Adhesive Debonding* fault of the automobile door is defined as the top event, which is a system fault. The settings of top events, intermediate events, and basic events are shown in Table 1.

2.3 Failure probability assessment

G Group established a fault analysis team comprising field technicians and engineers to solve the *Expansion Adhesive Debonding* fault of automobile doors by evaluating the possible risk probability of the basic events of the fault tree. Moreover, Table 2 is the standard table of the problem level and probability assessment found in the production process investigation, and Table 3 is the probability assessment table of the basic event N_1-N_{15} of the *Expansion Adhesive Debonding* fault of automobile doors.

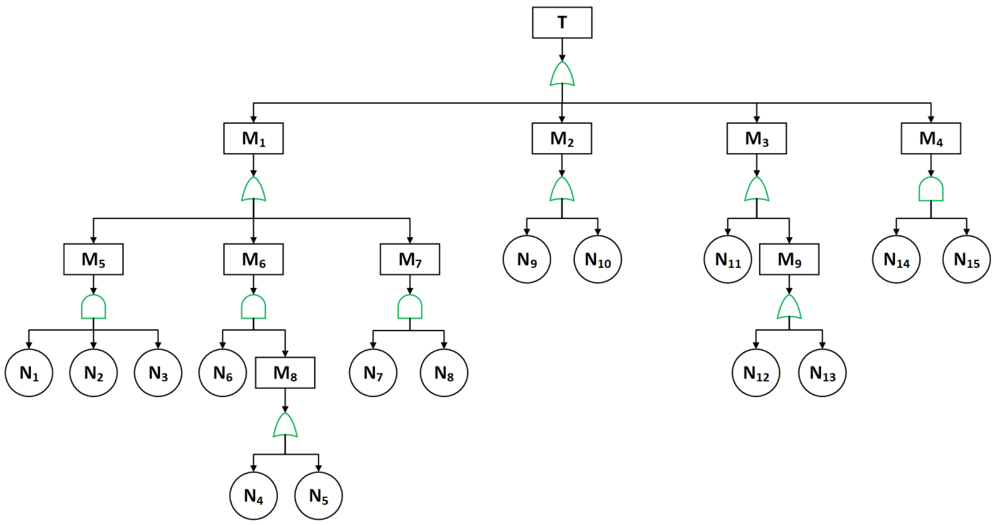

Fig. 2 Fault Tree Analysis of *Expansion Adhesive Debonding* of automobile doors

Table 1 Events represented by letters in Fault Tree Analysis

Event Type	Event Code	Fault event
Top event	T	Automobile doors *Expansion Adhesive Debonding*
Intermediate events	M_1	Excessive spacing between sheet metal parts
	M_2	Gluing equipment error
	M_3	The properties of the expansion adhesive do not meet the requirements
	M_4	Improper transportation and storage methods for assembly parts
	M_5	Part size deviation exceeding standard
	M_6	Excessive deformation of parts
	M_7	Part position deviation exceeds the standard
	M_8	Part deformation caused by the process of transferring parts
	M_9	The properties of the expansion adhesive are not sufficient to meet the needs of sea transportation
Elementary event	N_1	Dimensional deviation of outer door panel
	N_2	Dimensional deviation of door inner panel
	N_3	Dimensional deviation of reinforced beams
	N_4	Deformation of the reinforced beam caused by taking it
	N_5	Deformation of parts caused by handling after welding completion
	N_6	Overpressure of fixture pressure arm
	N_7	Poor consistency of solder joints leads to fluctuations in part position
	N_8	Excessive wear of fixture positioning pins
	N_9	Deviation of gluing point position
	N_{10}	Insufficient glue application
	N_{11}	Insufficient viscosity of expansion adhesive
	N_{12}	Deterioration of expanded adhesive due to prolonged exposure time
	N_{13}	Expansion adhesive deteriorates under high temperature and humidity conditions
	N_{14}	The coated expansion adhesive is shipped by sea to M country for baking
	N_{15}	Improper storage method for expansion adhesive during transportation

Table 2 Standard table of problem level and probability assessment found in production process investigation

Level	Probability of failure occurrence	
	Possibility of occurrence	Probability of occurrence value
one	great	0.9
two	more	0.8
three	secondary	0.6
four	less	0.2
five	very small	0.1

Table 3 Probability assessment table of basic event N_1-N_{15} of *Expansion Adhesive Debonding* failure of automobile

Elementary event N_i	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8
Probability P (N_i)	0.05	0.05	0.2	0.05	0.1	0.4	0.3	0.2
Elementary event N_i	N_9	N_{10}	N_{11}	N_{12}	N_{13}	N_{14}	N_{15}	
Probability P (N_i)	0.2	0.3	0.05	0.7	0.7	0.5	0.4	

3 Analysis based on Fault Tree Analysis of Expansion Adhesive Debonding

3.1 Top event probability

If the basic events in the fault tree are independent of each other, then the probability of the top event can be obtained from Eqs. 1 and 2 [17].

The probability of occurrence of the output event of the and gate structure：

$$P(T) = \prod_{i=1}^{n} P_i \tag{1}$$

The probability of occurrence of the output event of the or gate structure：

$$P(T) = 1 - \prod_{i=1}^{n} (1 - P_i) \tag{2}$$

According to Eqs. 1 and 2, the probability of top event occurrence can be calculated as P (T) = 0.9750.

3.2 Minimum cut set of fault tree analysis

If a set of basic events occur in the fault tree, the top event is bound to occur. The set of basic events is called the cut set; the minimum cut set represents a minimum failure mode that causes the top event of the fault tree to occur. The study of the minimum cut set helps identify the fault tree's weaknesses [18].

This study adopts the upward method to calculate the minimum cut sets in the fault tree of the automobile doors *Expansion Adhesive Debonding*. The calculation process is shown in the Eq. 3 [19].

$$\begin{aligned} T &= M_1 + M_2 + M_3 + M_4 \\ &= M_5 + M_6 + M_7 + N_9 + N_{10} + N_{11} + M_9 + N_{14} + N_{15} \\ &= N_1N_2N_3 + N_6M_8 + N_7N_8 + N_9 + N_{10} + N_{11} + N_{12} + N_{13} + N_{14} + N_{15} \\ &= N_1N_2N_3 + N_6N_4 + N_6N_5 + N_7N_8 + N_9 + N_{10} + N_{11} + N_{12} + N_{13} + N_{14} + N_{15} \end{aligned} \tag{3}$$

It should be noted that $N_1N_2N_3$ is a simplified expression of $\{N_1, N_2, N_3\}$, representing a fault event, and the rest are the same. According to Eq. 3, the corresponding minimum cut sets labelled C_1-C_{11} are: $C_1 = \{N_1, N_2, N_3\}$, $C_2 = \{N_4, N_6\}$, $C_3 = \{N_5, N_6\}$, $C_4 = \{N_7, N_8\}$, $C_5 = \{N_9\}$, $C_6 = \{N_{10}\}$, $C_7 = \{N_{11}\}$, $C_8 = \{N_{12}\}$, $C_9 = \{N_{13}\}$, $C_{10} = \{N_{14}\}$, $C_{11} = \{N_{15}\}$.

3.3 Minimum cut sets probability

Based on the probability of each basic event, each minimum cut set probability is calculated using Eq. 4 [20].

$$P(C_i) = \prod_{j \in C_i} P(x_j) \tag{4}$$

According to the calculation of Eq. 4, the minimum cut sets probability of the automobile doors *Expansion Adhesive Debonding* fault is obtained, as shown in Table 4.

Table 4 Minimum cut sets probability of *Expansion Adhesive Debonding* fault in automobile doors

Min. cut set C_i	C_1	C_2	C_3	C_4	C_5	C_6
Probability $P(C_i)$	0.0005	0.02	0.04	0.06	0.2	0.3
Min. cut set C_i	C_7	C_8	C_9	C_{10}	C_{11}	
Probability $P(C_i)$	0.05	0.7	0.7	0.5	0.4	0.05

4. Generalized Grey Relation Analysis

4.1 Importance of basic event

The basic event importance indicates the degree of influence of the basic event on the top event and is calculated by Eq. 5 [19].

$$I_j = \frac{\sum_{j \in C_i} P(C_i)}{P(T)} \tag{5}$$

According to the calculation of Eq. 5, the importance of basic events of *Expansion Adhesive Debonding* failure of automobile doors is shown in Table 5.

Table 5 Importance of basic events of *Expansion Adhesive Debonding* failure of automobile doors

Elementary event N_i	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8
Importance I_j (N_0)	0.0005	0.0005	0.0005	0.0205	0.0410	0.0615	0.0615	0.0615
Elementary event N_i	N_9	N_{10}	N_{11}	N_{12}	N_{13}	N_{14}	N_{15}	
Importance I_j (N_0)	0.2051	0.3077	0.0513	0.7179	0.7179	0.5128	0.4103	

4.2 Fault Analysis based on F-AGRA method

The F-AGRA method can reflect each factor's degree of influence on the system. The fault tree concept indicates that this degree of influence is the longitudinal difference between the initial feature matrix of the minimum cut sets and the vector of patterns to be examined consisting of the importance of the underlying events. It ultimately reflects the fault patterns' degree of contribution represented by each minimum cut set to the top event fault.

X_i represents the basic event probability of the i-th minimum cut set, $X = \{X_1, X_2, \cdots, X_i, \cdots, X_m\}^T$, $X_i = \{x_i(1), x_i(2), \cdots, x_i(n)\}$, here m is the number of minimum cut sets, and n is the number of elementary events in the fault system. The characteristic matrix X consisting of subsequences can be expressed as the Eq. 6.

$$X = \begin{Bmatrix} X_1 \\ X_2 \\ \cdots \\ X_i \\ \cdots \\ X_m \end{Bmatrix} = \left\{ \begin{pmatrix} x_1(1) & x_1(2) & \cdots & x_1(n) \\ x_2(1) & x_2(2) & \cdots & x_2(n) \\ \cdots & \cdots & \cdots & \cdots \\ x_i(1) & x_i(2) & \cdots & x_i(n) \\ \cdots & \cdots & \cdots & \cdots \\ x_m(1) & x_m(2) & \cdots & x_m(n) \end{pmatrix} \right\} \tag{6}$$

In the characteristic matrix X, if a basic event is not in some minimum cut sets, then the vector value for that position is 0. Otherwise, the vector value for that position is the probability of this basic event. Therefore, the characteristic matrix of the basic event probability column can be expressed as the Eq. 7.

$$\begin{pmatrix}
0.05 & 0.05 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.05 & 0 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.2 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.3 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.05 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4
\end{pmatrix} \tag{7}$$

Define the $X_0 = \{x_0(1), x_0(2), \ldots, x_0(n)\}$ as the primary sequence, the vector values corresponding to $x_0(1)$, $x_0(2)$, ..., $x_0(n)$ are the importance of each basic event respectively, and get pending pattern vector X_0 = {0.0005, 0.0005, 0.0005, 0.0205, 0.0410, 0.0615, 0.0615, 0.0615, 0.2051, 0.3077, 0.0513, 0.7179, 0.7179, 0.5128, 0.4103}.

Assuming that $X_0 = \{x_0(1), x_0(2), \ldots, x_0(n)\}$ is the primary sequence and X_0 and X_i have the same length, the absolute Grey Relation Degree ε_{0i} can be derived from Eq. 8 [21].

$$\left.\begin{aligned}
\varepsilon_{0i} &= \frac{1 + |S_0| + |S_i|}{1 + |S_0| + |S_i| + |S_i - S_0|}, \quad i = 1,2,\cdots,n \\
|S_0| &= \left| \sum_{k=1}^{n-1} x_0(k) + \frac{1}{2} x_0(n) \right| \\
|S_i| &= \left| \sum_{k=1}^{n-1} x_i(k) + \frac{1}{2} x_i(n) \right| \\
|S_i - S_0| &= \left| \sum_{k=1}^{n-1} (x_i(k) - x_0(k)) + \frac{1}{2} (x_i(n) - x_0(n)) \right|
\end{aligned}\right\} \tag{8}$$

$|S_0|$ is the fluctuation amplitude sum of the primary feature series, $|S_i|$ is the fluctuation amplitude sum of the feature series composed of the factors in the system, and $|S_i\text{-}S_0|$ is the difference between the amplitude of fluctuations of factors in the main series and subseries. According to the equation, the absolute grey relational degree of the minimum cut sets for the *Expansion Adhesive Debonding* fault of automobile doors is shown in Table 6.

According to Table 6, we get the Importance Ranking of the minimum cut sets for *Expansion Adhesive Debonding* fault of automobile doors based on F-AGRA method, as:

$$C_8 = C_9 > C_3 > C_4 = C_{10} > C_2 > C_{11} > C_1 = C_6 > C_5 > C_7$$

Fig. 3 is histogram of Absolute Grey Relation of minimum cut sets.

Table 6 Absolute Grey Relation Degree of minimum cut sets for *Expansion Adhesive Debonding* fault of automobile doors

Min cut set C_i	C_1	C_2	C_3	C_4	C_5	C_6
Absolute Grey Relation Degree ε_{0i}	0.6154	0.6371	0.6587	0.6443	0.6010	0.6154
Min cut set C_i	C_7	C_8	C_9	C_{10}	C_{11}	
Absolute Grey Relation Degree ε_{0i}	0.5794	0.6731	0.6731	0.6443	0.6299	

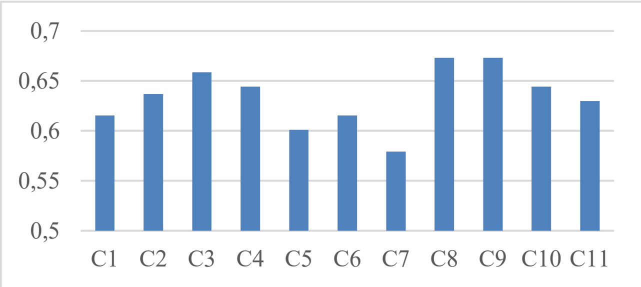

Fig. 3 Histogram of Absolute Grey Relation of minimum cut sets

4.3 Fault Analysis based on F-RGRA method

The F-RGRA method is used to analyse the differences in the longitudinal variation of the subsequence and the primary sequence. Under the fault tree concept, it can be understood as the degree of conformity of the change in each value within each row of the initial feature matrix of the minimum cut sets with the change in the value of the vector of factors of the pattern to be examined consisting of the importance of the basic events. It can eventually reflect the degree of influence of the change in the failure mode of each minimum cut set on the change of the top event.

In the calculation of the relative grey relational degree, the initial value is generally calculated by dividing the first term of the sequence, as shown in the Eq. 9.

$$X_i' = \{x_i'(1), x_i'(2), \dots, x_i'(n)\} = \left\{\frac{x_i(1)}{x_i(1)}, \frac{x_i(2)}{x_i(1)}, \dots, \frac{x_i(n)}{x_i(1)}\right\} \tag{9}$$

However, this approach becomes problematic if the first term in the data set is 0. Therefore, the equation can express the definition of the initial value by replacing the first term with the largest value in the data set (Eq. 10),

$$X_i' = \{x_i'(1),\ x_i'(2), \dots, x_i'(n)\} = \left\{\frac{x_i(1)}{x_i(d)}, \frac{x_i(2)}{x_i(d)}, \dots, \frac{x_i(n)}{x_i(d)}\right\} \tag{10}$$

where $x_i(d) = max\{x_i\}$. The characteristic matrix of the initial value image can be expressed as Eq. 11.

$$X = \begin{Bmatrix} X_1' \\ X_2' \\ \dots \\ X_i' \\ \dots \\ X_m' \end{Bmatrix} = \left\{\begin{pmatrix} \frac{x_1(1)}{x_1(d)} & \frac{x_1(2)}{x_1(d)} & \dots & \frac{x_1(n)}{x_1(d)} \\ \frac{x_2(1)}{x_2(d)} & \frac{x_2(2)}{x_2(d)} & \dots & \frac{x_2(n)}{x_2(d)} \\ \dots & \dots & \dots & \dots \\ \frac{x_i(1)}{x_i(d)} & \frac{x_i(2)}{x_i(d)} & \dots & \frac{x_i(n)}{x_i(d)} \\ \dots & \dots & \dots & \dots \\ \frac{x_m(1)}{x_m(d)} & \frac{x_m(2)}{x_m(d)} & \dots & \frac{x_m(n)}{x_m(d)} \end{pmatrix}\right\} \tag{11}$$

The characteristic matrix of the data after the initial image processing:

$$\begin{pmatrix} 0.25 & 0.25 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.125 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.67 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{12}$$

Similarly, in the primary sequence $X_0' = \{x_0'(1), x_0'(2), \cdots, x_0'(n)\}$, $x_0'(1), x_0'(2), \cdots, x_0'(n)$ can be obtained from Eq. 13.

$$X_0' = \{x_0'(1), x_0'(2), \ldots, x_0'(n)\} = \left\{\frac{x_0(1)}{x_0(d)}, \frac{x_0(2)}{x_0(d)}, \ldots, \frac{x_0(n)}{x_0(d)}\right\} \tag{13}$$

The pending pattern vector X_0'= {0.0007, 0.0007, 0.0007, 0.0256, 0.0571, 0.0857, 0.0857, 0.0857, 0.2857, 0.4286, 0.0715, 1, 1, 0.7143, 0.5715}.

If X_0' and X_i' are the initial value images of X_0 and X_i, then the absolute grey relational degree between X_0' and X_i' is the relative relation degree between X_0 and X_i, denoted as r_{0i} ,which can be obtained from the Eq. 14 [22].

$$\left.\begin{aligned} r_{0i} &= \frac{1 + |S_0'| + |S_i'|}{1 + |S_0'| + |S_i'| + |S_i' - S_0'|}, \quad i = 1,2,\cdots,n \\ |S_0'| &= \left|\sum_{k=1}^{n-1} x_0'(k) + \frac{1}{2} x_0'(n)\right| \\ |S_i'| &= \left|\sum_{k=1}^{n-1} x_i'(k) + \frac{1}{2} x_i'(n)\right| \\ |S_i' - S_0'| &= \left|\sum_{k=1}^{n-1} (x_i'(k) - x_0'(k)) + \frac{1}{2} (x_i'(n) - x_0'(n))\right| \end{aligned}\right\} \tag{14}$$

$|S_0'|$ is the sum of the rates of change of the volatility of the total characteristic sequence, $|S_{\mathrm{i}}'|$ is the sum of the rates of change of the volatility of the characteristic sequence of the factors in the system, and $|S_{\mathrm{i}}' - S_0'|$ is the sum of the differences in the rates of change in the volatility of the corresponding factors in the primary sequence and the subseries. According to the calculation of the Eq. 14, the relative grey relational degree of the minimum cut sets for the *Expansion Adhesive Debonding* fault of automobile doors is shown in Table 7.

Table 7 Relative Grey Relation Degree of minimum cut sets for *Expansion Adhesive Debonding* faults in automobile doors

Min cut set C_i	C_1	C_2	C_3	C_4	C_5	C_6
Relative Grey Relation Degree r_{0i}	0.7161	0.6756	0.7161	0.7345	0.6621	0.6621
Min cut set C_i	C_7	C_8	C_9	C_{10}	C_{11}	
Relative Grey Relation Degreer_{0i}	0.6621	0.6621	0.6621	0.6621	0.6080	

According to Table 7, we get the Importance Ranking of the minimum cut sets for *Expansion Adhesive Debonding* fault of automobile doors based on F-RGRA method, as:

$$C_4 > C_1 = C_3 > C_2 = C_5 = C_6 = C_7 = C_8 = C_9 = C_{10} > C_{11}$$

Fig. 4 is histogram of Relative Grey Relation of minimum cut sets.

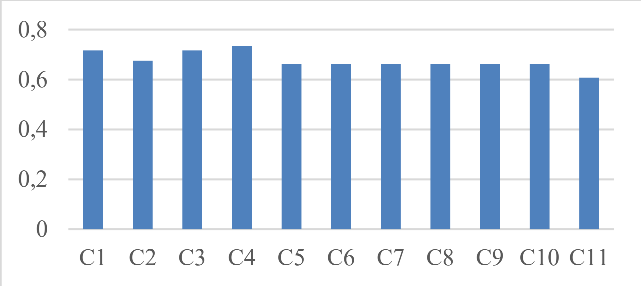

Fig. 4 Histogram of Relative Grey Relation of minimum cut sets

4.4 Fault Analysis based on F-CGRA method

The F-CGRA method is a combination of the F-AGRA method and the F-RGRA method. It integrates the longitudinal differences and rates of change of the selected indicators (called: primary sequence) and related factors (called: subseries). Finally, it realizes the relation analysis of the selected indicators and system-related factors.

In general, the Comprehensive Grey Relation Degree ρ_{0i} is defined by combining the Absolute Grey Relation Degree and the Relative Grey Relation Degree as shown in Eq. 15 [23].

$$\rho_{0i} = \theta_i \varepsilon_{0i} + (1 - \theta_i) r_{0i} \tag{15}$$

In the equation, θ_i is the distribution coefficient, which can be determined by the maximum deviation method proposed by Sun *et al*. as in Eq. 16 [24].

$$\theta_i = \frac{B_{\varepsilon i}}{B_{\varepsilon i} + B_{ri}} \tag{16}$$

Total deviation of Absolute Grey Relation Degree [25]:

$$B_{\varepsilon i} = \sum_{k=1}^{n} |\varepsilon_{0i} - \varepsilon_{0k}| \tag{17}$$

Total deviation of Relative Grey Relation Degree [25]:

$$B_{ri} = \sum_{k=1}^{n} |r_{0i} - r_{0k}| \tag{18}$$

According to the calculation of Eqs. 16, 17, 18, the Comprehensive Grey Relation Degree of each minimum cut set of automobile doors *Expansion Adhesive Debonding* fault is shown in Table 8.

Table 8 Comprehensive Grey Relation Degree of minimum cut sets for *Expansion Adhesive Debonding* faults in automobile doors

Min cut set C_i	C_1	C_2	C_3	C_4	C_5	C_6
Total dispersion of Absolute Grey Relation Degree $B_{\varepsilon i}$	0.3031	0.2524	0.3316	0.2596	0.4039	0.3031
Total dispersion of Relative Grey Relation Degree B_{ri}	0.4910	0.2885	0.4910	0.6566	0.2480	0.2480
partition coefficient θ_i	0.3817	0.4666	0.4031	0.2833	0.6196	0.5500
Comprehensive Grey Relation Degree ρ_{0i}	0.6738	0.6576	0.6930	0.7089	0.6242	0.6364
Min cut set C_i	**C_7**	**C_8**	**C_9**	**C_{10}**	**C_{11}**	
Total dispersion of Absolute Grey Relation Degree $B_{\varepsilon i}$	0.5983	0.4342	0.4342	0.2596	0.2596	
Total dispersion of Relative Grey Relation DegreeB_{ri}	0.2480	0.2480	0.2480	0.2480	0.7349	
partition coefficient θ_i	0.7070	0.6365	0.6365	0.5114	0.2610	
Comprehensive Grey Relation Degree ρ_{0i}	0.6036	0.6691	0.6691	0.6530	0.6137	

According to Table 8, we get the Importance Ranking of the minimum cut sets for *Expansion Adhesive Debonding* fault of automobile doors based on F-CGRA method, as:

$$C_4 > C_3 > C_1 > C_8 = C_9 > C_2 > C_{10} > C_6 > C_5 > C_{11} > C_7$$

Fig. 5 is minimum cut sets Comprehensive Grey Relation Degree histogram.

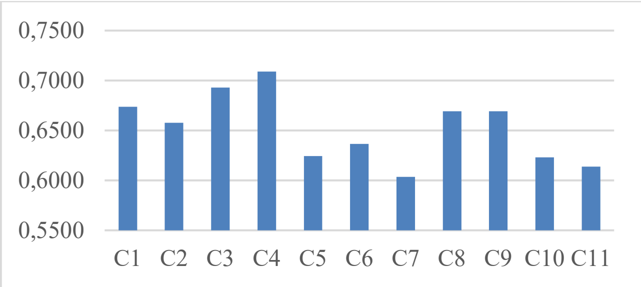

Fig. 5 Minimum cut sets Comprehensive Grey Relation Degree histogram

5. Result analysis and discussion

The FTA results of the three methods are compared. The results show that if a basic event occurs in a different minimum cut set, the probability of occurrence of the top event is greatly reduced after that event is excluded, even if the probability of occurrence of that event is low. This finding is also proved by the practice in the actual production. Unlike the FTA method alone, the combination of FTA and GRA can effectively analyse the relationship between system fault characteristics and intuitively reflect the likelihood magnitude of each basic event fault in the system.

In the actual production, the fault analysis team of G Group found the *Expansion Adhesive Debonding* fault in the automobile doors after 13 days through several checks and experiments, such as measurement of sheet metal parts, adjustment of the spacing between sheet metal parts, adjustment of glue application position and glue application amount, and high-temperature exposure of expansion adhesive placement, using inspection tools. The group found that the main cause of the problem is the intermediate events M_1 (the nature of expansion adhesive was not enough to meet the shipping demand) and M_3 (the spacing between the sheet metal parts was excessive), which include the basic events N_1-N_8, N_{11}-N_{13}. After the improvement of the basic events N_5, N_6, N_7, N_8, N_{12}, N_{13}, N_{14}, the *Expansion Adhesive Debonding* fault of the automobile doors was prevented from occurring.

The FTA in the third part shows that the basic events N_5, N_6, N_7, N_8, N_{12}, N_{13} and N_{14} constitute the basic event combination $\{N_5, N_6\}$, $\{N_7, N_8\}$, $\{N_{12}\}$, $\{N_{13}\}$ and $\{N_{14}\}$ corresponding to the five failure modes C_8, C_9, C_3, C_4 and C_{10}. This order is consistent with the F-AGRA method's ranking of fault levels in this study. $C_8 = C_9 > C_3 > C_4 = C_{10} > C_2 > C_{11} > C_1 = C_6 > C_5 > C_7$ indicates a high degree of conformity. Therefore, the F-AGRA method is the most consistent with the actual production situation. It is followed by the F-CGRA method and the F-RGRA method in the analysis of the *Expansion Adhesive Debonding* fault of automobile doors.

The comparison result of the three methods indicates that F-AGRA focuses on factors with a great influence on the system. The contribution of the failure modes represented by each minimum cut set to the top event is calculated. F-RGRA obtains the initial value of the vector set $X'_0 - X'_{11}$ by dividing the largest term in the vector max $\{x_i\}$, which amplifies the influence of the failure mode changes in each minimum cut set on the top event. The degree of influence on the change of the top event is focused on reflecting the trend of factors and system changes. However, the direct influence of the failure mode represented by its minimum cut sets on the top event is not as obvious as F-AGRA. F-CGRA calculates the total deviation of the two correlations' balance reflects the direct contribution and variable influence of the fault mode represented by the minimum cut set to the top event.

For the direct contribution and change in the impact of the ranking, the unilateral accuracy and embodiment of the trend in the combined method are not as obvious as those in the two separate methods. Therefore, the method failure analysis between the above two methods is conducted.

The characteristics and applicable scenarios of the three methods are summarized in Table 9.

Table 9 Characteristics and applicable occasions of F-AGRA, F-RGRA, and F-CGRA methods

Method	Characteristic	Applicable occasions
F-AGRA	Visually reflect the contribution of the fault modes represented by each minimum cut set to the top event	The fault tree analysis is mainly composed of or gate structure events. and there are few and gate structure events, the interaction between elementary event is less
F-RGRA	The focus is on reflecting which factors are consistent with the system change trend, but the direct impact of the fault mode represented by the minimum cut set on the top event is not obvious	The fault tree analysis is mainly composed of and gate structure events, or there are few or gate structure events
F-CGRA	Balance reflects the direct contribution and changing impact of the fault mode represented by the minimum cut set on the top event	The fault tree analysis has many levels and is complex, and there are and gate events close to the root of the fault tree analysis

6. Conclusion

This study analyses the occasional and difficult-to-diagnose fault problems in the automotive industry. It uses three methods for fault analysis, explores the applicable scenarios of the three methods, improves the efficiency of fault analysis, and provides the automotive manufacturing industry with new ideas for fault analysis and fault diagnosis. Thus, the causes of product failure can be efficiently targeted for improvement, and the efficiency of improvement can be improved. This study can also be extended to the failure analysis of other complex products or systems by providing an efficient, accurate, and scientific theoretical basis for handling accident priorities, controlling the occurrence of accidents, and improving the reliability and safety of systems.

In this study, the method is only used to study the *Expansion Adhesive Debonding* fault of automobile doors. The method will be verified using other cases in the future. The analysis of the top event relies on the accurate construction of the fault tree and the comprehensive investigation of the cause of the failure. The probability of failure of each basic event must be accurately determined; otherwise, accurate conclusions may not be drawn. The fault diagnosis analysis method based on fault tree and generalized grey theory remains complicated in the calculation process when used for complex fault tree structures. Future research should focus on the sensitivity analysis of fault events while designing the program software using the three methods matching the operation to solve the problem of complex operation.

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