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Incentive modeling analysis in engineering applications and projects with stochastic duration time

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A B S T R A C T

Incentives are quite common to be utilized in engineering applications such as some infrastructure development projects or construction projects. Due to the increasing complexity of construction management and the continuing trend towards outsourcing of component or engineering outsourcing activities, we focus on the issue of incentive design. Time collaboration is one of the main focuses of random project duration time in parallel projects. In this article, we consider a setting where a manufacturer outsources two parallel subtasks to two different suppliers, and the manufacturer is time sensitive. On the premise that the project completion time follows the exponential distribution, some models are established to compare the proposed incentives and we get the comparative analysis of the proposed incentives. This paper puts forward three kinds of time-based incentive mechanisms, namely, deadline incentive mechanism, competition mechanism and mixed incentive mechanism. We do modeling analysis for all incentive mechanisms. We get the optimal work rates determined by suppliers and compare various incentive mechanisms to maximize manufacturers' profits.

A R T I C L E I N F O

Keywords:
Engineering applications;
Incentive mechanism;
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Stochastic duration time;
Deadline incentive model;
Competition model;
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1. Introduction

In recent years, with the increasing complexity of cross-industry projects, scholars have paid more and more attention to the importance and significance of engineering and project management research. In traditional construction or modern product development management, there is an increasing tendency to explore the role of incentive mechanisms in different environments. At the same time, project management and engineering applications have expanded dramatically from traditional building and infrastructure management to include new product development, information technology, pharmaceuticals, and service development. Project-based organizations are increasingly dependent on suppliers, and as a result, supply chain risks can arise. These new applications create different characteristics, and we need to develop new and innovative ways to solve the problems that may arise in these areas.

Project management includes planning and controlling the uncertainty in the duration of various components or tasks. Tools such as CPM, PERT and cost-time trade analysis are often used for project management. Therefore, the project owner can have a detailed overview when managing projects. These tools are effective and efficient when there is little uncertainty about the project completion time or operating costs. However, uncertainty arises in industries such as architecture, hardware and software development, national defense and management due to the increasing complexity of project management. For example, new power station, aircraft, and telecommunications network are all characterized by long construction period, high risks, and key links with engineering and commercial teams. We know little about how to deal with and manage the risks arising from project uncertainty.

In the field of consumer electronics, for example, more and more enterprises cooperate with Foxconn, Flextronics and other electronic manufacturing service providers to conduct research, development, design and production of electronic products. In a setting where the manufacturer cannot begin to work until all parts are delivered, the manufacturer's schedule depends on the slowest delivery. Project delivery delays can have a significant negative impact on the interests of manufacturers, so manufacturers need to devise mechanisms to encourage suppliers to work faster so as to speed up project completion time.

This article considers a setting where a manufacturer outsources two parallel subtasks to two suppliers with random project completion times. The manufacturer (she) plays the role of Stackelberg leader, making decisions about the best price she can offer to the suppliers. Each supplier (he) must determine the best work rate to obtain the best expected discount profit. When the manufacturer is time urgent to launch new products, she is very sensitive to the completion time of the entire project. We assume that the supplier's work rate is not adjustable during the whole period due to some objective factors or external conditions. We mainly focus on time-based incentive mechanisms in this paper.

Time-based incentive mechanism is common in practice and is a tool for manufacturers to motivate suppliers to complete tasks on time while managing random projects. Construction projects involve complex processes, so avoiding delays during the construction phase is very important. For example, after the Northridge earthquake in 1994, the city of Los Angeles (the project manager) wanted to repair the Santa Monica Freeway within 180 days. The city of Los Angeles quickly offered Clint Meyers (the contractor) an incentive contract. Specifically, if the project is completed 180 days ahead of schedule, the contractor will be paid an additional \$200,000 per day; However, a penalty of \$200,000 per day will be paid if the contractor is late [1, 2]. We propose three types of time-based incentives in our article.

In the case of deadline incentive mechanism, the project contractor is usually given a deadline in traditional project management and is responsible for the penalty charged by the project manager if the deadline is not met. The project owner often provides incentives/disincentives to the contractors. Deadlines are also commonly used in product development processes [3, 4]. In an empirical study of the global computer industry, Eisenhardt and Tabrizi [5] found that frequent deadlines help accelerate product development.

Due to the randomness of the task completion time, the two suppliers may end the tasks with different task duration times. The result could be that one supplier finishes earlier (hereinafter referred as faster supplier) and one later (hereinafter referred as slower supplier), and this asynchronization may harm the manufacturer's profit. Thus, we propose a competition incentive mechanism, which means that the faster supplier will be awarded, and the slower supplier will be punished.

On the problem of designing incentives for parallel random subprojects, our paper is the first one to integrate reward and punishment incentives design together with project structure. We also incorporate the criteria for rewards and penalties in those different incentive mechanisms, and we do a more comprehensive comparative analysis of proposed incentive mechanisms.

We then combine the above two incentive mechanisms together, thus getting a mixed incentive mechanism. We then examine its influence on manufacturer's benefits. By comparing those three different incentive mechanisms, we get some illuminating results to shed light on project management and our analyses solve the following questions:

Q1: What is the supplier's optimal work rate under different incentive mechanisms?
Q2: What incentives will result in higher profits for the manufacturer?

The remainder of this paper is organized as follows: in Section 2, we provide a related literature review on project supply chain management. Section 3 presents our base model settings. In Section 4, we compare three kinds of incentive mechanisms together with the benchmark case. In Section 5, by some numerical analysis, we give manufacturer's optimal choice of incentive mechanisms under different conditions. In Section 6, we summarize our work and give a conclusion of this paper. All proofs are relegated to Appendix.

2. Literature review

Scholars and practitioners have extensively studied project management. In traditional project management, scheduling, and planning activities are the main concerns, and optimization is the main way to solve these problems. We recommend that readers refer to the study [6, 7] for further information. Hall [8] provides a comprehensive review and discussion of research and teaching opportunities in this area. In this section, we mainly focus on the problem of project contracting and incentive mechanism design within multi-agents. We review some important contributions that researchers have achieved. This paper mainly involves the following four research directions.

2.1 Deadlines in project

Deadlines are often used in product development processes and construction projects. A deadline means that the project must be stopped when the timeline reaches the stop point, or agents who do not meet the deadline will be penalized. Zhang [9] examines the value of deadlines from the agency theory perspective and considers that a company pays an agent to lead product development activities. It mainly focuses on the trade-off between the project's return and the project's labor cost. The paper concludes that deadlines are good, because they diminsh the agent's incentive to procrastinate dynamically. Bordley *et al*. [10] consider a deadline problem with uncertainty. Traditional project management deals with unexpected changes in project deadlines through an external change control process. They introduce a new counter-intuitive idea of recognizing uncertainty in project deadlines and show that it can greatly enhance the value of managers' decisions. Du *et al*. [11] studied the optimal timing for occasional placement of 'fulcrum' in crowdfunding. The project will be funded successfully only if the target is met within the specified deadline. They evaluated three strategies in detail, namely, seeding, feature upgrading and time-limited offering, to increase the likelihood of turning the random pledge process from failure to success.

Our paper differs from the study [9, 11], in which the deadline serves as a stopping point of the project. However, ours mainly focuses on the reward or penalty associated with the specific deadline.

2.2 Time-based incentive contract

In the traditional supply contract, the uncertainty of demand is the main focus, and the quantity of order is the main decision variable. However, in project contracting, time uncertainty is the main focus, and the contractor's work rate decision is the main decision variable. Therefore, various incentive contracts based on time are studied by scholars in the field of project management. Gupta *et al*. [12] studied A+B infrastructure procurement mechanism, which state transportation agencies use to provide incentives for faster completion. Early completion will be rewarded, late will be penalized. Managers use time-based incentive contracts to encourage contractors to make greater efforts to complete tasks faster. Tang *et al*. [13] compared two different time-dependent project management contracts (C1 and C2) when managers conducted reverse auctions. In each incentive contract, completion earlier than the due date will be incentivized and completion later than the due date will be discouraged.

Chen and Lee [14] consider a coordination problem where the manufacturer carries out a series of tasks, and each task needs to procure a certain material from a supplier. They propose a delivery-schedule-based incentive contract to mitigate incentive misalignment between firms in a project supply chain. The delivery delay than the due date will be punished and early delivery will be awarded. Bayiz and Corbett [2] provide a framework to integrate contracting and asymmetric information into a project management context and study the value of incentive contracts. They derived the optimal incentive contracts the project manager offers to the subcontractors when the two subprojects are conducted in parallel or in serial, respectively. Kwon *et al.* [15] consider a coordination problem when managing a project with uncertain completion and an unobservable contractor's work rate. Fixed price, time-based, cost-based, and rate-based incentive contracts are examined in their work. Zhao and Mu [16] consider a situation where a manufacturer outsources two parallel tasks to two different suppliers, and they propose nine kinds of time-based incentive contracts a manufacturer can offer to the suppliers.

2.3 Reward-penalty mechanism

A project usually consists of a set of activities that introduce delays or expeditions related to penalty or reward issues when the planned completion time is incorrectly estimated. Bergantiños and Lorenzo [17] consider a situation where a planner wants to execute a project involving several companies. If the deadline is missed, the company will be penalized. They discussed two ways to impose penalties: they would apply only if the entire project was delayed; Even if the project is completed on time, the penalties apply to every company that causes delays. Estévez-Fernández [18] analyzed the situation where a project with multiple activities was not realized as planned. If the project is accelerated, there is a reward. Similarly, if the project is delayed, there is a penalty. In this paper, we consider the existence of any non-decreasing reward function and penalty function for total exploration and delay, respectively, focusing on the distribution of total reward (penalty) function between activities. Chen *et al.* [19] proposed an incentive payment contract for a series of random items. Their proposed contract reflects the convex time-cost trade-off that is well known in the project literature. In contrast to fixed-price contracts, such incentive contracts imply penalties for suppliers.

2.4 Parallel projects

Project structure is a key focus when conducting product development activities or outsourcing activities. These sub-projects are usually performed in parallel (tasks have to be performed in parallel), or in serial (tasks have to be performed sequentially) or in network (tasks have to be performed in parallel-serial). Our work mainly considers a scenario where the sub-projects are conducted in parallel. Kwon *et al.* [20] studied a delayed payment scheme where multiple concurrent subprojects were outsourced to different suppliers. Under the delayed payment scheme, each supplier will be paid after all tasks are completed, that is, faster supplier payments are delayed. Song *et al.* [21] explored incentives for firms under risk-sharing partnerships in the context of project management. In this partnership, each partner pays its own costs and shares project completion results (rewards or losses) between faster and slower suppliers. They examine the project network in serial, parallel, and assembled ways. Dawande *et al.* [22] studied the coordination problems faced by enterprises when undertaking projects consisting of multiple tasks. Under the assumption of exponential completion time, the optimal coordination contract of parallel and sequential tasks is studied.

The above literatures only consider the incentive mechanism design problem under partial factors. However, our paper considers the time-based incentive mechanism design problem faced by a manufacturer who carries out outsourcing activities in more complicated cases. Our proposed incentive mechanisms take deadlines in projects issue and reward-penalty incentives into consideration, and we also consider the time synergy problem in parallel projects. Our paper is a more comprehensive one that considers the incentive mechanisms design issue by the project owner.

3. Model settings

In our context, we consider a scenario where the manufacturer outsources two parallel subtasks with stochastic project completion time to two different suppliers. The duration of each subtask X_i $(i = 1, 2)$ is exponentially distributed with parameter μ_i, where the work rate $\mu_i > 0$ is selected by supplier i at the beginning of the project. Throughout our analysis, we assume that the two subtasks are of the same workloads and difficulties. The suppliers cannot change the work rates once selected at the beginning, due to some technical or practical reasons. The manufacturer cannot start to work until the suppliers deliver all parts.

We consider the time value of revenue gained and cost incurred by the supplies into our context. Thus, our model takes the discounting issue into our analysis. Let the continuous-time discount rate be $\alpha > 0$, which captures the fact that the suppliers want to receive his payment earlier and incur costs later.

We consider a situation where the manufacturer is time sensitive to the completion time of the total project. For example, while developing some new products, the manufacturer is urgent to launch or release new products to occupy the market earlier. The total completion time of the project satisfies that $T = \max(X_1, X_2)$. For ease of exposition, we assume that the project's value $V(T)$ to the manufacturer is a linear, decreasing function of the project completion time, specifically, we define $V(T) = A - BT$ $(A, B > 0)$.

By the properties of exponential distributions, we have that the probability density function of random variable X_i $(i = 1,2)$ is $\mu_i e^{-\mu_i t}$ and the cumulative distribution function of a random variable X_i is $F_i(t) = 1 - e^{-\mu_i t}$. We can infer that the discount factor satisfies that $E[e^{-\alpha X_i}] = \int_0^\infty \mu_i e^{-\alpha t} e^{-\mu_i t}\, dt = \frac{\mu_i}{\alpha+\mu_i}$.

While working at the rate of μ_i $(i = 1,2)$, each supplier incurs an operating cost of $\kappa(\mu_i)$ per unit time. We assume that the suppliers' operating cost $\kappa(\mu_i)$ per unit time associating with the work rate μ_i is a convex-increasing function, and it is given by $\kappa(\mu_i) = k\mu_i^2$ with $k > 0$. Furthermore, we have that supplier i 's expected discounted operating costs equal that $E\left[\int_0^{T_i} \kappa(\mu_i) e^{-\alpha t} dt\right] = \int_0^\infty [\int_0^{t_i} \kappa(\mu_i) e^{-\alpha t} dt]\, \mu_i e^{-\mu_i t_i} dt_i = \frac{k\mu_i^2}{\alpha+\mu_i}$.

We compare four kinds of mechanisms manufacturer can offer to the suppliers (three timed-based incentive models together with the benchmark case). For the sake of perceptual intuition, we list some characteristics among the four mechanisms. Table 1 shows the specific elements contained in these four kinds of models.

In the base model, we consider a setting where the manufacturer does not offer any incentive to the suppliers. The two suppliers start to work simultaneously and make the work rate decisions $\mu_i (i = 1,2)$. Each supplier can get a payment of ω after his own subtask is finished, and the two suppliers make their decision independently.

In the deadline incentive model, the manufacturer imposes a specific due date, which we also referred as a deadline D. In addition, to paying a base price ω, the manufacturer awards the supplier a reward at r per unit time of early completion than the deadline, and charges the supplier a penalty at p per unit time of delay than the deadline.

In the competition incentive model, the manufacturer awards the faster supplier a reward of r per unit time earlier than the slower supplier; she also charges the slower supplier a penalty of p per unit time later than the faster supplier, together with a base price ω.

Table 1 Classification of different mechanisms

Base Model	Deadline Incentive Model
• A base price	• A base price • A specific due date. • An incentive/disincentive scheme
Competition Model	**Mixed Model**
• A base price • A comparison of the faster and slower • An incentive/disincentive scheme	• A base price • A specific due date. • A comparison of the faster and slower • An incentive/disincentive scheme

In the mixed incentive model, the manufacturer imposes a specific due date D. The manufacturer awards the faster supplier a reward of r per unit time earlier than the deadline and charges the slower supplier a penalty of p per unit time later than the deadline, together with a base price ω.

4. The model

4.1 Base model

In our base model, there is no incentive mechanisms offered to the suppliers by the manufacturer. The manufacturer outsources two subtasks to two different suppliers. The two suppliers start to work simultaneously, and each supplier does not take the other's decision into his own consideration. Thus, each supplier makes his work rate decision μ_i independently. When supplier i finishes his own subtask, he will get a payment of ω, and his expected discounted profit can be expressed as:

$$\Pi_i = \omega E[e^{-\alpha X_i}] - \frac{k\mu_i^2}{\alpha+\mu_i} \; (i = 1,2) \tag{1}$$

From the concavity of Eq. 1, we can get:

Proposition 1. Both suppliers' optimal work rates are the same in equilibrium, and they are characterized by

$$\mu^* = -\alpha + \sqrt{\alpha^2 + \frac{\omega\alpha}{k}} \tag{2}$$

We have that μ^* is strictly increasing with ω.

When both subtasks are finished and delivered to the manufacturer, the outsourcing activity ends. Because we have that $E[T] = E[\max(X_1, X_2\,)] = \frac{1}{\mu_i} + \frac{1}{\mu_{-i}} - \frac{1}{\mu_i+\mu_{-i}}$, thus, in equilibrium, we can get that $E[T] = \frac{3}{2\mu^*}$.

The manufacturer makes an optimal price decision ω, and her profit can be expressed as:

$$\Pi_m = A - BE[T] - 2\omega. \tag{3}$$

By the concavity of Eq. 3, we can obtain:

Proposition 2. The manufacturer's optimal price decision satisfies that:

$$\sqrt{\alpha^2 + \frac{\omega^*\alpha}{k}}\left(\sqrt{\alpha^2 + \frac{\omega^*\alpha}{k}} - \alpha\right)^2 = \frac{3B\alpha}{8k} \tag{4}$$

The specific certification process is in Appendix.

The above proposition provides the manufacturer's optimal price decision, and we can obtain the manufacturer's optimal profit under the benchmark case.

4.2 Deadline incentive mechanism

In this case, we consider a situation where the manufacturer offers a deadline incentive mechanism to the suppliers. The manufacturer decides optimal price ω paid to the suppliers, and the suppliers make the optimal work rate decision $\mu_i (i = 1,2)$. Unlike sub-section 4.1, the two suppliers' decisions are no longer independent of each other under the incentive mechanisms.

Consistent with the above analysis, this incentive mechanism contains the following three components:

A base price. The price manufacturer pays to the supplier is ω, which is irrelevant to the performance of supplier's delivery time.

A specific due date. The manufacturer sets a pre-determined delivery date, which we referred as a deadline D.

An incentive/disincentive scheme. The supplier will be awarded a reward at r per unit time of early completion than the deadline and will be charged a penalty at p per unit time of delay than the deadline.

Under the deadline incentive mechanism, one supplier makes his optimal work rate decision regardless of the other supplier's decision. After the completion of his own subtask, supplier i $(i = 1{,}2)$ will get an expected payment of $W^d(i)$, which satisfies that:

$$W^d(i) = \omega + rE(D - X_i)^+ - pE(X_i - D)^+ \tag{5}$$

By some calculations, we can get the following:

$$W^d(i) = \omega + pD - pE(X_i) + (r - p)\left[D\int_0^D f(t_i)dt_i - \int_0^D t_i f(t_i)dt_i\right]$$

Lemma 1. The expected payment $W^d(i)$ satisfies that:

$$W^d(i) = \omega + rD - r\frac{1}{\mu_i} + (r - p)\frac{1}{\mu_i}e^{-\mu_i D} \tag{6}$$

See Appendix for the specific certification process.

Furthermore, we can express supplier i's expected discounted profit as follows:

$$\Pi_i^d = W^d(i)E[e^{-\alpha X_i}] - \frac{k\mu_i^2}{\alpha+\mu_i} \tag{7}$$

By some calculations, we can get the following proposition.

Proposition 3. The suppliers' optimal work rates are the same in equilibrium and it is characterized by:

$$k(\mu^{d*})^2 + 2k\alpha\mu^{d*} + (r - p)e^{-\mu^{d*}D}\left(1 + D\alpha + D\mu^{d*}\right) = \omega\alpha + rD\alpha + r \tag{8}$$

The specific certification process is in Appendix.

We define $G = r - p$. From proposition 3, we have that if $G \le 0$, we can get that $\mu^{d*} > \mu^*$. If $G > 0$, when $e^{-\mu^{d*}D}\left(1 + D\alpha + D\mu^{d*}\right) < \frac{rD\alpha+r}{r-p}$, we have $\mu^{d*} > \mu^*$. When $e^{-\mu^{d*}D}\left(1 + D\alpha + D\mu^{d*}\right) > \frac{rD\alpha+r}{r-p}$, we have $\mu^{d*} < \mu^*$. In equilibrium, we have $W^d(1) = W^d(2) = W^d$.

We then make some comparative static analyses concerning on the supplier's optimal work rate under the deadline incentive mechanism. Table 2 shows the relationship between supplier's optimal work rate and the pre-determined specific due date.

We then show the relationship between supplier's optimal work rate and the disincentive mechanism factor, and the relationship between supplier's optimal work rate and the base price in Table 3.

As to the relationship between supplier's optimal work rate and the incentive mechanism factor, the results are given in Table 4.

Table 2 The relationship between μ^{d*} and D

$G < 0$	$G = 0$	$G > 0$
$\frac{d}{dD}\mu^{d*} > 0$, if $\mu^{d*} < -\frac{1}{D}\ln\frac{2k}{D^2(r-p)}$	$\frac{d}{dD}\mu^{d*} > 0$	$\frac{d}{dD}\mu^{d*} > 0$, i. if $\mu^{d*} < -\frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} + \frac{\alpha}{D}}$. ii. if $\mu^{d*} > -\frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} + \frac{\alpha}{D}}$ and $e^{-\mu^{d*}D}(D\alpha\mu^{d*} + D(\mu^{d*})^2 - \alpha) < \frac{r\alpha}{p-r}$
$\frac{d}{dD}\mu^{d*} < 0$, if $\mu^{d*} > -\frac{1}{D}\ln\frac{2k}{D^2(r-p)}$		$\frac{d}{dD}\mu^{d*} < 0$, i. if $\mu^{d*} > -\frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} + \frac{\alpha}{D}}$ and $e^{-\mu^{d*}D}(D\alpha\mu^{d*} + D(\mu^{d*})^2 - \alpha) > \frac{r\alpha}{p-r}$

Table 3 The relationship between μ^{d*} and p, ω

$G \le 0$	$G > 0$
$\frac{d}{dp}\mu^{d*} > 0$	$\frac{d}{dp}\mu^{d*} > 0$, if $\mu^{d*} < -\frac{1}{D}\ln\frac{2k}{D^2(r-p)}$ $\frac{d}{dp}\mu^{d*} < 0$, if $\mu^{d*} > -\frac{1}{D}\ln\frac{2k}{D^2(r-p)}$
$\frac{d}{d\omega}\mu^{d*} > 0$	$\frac{d}{d\omega}\mu^{d*} > 0$, if $\mu^{d*} < -\frac{1}{D}\ln\frac{2k}{D^2(r-p)}$ $\frac{d}{d\omega}\mu^{d*} < 0$, if $\mu^{d*} > -\frac{1}{D}\ln\frac{2k}{D^2(r-p)}$.

Table 4 The relationship between μ^{d*} and r

$G \leq 0$	$G > 0$
$\frac{d}{dr}\mu^{d*} > 0$, if $\mu^{d*} < -\alpha + \sqrt{\alpha^2 + \frac{p+D\alpha p+\omega\alpha}{k}}$	$\frac{d}{dr}\mu^{d*} > 0$, i. if $\mu^{d*} < \min\left\{-\alpha + \sqrt{\alpha^2 + \frac{p+D\alpha p+\omega\alpha}{k}}, -\frac{1}{D}\ln\frac{2k}{D^2(r-p)}\right\}$ ii. if $\mu^{d*} > \max\left\{-\alpha + \sqrt{\alpha^2 + \frac{p+D\alpha p+\omega\alpha}{k}}, -\frac{1}{D}\ln\frac{2k}{D^2(r-p)}\right\}$
$\frac{d}{dr}\mu^{d*} < 0$, if $\mu^{d*} > -\alpha + \sqrt{\alpha^2 + \frac{p+D\alpha p+\omega\alpha}{k}}$	$\frac{d}{dr}\mu^{d*} < 0$, i. if $\mu^{d*} \sim \left[-\alpha + \sqrt{\alpha^2 + \frac{p+D\alpha p+\omega\alpha}{k}}, -\frac{1}{D}\ln\frac{2k}{D^2(r-p)}\right]$ ii. if $\mu^{d*} \sim \left[-\frac{1}{D}\ln\frac{2k}{D^2(r-p)}, -\alpha + \sqrt{\alpha^2 + \frac{p+D\alpha p+\omega\alpha}{k}}\right]$

After the delivery of subtasks by the two suppliers, the outsourcing activities ends. The manufacturer's profit can be expressed as

$$\Pi_m^d = A - BE[T] - 2W^d. \tag{9}$$

In order to solve the above problem, we transfer (8) as $L(\mu^{d*}, \omega) = 0$. Manufacturer's profit maximizing problem in Eq. 9 can be redeemed as an optimization problem, which can be recognized as follows:

$$\max_{\mu^{d*},\omega} \Pi_m^d = A - BE[T] - 2W^d$$
$$\text{s.t. } L(\mu^{d*}, \omega) = 0$$

Because of the implicit function form of Eq. 8, it's hard to get the analytical expression of optimal price decision made by the manufacturer. We do some numerical examples in Section 5 to determine if deadline incentive mechanisms dominate the others. We use optimization algorithms to determine the optimal (μ^{d*}, ω). We do some comparative analyses concerning supplier's optimal work rate decision in this sub-section and discuss the other two incentive mechanisms in the following analysis.

4.3 Competition mechanism

In this section, we consider a situation where the manufacturer offers a competition mechanism to the suppliers. The manufacturer makes a decision of optimal price ω paid to the suppliers, and the suppliers make the optimal work rate decision $\mu_i (i = 1,2)$. The two suppliers also have to take the other's decision into his own consideration under this incentive mechanisms.

This incentive mechanism manufacturer offers to the suppliers contains the following three components:

A base price. The manufacturer pays each supplier a base price of ω.

A comparison of the faster and slower. Because of the stochasticity of project duration times, the result ex post can be distinguished from a faster supplier and a slower supplier.

An incentive/disincentive scheme. The faster supplier will be awarded a reward of r per unit time earlier than the slower supplier; the slower supplier will be charged a penalty of p per unit time later than the faster supplier.

For supplier i ($i = 1,2$), we can get that his expected payment gained from the manufacturer after his completion of the subtask satisfies:

$$W^c(i) = \omega + rE[X_{-i} - X_i]^+ - pE[X_i - X_{-i}]^+ \tag{10}$$

Lemma 2. With simple calculations, we can infer that:

$$W^c(i) = \omega + r\frac{\mu_i}{\mu_{-i}(\mu_i+\mu_{-i})} - p\frac{\mu_{-i}}{\mu_i(\mu_i+\mu_{-i})}$$

The specific certification process is in Appendix.

Furthermore, we have that supplier i's expected discounted profit can be expressed as:

$$\Pi_i^c = W^c(i)E[e^{-\alpha X_i}] - \frac{k\mu_i^2}{\alpha+\mu_i} \tag{11}$$

Proposition 4. In equilibrium, supplier's optimal work rate is given by:

$$4k(\mu^{c*})^4 + 8k\alpha(\mu^{c*})^3 - (3p + 4\omega\alpha)(\mu^{c*})^2 + (r - p\alpha)\mu^{c*} = r\alpha \tag{12}$$

See Appendix for the specific certification process.

From the above result, we can see that if $p = r = 0$, we have $\mu^{c*} = \mu^*$. Furthermore, we have the following:

If $\frac{7-4\sqrt{3}}{\alpha} < \frac{p}{r} < \frac{7+4\sqrt{3}}{\alpha}$, we have $\mu^{c*} > \mu^*$.

If $\frac{p}{r} < \frac{7-4\sqrt{3}}{\alpha}$ or $\frac{p}{r} > \frac{7+4\sqrt{3}}{\alpha}$, we have when $\mu^{c*} < \frac{r-p\alpha-\sqrt{p^2\alpha^2+r^2-14p\alpha r}}{6p}$ or $\mu^{c*} > \frac{r-p\alpha+\sqrt{p^2\alpha^2+r^2-14p\alpha r}}{6p}$, then $\mu^{c*} > \mu^*$; when $\frac{r-p\alpha-\sqrt{p^2\alpha^2+r^2-14p\alpha r}}{6p} < \mu^{c*} < \frac{r-p\alpha+\sqrt{p^2\alpha^2+r^2-14p\alpha r}}{6p}$, then $\mu^{c*} < \mu^*$. And in equilibrium, we have $W^c(1) = W^c(2) = W^c$.

We then make some comparative static analyses under the competitive mechanism, and show the relationship between supplier's optimal work rate and the base price, the incentive factor and the disincentive factor in Table 5.

Table 5 comparative static analyses under competition model

$\frac{d}{dr}\mu^{c*}$	$\frac{d}{dr}\mu^{c*} > 0$, i. if $\mu^{c*} < \alpha$ and $16k(\mu^{c*})^3 + 24kr(\mu^{c*})^2 - 8\omega\alpha\mu^{c*} - 6p\mu^{c*} > p\alpha - r$ ii. if $\mu^{c*} > \alpha$ and $16k(\mu^{c*})^3 + 24kr(\mu^{c*})^2 - 8\omega\alpha\mu^{c*} - 6p\mu^{c*} < p\alpha - r$ $\frac{d}{dr}\mu^{c*} < 0$, i. if $\mu^{c*} < \alpha$ and $16k(\mu^{c*})^3 + 24kr(\mu^{c*})^2 - 8\omega\alpha\mu^{c*} - 6p\mu^{c*} < p\alpha - r$ ii. if $\mu^{c*} > \alpha$ and $16k(\mu^{c*})^3 + 24kr(\mu^{c*})^2 - 8\omega\alpha\mu^{c*} - 6p\mu^{c*} > p\alpha - r$
$\frac{d}{dp}\mu^{c*}$	$\frac{d}{dp}\mu^{c*} > 0$, if $16k(\mu^{c*})^3 + 24kr(\mu^{c*})^2 - 8\omega\alpha\mu^{c*} - 6p\mu^{c*} > p\alpha - r$ $\frac{d}{dp}\mu^{c*} < 0$, if $16k(\mu^{c*})^3 + 24kr(\mu^{c*})^2 - 8\omega\alpha\mu^{c*} - 6p\mu^{c*} < p\alpha - r$
$\frac{d}{d\omega}\mu^{c*}$	$\frac{d}{d\omega}\mu^{c*} > 0$, if $16k(\mu^{c*})^3 + 24kr(\mu^{c*})^2 - 8\omega\alpha\mu^{c*} - 6p\mu^{c*} > p\alpha - r$ $\frac{d}{d\omega}\mu^{c*} < 0$, if $16k(\mu^{c*})^3 + 24kr(\mu^{c*})^2 - 8\omega\alpha\mu^{c*} - 6p\mu^{c*} < p\alpha - r$

After both subtasks are completed, the outsourcing activity ends. We can express the manufacturer's profit as follows:

$$\Pi_m^c = A - BE[T] - 2W^c \tag{13}$$

We redefine Eq. 12 as $L(\mu^{c*}, \omega) = 0$. Furthermore, the manufacturer's problem can be re-deemed as:

$$\max_{\mu^{c*},\omega} \Pi_m^c = A - BE[T] - 2W^c$$
$$\text{s.t. } L(\mu^{c*}, \omega) = 0$$

Due to the complexity of Eq. 12, the analytical expression of the optimal price decision made by the manufacturer is hard to determine. Some numerical expression is provided in Section 5. We seek to compare the results among different models with respect to manufacturer's profit function.

4.4 Mixed mechanism

In this case, we consider an incentive mechanism combining the above analyzed deadline and competition incentives. The manufacturer makes a decision of optimal price ω paid to the suppliers, and the suppliers make the optimal work rate decision $\mu_i (i = 1,2)$.

The mixed incentive mechanism manufacturer can offer to the suppliers contains the following four components:

A base price. The manufacturer pays a base price of ω to the suppliers.

A specific due date. The manufacturer sets a pre-determined delivery due date D.

A comparison of the faster and slower. Due to the randomness of project completion times, the result ex post can be distinguished from a faster supplier and a slower supplier.

An incentive/disincentive scheme. The faster supplier will be awards a reward of r per unit time earlier than the deadline; the slower supplier will be charged a penalty of p per unit time later than the deadline.

For supplier i $(i = 1{,}2)$, after his completion of his own subtasks, he receives the pre-agreed base price ω. We can get that his expected payment gained from the manufacturer satisfies:

$$W^m(i) = \omega + rE[(D - X_i)^+ | X_i < X_{-i}] - pE[(X_i - D)^+ | X_i > X_{-i}] \tag{14}$$

Lemma 3. With some algebra calculations, we can get the following:

$$W^m(i) = \omega + rD - r\frac{1}{\mu_i + \mu_{-i}} + r\frac{1}{\mu_i + \mu_{-i}}e^{-(\mu_i + \mu_{-i})D} - p\frac{1}{\mu_{-i}}\left[\frac{\mu_i + \mu_{-i}}{\mu_i}e^{-\mu_i D} - \frac{\mu_i}{\mu_i + \mu_{-i}}e^{-(\mu_i + \mu_{-i})D}\right]$$

See Appendix for the certification process.

Furthermore, supplier i's expected discounted profit can be expressed as follows:

$$\Pi_i^m = W^m(i)E[e^{-\alpha X_i}] - \frac{k\mu_i^2}{\alpha + \mu_i}. \tag{15}$$

Proposition 5. The first-order condition of supplier's profit satisfies that:

$$4k(\mu^{m*})^4 + \Phi(\mu^{m*})^3 - \Psi(\mu^{m*})^2 - \Upsilon\mu^{m*} = r\alpha - r\alpha e^{-2\mu^{m*}D} \tag{16}$$

in which $\Phi = 8k\alpha - 8pDe^{-\mu^{m*}D} + 2pDe^{-2\mu^{m*}D}$,
$\Psi = 4\omega\alpha + 4rD\alpha - 2rDe^{-2\mu^{m*}D} + 8p\alpha De^{-\mu^{m*}D} - 2p\alpha De^{-2\mu^{m*}D} + 4pe^{-\mu^{m*}D} + pe^{-2\mu^{m*}D}$, and
$\Upsilon = 3r - 3re^{-2\mu^{m*}D} - 2rD\alpha e^{-2\mu^{m*}D} - 4p\alpha e^{-\mu^{m*}D} + 3p\alpha e^{-2\mu^{m*}D}$.

The specific certification process is in Appendix.

If $r = p = 0$, we have that $\mu^{m*} = \mu^*$. In equilibrium, we have $W^m(1) = W^m(2) = W^m$. We can infer that if formula Eq. 15 has a maximum value, then supplier's optimal work rate satisfies Eq. 16; otherwise, we can find the maximum value of Eq. 15 by limiting the scope of parameter ω.

Furthermore, we have that the manufacturer's profit satisfies that:

$$\Pi_m^m = A - BE[T] - 2W^m \tag{17}$$

In the next section, we also provide the manufacturer's optimal price decision through some numerical examples. Section 4 provides three kinds of incentive mechanisms to compare with the benchmark case. Deadline, competition, and mixed incentive mechanisms are analyzed and the comparison results are given in the following analysis.

5. Numerical analysis

Due to the complexity of the manufacturer's profit function, we cannot get the analytical expression of manufacturer's optimal price decision under the three proposed incentive mechanism models. We compare the results among different models through some numerical examples.

In our numerical examples, we set $\alpha = 1, k = 1, A = 40$ and $B = 1$. We use some abbreviations to represent different incentive models, i.e., BM is short for base model, DM is short for deadline incentive model, CM is short for competition model and MM short for mixed model.

Fig. 1 shows the results among different models when parameter p varies from 0 to 10 given parameter $D = 0.5$. With different values of parameter r, i.e., $r = 0, 2, 4, 6, 8, 10$, we get six numerical examples, and manufacture's optimal profit among different incentive mechanisms differs under different circumstances.

Observation 1. The comparison results among different incentive models differ concerning different absolute value and relative relationship between parameter r and p. All four mechanisms may be in the domination position under different circumstances.

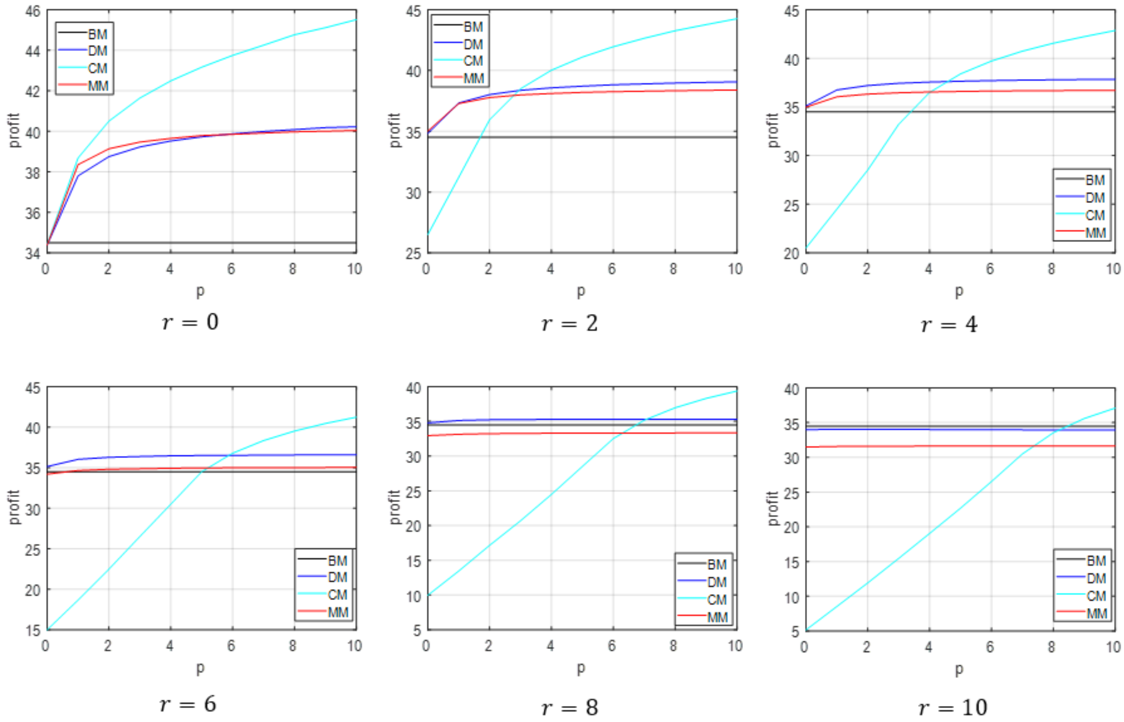

Fig. 1 Numerical examples with varying parameter r with given $D = 0.5$

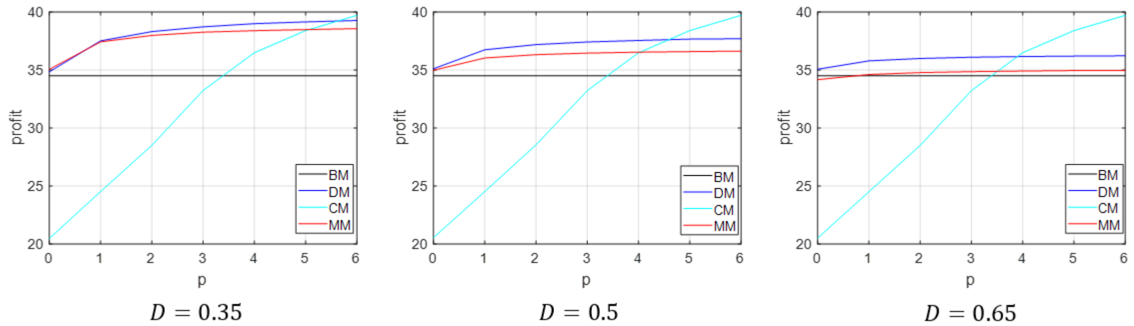

Fig. 2 Numerical examples with varying parameter D with given $r = 4$

We then show the comparison results among different models with a different specific deadline. Fig. 2 gives the specific results with given parameter $r = 4$, and p varies from 0 to 10. By choosing parameter D among 0.35, 0.5 and 0.65, we get three numerical examples.

Observation 2. Given the absolute and relative value of parameters r and p, the influence of specific due date D has a different impact on the comparison results among different incentive models. A smaller D has a same impact on the results with a smaller r when other parameters are the same; a larger D has a same impact on the results with a lager r when other parameters are the same.

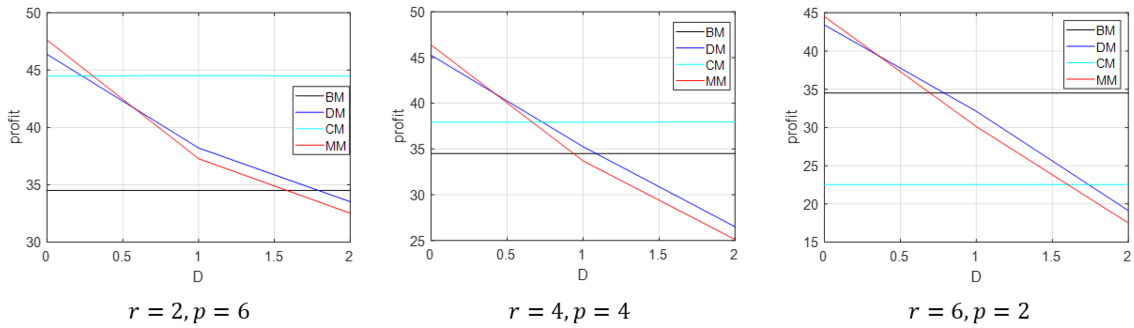

Fig. 3 Numerical examples with varying parameter r and p

Fig. 3 shows the results with varying parameter D from 0 to 2. With 3 different pairs of parameters r and p, we get different numerical results on manufacturer's optimal choice from choosing different incentive mechanisms.

Observation 3. Different combination pairs of parameters r and p has a bigger influence on the performance of incentive model CM. The performance of incentive models DM and MM decreases with the specific due date D. Model MM dominates DM when parameter D is smaller and model DM nominates MM when parameter D is larger.

In this Section, we show the comparative results through some numerical examples, and the meaning results can shed light on some managerial implications for the real-world business practice.

6. Conclusion

We consider an incentive mechanism design problem faced by a manufacturer when conducting outsourcing activities. Three different kinds of incentive mechanisms are proposed in our article. We get some comparative results and management implications for the real-world business practice.

6.1 Findings

We proposed three kinds of incentive mechanisms. The first one is deadline incentive mechanism, in which the manufacturer impose a specific deadline and when the supplier completes earlier than the deadline will be rewarded and will be penalized when completes later than the deadline.

The second incentive mechanism is competition mechanism. In our setting where the manufacturer outsources two parallel subtasks to two different suppliers, the two suppliers may end the corresponding subtask with different duration times due to the stochasticity of the project.

The third incentive mechanism combines the above two mechanisms, which we referred to as mixed incentive mechanism. It considers both the deadline in projects and competitiveness in parallel projects. Only the faster supplier is awarded when earlier than the deadline and only the slower supplier is penalized when later than the deadline.

We find that the results of the comparison of the three mechanisms depend on the related parameters settings, and are all likely to be in a dominant position. We give the specific results through some numerical examples.

We assume that the supplier can get a base price ω in the above analysis, and different base price the manufacturer paid to the supplier is related to different project size. Thus, our findings can be generalized to projects of different magnitudes, which is meaningful to the real-world business practice.

6.2 Future research

There are some extensions that we can continue to work on in the future. We list some possible research schemes in the following content.

In our work, we consider a two parallel sub-projects structure, however, what happens when there are more than two tasks in parallel? It could be a lot more complicated and needs further study by scholars. We can figure that the deadline incentive mechanism can gain the same result as we show in sub-section 4.2. Regarding to competition incentive mechanism, different from two parallel sub-projects structure, when there are n $(n \geq 2)$ tasks in parallel, we can set that the fastest supplier can get a reward and the slowest supplier will get a penalty. The specific incentive result needs our further study. As to the mixed incentive mechanism, we can also consider that the fastest supplier can get a reward if his completion time is earlier than the deadline and vice versa. However, in this case, the modeling process may be very complicated and whether we can gain some insightful results is unclear.

Serial project structure is another direction we can pay attention to. Also, we are wondering that what happens when there are more than two tasks in a general assembly network? Do any of the results reported in this paper continue to hold? When involving network structure, the study will be more complicated and difficult.

The above discussion prompts us to pay more attention to the problems in project supply chain management and try to adopt new methods to solve these problems. In addition, emerging applications in project management will certainly identify further important research questions and opportunities in addition to the issues we discuss here. With the increasing complexity of project supply chain and the increasing subcontracting activities, how to solve this problem is a great challenge. Therefore, it requires the attention and time of our scholars to conduct research and demonstrate some management implications to guide guidance into the real world.

We believe that in the future, scholars will make great progress in academic research in this field, which will further guide the practical business application of project management. In the next several years we will have more and more important results, which are of course the basis for future research.

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References

[1] Boarnet, M.G. (1998). Business losses, transportation damage and the Northridge earthquake, *Journal of Transportation and Statistics*, Vol. 1, No. 2, 49-64, doi: 10.21949/1501575.
[2] Bayiz, M., Corbett, C.J. (2005). Coordination and incentive contracts in project management under asymmetric information, Working paper, UCLA Anderson School, doi: 10.2139/ssrn.914227.
[3] Lindkvist, L., Soderlund, J., Tell, F. (1998). Managing product development projects: On the significance of fountains and deadlines, *Organization Studies*, Vol. 19, No. 6, 931-951, doi: 10.1177/017084069801900602.
[4] Kerzner, H. (2017). *Project management: A systems approach to planning, scheduling, and controlling*, John Wiley & Sons, Hoboken, New Jersey, USA.
[5] Eisenhardt, K.M., Tabrizi, B.N. (1995). Accelerating adaptive processes: Product innovation in the global computer industry, *Administrative Science Quarterly*, Vol. 40, No. 1, 84-110, doi: 10.2307/2393701.
[6] Su, J., Zhang, F., Wang, D., Sindakis, S., Xiao, Y., Herrera-Viedma, E. (2023). Examining the influence of knowledge spillover on partner selection in knowledge Alliances: The role of benefit distribution, *Computers & Industrial Engineering*, Vol. 180, Article No. 109245, doi: 10.1016/j.cie.2023.109245.
[7] Nahmias, S., Olsen, T.L. (2015). *Production and operations analysis*, Seventh Edition, Waveland Press, Long Grove, Illinois, USA.
[8] Hall, N.G. (2016). Research and teaching opportunities in project management, *INFORMS TutORials in Operations Research,* 329-388, doi: 10.1287/educ.2016.0146.
[9] Zhang, J. (2016). Deadlines in product development, *Management Science*, Vol. 62, No. 11, 3310-3326, doi: 10.1287/mnsc.2015.2300.
[10] Bordley, R.F., Keisler, J.M., Logan, T.M. (2019). Managing projects with uncertain deadlines, *European Journal of Operational Research*, Vol. 274, No. 1, 291-302, doi: 10.1016/j.ejor.2018.09.036.
[11] Du, L., Hu, M., Wu, J. (2022). Contingent stimulus in crowdfunding, *Production & Operations Management*, Vol. 31, No. 9, 3543-3559, doi: 10.1111/poms.13782.
[12] Gupta, D., Snir, E.M., Chen, Y. (2015). Contractors' and agency decisions and policy implications in A+B bidding, *Production and Operations Management*, Vol. 24, No. 1, 159-177, doi: 10.1111/poms.12217.
[13] Tang, C.S., Zhang, K., Zhou, S.X. (2015). Incentive contracts for managing a project with uncertain completion time, *Production and Operations Management*, Vol. 24, No. 12, 1945-1954, doi: 10.1111/poms.12387.
[14] Chen, S., Lee, H. (2016). Incentive alignment and coordination of project supply chains, *Management Science*, Vol. 63, No. 4, 1011-1025, doi: 10.1287/mnsc.2015.2373.
[15] Kwon, H.D., Lippman, S.A., Tang, C.S. (2010). Optimal time-based and cost-based coordinated project contracts with unobservable work rates, *International Journal of Production Economics*, Vol. 126, No. 2, 247-254, doi: 10.1016/j.ijpe.2010.03.013.
[16] Zhao, J., Mu, Y. (2022). Managing projects under different payment schemes, In: *Proceedings of the 5th European International Conference on Industrial Engineering and Operations Management,* Rome, Italy, doi: 10.46254/EU05.20220282.
[17] Bergantiños, G., Lorenzo, L. (2019). How to apply penalties to avoid delays in projects, *European Journal of Operational Research*, Vol. 275, No. 2, 608-620, doi: 10.1016/j.ejor.2018.11.056.
[18] Estévez-Fernández, A. (2012). A game theoretical approach to sharing penalties and rewards in projects, *European Journal of Operational Research*, Vol. 216, No. 3, 647-657, doi: 10.1016/j.ejor.2011.08.015.

[19] Chen, T., Klastorin, T., Wagner, M.R. (2015). Incentive contracts in serial stochastic projects, *Manufacturing & Service Operations Management*, Vol. 17, No. 3, 290-301, doi: 10.1287/msom.2015.0528.

[20] Kwon, H.D., Lippman, S.A., McCardle, K., Tang, C.S. (2010). Managing project contracts with delayed payments, *Manufacturing & Service Operations Management,* Vol. 12, No. 4, 692-707, doi: 10.1287/msom.1100.0301.

[21] Song, J.M., Zhao, Y., Xu, X. (2020). Incentives and gaming in collaborative projects under risk-sharing partnerships, *Manufacturing & Service Operations Management*, Vol. 23, No. 2, 453-470, doi: 10.1287/msom.2019.0840.

[22] Dawande, M., Janakiraman, G., Qi, A., Wu, Q. (2019). Optimal incentive contracts in project management, *Production and Operations Management*, Vol. 28, No. 6, 1431-1445, doi: 10.1111/poms.12997.

Appendix

Proof of Proposition 2. By taking the first-order condition of manufacturer's profit function, we have:

$$\frac{d}{d\omega}\Pi_m = -2 + \frac{3B}{2\left(-\alpha+\sqrt{\alpha^2+\frac{\omega\alpha}{k}}\right)^2}\cdot\frac{\alpha}{2k\sqrt{\alpha^2+\frac{\omega\alpha}{k}}}.$$

By taking the second-order condition of Π_m, we can obtain:

$$\frac{d^2}{d\omega^2}\Pi_m = \frac{3B\alpha}{4k}\cdot\frac{-1}{2\left(-\alpha+\sqrt{\alpha^2+\frac{\omega\alpha}{k}}\right)^3}\cdot\left(\frac{\alpha}{2c\sqrt{\alpha^2+\frac{\omega\alpha}{k}}}\right)^2 + \frac{3B\alpha}{4k}\cdot\frac{1}{\left(-\alpha+\sqrt{\alpha^2+\frac{\omega\alpha}{k}}\right)^2}\cdot\frac{-1}{2\left(\alpha^2+\frac{\omega\alpha}{k}\right)^{\frac{3}{2}}}.$$

We can get that $\frac{d^2}{d\omega^2}\Pi_m < 0$. Thus, we can infer that Π_m is a concave function with respect to parameter ω.

Proof of Lemma 1. From $W^d = \omega + rE(D - X_i)^+ - pE(X_i - D)^+$, we can get:

$$W^d = \omega + r\int_0^D (D-t_i)f(t_i)dt_i - p\int_D^\infty (t_i - D)f(t_i)dt_i = \omega + pD - pE(X_i) + (r - p)\left[D\int_0^D f(t_i)dt_i - \int_0^D t_i f(t_i)dt_i\right].$$

Thus, we have $W^d = +rD - r\frac{1}{\mu_i} + (r-p)\frac{1}{\mu_i}e^{-\mu_i D}$, and the lemma is proved.

Proof of Proposition 3. By some calculations, we can obtain:

$$\frac{d}{d\mu_i}\Pi_i^d = \frac{1}{(\alpha+\mu_i)^2}[\omega\alpha + rD\alpha + r - (r-p)e^{-\mu_i D}(1 + D\alpha + D\mu_i) - k(\mu_i)^2 - 2k\alpha\mu_i].$$

$$\frac{d^2}{d(\mu_i)^2}\Pi_i^d = \frac{1}{(\alpha+\mu_i)^3}[(r-p)e^{-\mu_i D}(D\alpha + D\mu_i)D(\alpha+\mu_i) + 2(r-p)e^{-\mu_i D}(1 + D\alpha + D\mu_i) - 2c\alpha^2 - 2(\omega\alpha + rD\alpha + r)].$$

If $r \le p$, we can get that Π_i^d is a concave function. Thus, we can get the result by taking the first order condition of (7).

If $r > p$, for sufficiently large μ_i, we have $\frac{d}{d\mu_i}\Pi_i^d < 0$. And we can infer that $\lim_{\mu_i\to 0}\frac{d}{d\mu_i}\Pi_i^d = \frac{1}{\alpha^2}[\omega\alpha + rD\alpha + r - (r-p)(1 + D\alpha)] = \omega\alpha + p + pD\alpha > 0$.

Therefore, we can infer that there exists a maximum value of the supplier's profit function and the proposition is proved.

Proof of Lemma 2. Let $Z = X_i - X_{-i}$, thus, the probability density function of random variable Z when $Z > 0$ satisfies that

$$f(z) = \int_{t_i=z}^\infty \mu_i e^{-\mu_i t_i}\mu_{-i}e^{-\mu_{-i}(t_i - z)}dt_i = \frac{\mu_i\mu_{-i}}{(\mu_i+\mu_{-i})}e^{-\mu_i z}.$$

Therefore, we can get that

$$E[X_i - X_{-i}]^+ = \int_0^\infty z f(z)dz = \frac{\mu_{-i}}{\mu_i(\mu_i+\mu_{-i})}.$$

Thus, the lemma is proved.

Proof of Proposition 4. From formula Eq. 12, we have

$$\frac{d}{d\mu_i}\Pi_i^c = \frac{1}{(\alpha+\mu_i)^2(\mu_i+\mu_{-i})^2}\left[\omega\alpha(\mu_i+\mu_{-i})^2 + r(\alpha+\mu_i) - r\frac{\mu_i}{\mu_{-i}}(\mu_i+\mu_{-i}) + p\mu_{-i}(\alpha + 2\mu_i + \mu_{-i}) - (k\mu_i^2 + 2k\alpha\mu_i)(\mu_i+\mu_{-i})^2\right].$$

For sufficiently large μ_i, we have $\frac{d}{d\mu_i}\Pi_i^c < 0$. And we can get that $\lim_{\mu_i \to 0} \frac{d}{d\mu_i}\Pi_i^c = \frac{1}{\alpha^2 \mu_{-i}^2}\left[\omega\alpha\mu_{-i}^2 + r\alpha + p\mu_{-i}(\alpha + \mu_{-i})\right] > 0$.

Thus, we can infer that there exists a maximum value of the supplier's profit function and the proposition is proved.

Proof of Lemma 3. The conditional distribution function $\mathrm{F}(X_i < t | X_i < X_{-i}) = \frac{\mathrm{Pro}(X_i<t, X_i<X_{-i})}{\mathrm{Pro}(X_i<X_{-i})}$. We first examine that $\mathrm{Pro}(X_i < t, X_i < X_{-i}) = \int_0^t \mu_i e^{-\mu_i t_i} \int_{t_i}^{\infty} \mu_{-i} e^{-u_{-i} t_{-i}} \, dt_{-i} dt_i = \frac{\mu_i}{\mu_i + \mu_{-i}}\left[1 - e^{-(\mu_i+\mu_{-i})t}\right]$.

By calculation, we can infer that $\mathrm{Pro}(X_i < X_{-i}) = \int_0^{\infty} \mu_{-i} e^{-\mu_{-i} t_{-i}} \int_0^{t_{-i}} \mu_i e^{-\mu_i t_i} \, dt_i dt_{-i} = \frac{\mu_i}{\mu_i+\mu_{-i}}$. Thus, we have $\mathrm{F}(X_i < t | X_i < X_{-i}) = 1 - e^{-(\mu_i+\mu_{-i})t}$.

Thus, we can get that $E[(D - X_i)^+ | X_i < X_{-i}] = (\mu_i + \mu_{-i}) \int_0^D (D - t)\, e^{-(\mu_i+\mu_{-i})t} dt = D + \frac{1}{\mu_i+\mu_{-i}}\left[e^{-(\mu_i+\mu_{-i})D} - 1\right]$.

By the same way, we can get that $\mathrm{F}(X_i < t | X_i > X_{-i}) = \frac{\mathrm{Pro}(X_i<t, X_i>X_{-i})}{\mathrm{Pro}(X_i>X_{-i})}$. Furthermore, we have that $\mathrm{Pro}(X_i < t, X_i > X_{-i}) = \int_0^t \mu_i e^{-\mu_i t_i} \int_0^{t_i} \mu_{-i} e^{-\mu_{-i} t_{-i}} \, dt_{-i} dt_i = \frac{\mu_{-i}}{\mu_i+\mu_{-i}} - e^{-\mu_i t} + \frac{\mu_i}{\mu_i+\mu_{-i}} e^{-(\mu_i+\mu_{-i})t}$, and we can infer that $\mathrm{Pro}(X_i > X_{-i}) = \frac{\mu_{-i}}{\mu_i+\mu_{-i}}$.

Thus, we can get that $E[(X_i - D)^+ | X_i > X_{-i}] = \frac{\mu_i}{\mu_{-i}}(\mu_i + \mu_{-i}) \int_D^{\infty} (t - D)\left[e^{-\mu_i t} - e^{-(\mu_i+\mu_{-i})t}\right] dt = \frac{1}{\mu_{-i}}\left[\frac{\mu_i+\mu_{-i}}{\mu_i} e^{-\mu_i D} - \frac{\mu_i}{\mu_i+\mu_{-i}} e^{-(\mu_i+\mu_{-i})D}\right]$.

Thus, the lemma is proved.

Proof of Proposition 5. From the supplier's profit function, we can get

$$\frac{d}{d\mu_i}\Pi_i^m = \frac{1}{(\alpha+\mu_i)^2(\mu_i+\mu_{-i})^2}\Big\{(\omega\alpha + rD\alpha)(\mu_i + \mu_{-i})^2 - r(\alpha + \mu_i)\left(e^{-(\mu_i+\mu_{-i})D} - 1\right) - r(\mu_i + \mu_{-i})\left(e^{-(\mu_i+\mu_{-i})D} - 1\right) - rD(\alpha + \mu_i)(\mu_i + \mu_{-i})e^{-(\mu_i+\mu_{-i})D} - p\frac{\alpha}{\mu_{-i}}(\mu_i + \mu_{-i})^2\left(\frac{\mu_i+\mu_{-i}}{\widetilde{\mu}_i} e^{-\mu_i D} - \frac{\mu_i}{\mu_i+\mu_{-i}} e^{-(\mu_i+\mu_{-i})D}\right) - p\frac{\mu_i}{\mu_{-i}}(\alpha + \mu_i)(\mu_i + \mu_{-i})^2\left[-D\frac{\mu_i+\mu_{-i}}{\mu_i} e^{-\mu_i D} - \frac{\mu_{-i}}{\mu_i^2} e^{-\mu_i D} + D\frac{\mu_i}{\mu_i+\mu_{-i}} e^{-(\mu_i+\mu_{-i})D} - \frac{\mu_{-i}}{(\mu_i+\mu_{-i})^2} e^{-(\mu_i+\mu_{-i})D}\right] - \left(k\mu_i^2 + 2k\alpha\mu_i\right)(\mu_i + \mu_{-i})^2\Big\}.$$

By letting $\mu_i = \mu_{-i} = \mu^{m*}$, we can get formula Eq. 16.

Thus, the proposition is proved.