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The investigation of production variance in a module-based assembly system: A Markovian Arrival Process approach

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A B S T R A C T

This paper provides an in-depth study of the assembly production variance problem through the largest supplier of General Motors China. We focus on the production variance problem in an unreliable assembly (MOBA) system with a finite inter-station buffer, focusing on two of the central issues, namely the output variance as well as the delivery schedule variance. We model every subsystem's departure procedure in the MOBA system using the Markov Arrival Process (MAP) approach. Through the approximate use of MAP, we successfully shorten the time needed to calculate the output variance as well as delivery schedule variance of a large-scale MOBA system, which improves the efficiency of the system while ensuring that it meets the customer's needs. The relationship between production variance and system parameters is also studied, which is of substantial significance for optimizing the productivity of MOBA systems and improving customer satisfaction.

A R T I C L E   I N F O

Keywords:
Manufacturing;
Assembly;
Production variance;
Markovian arrival process (MAP);
Module-based assembly (MOBA) system

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1. Background

The aim of this paper is to address the variance problem of an unreliable module-based assembly system (MOBA). The main assembly line, which includes M assembly stations and M sub-fabrication lines, is the component that makes up the MOBA system, each supplying modules to one assembly station with a specific fill rate. One assembly operation is performed at each assembly station, which involves attaching a module from a sub-line from a higher stream station to a subassembly of the stream level.

In China, most of suppliers for the car industry share this production mode. To illustrate it, a study was made by us for a famous bumper factory which is the biggest supplier for GM in Yantai City.

The factory uses a typical MOBA system after injection molding and painting. Each assembly line has 5 to 10 assembly stations, as well as sub-fabrication lines according to the parts of the bumpers. In today's business landscape, clients have increasingly high expectations for shorter delivery times. Given the fast-paced nature of the industry, we feel it is crucial to build models that analyze the variability in output or delivery schedule of MOBA systems. This will assist factories in effectively managing fluctuations in production.

Some interrupts such as set-up and module-changing were not included in this study since these conditions are totally under control in manufacturing process with a fixed time and procedure.

2. Introduction

The field of analytical modeling of manufacturing systems has been the subject of significant research and interest since the early 1960s. Dallery and Gershwin's [1], and Papadopoulos and Heavey's [2] contributions to this field are notable. Most of those research examine manufacturing systems in a condition of equilibrium, which provide certain performance metrics such as the mean production rate and the mean buffer levels.

The existing body of literature mostly concentrates on the analysis of production variation in manufacturing systems that employ serial production lines. The variability in output of manufacturing lines equipped with buffers between stations has been examined by Miltenberg [3], Martinčević and Kozina [4], Carrascosa [5], and Tan [6, 7]. Miltenburg [3] provided a technique for calculating the asymptotic variance of the output per unit time. This approach utilizes the outcomes derived for the asymptotic averages as well as variance of the overall duration of stay in Markov chains. Martinčević and Kozina [4] proposed a technique for calculating the variability of the output from a single machine during a certain time frame. This approach relies on calculating the derivative functions for the likelihood of generating n components at a specific moment, and subsequently solving these equations by including certain boundary equations. Gershwin further suggested a decomposition technique for calculating the output variation in extended manufacturing lines. Carrascosa [5] expanded upon Gershwin's approach as well as provided extensive numerical and simulation findings that investigate the impact of system factors on the output variance. Tan [6, 7] determined the rate of asymptotic variation for production lines with restricted buffers by using the averages as well as variances of Markov incentives systems. In terms of the amount of operations, his technique outperforms Miltenburg's by a factor of a thousand. In his work on serial production lines without inter-station buffers, Tan [8, 9] discussed the output variance and found closed-form equations for the asymptotic variance of the output. Tan [10], and Behmanesh and Rahimi [11] investigated the delivery schedule variation of serial manufacturing lines. For a solitary workstation characterized by foreseeable processing times and unforeseeable downtimes, a predetermined lot size was determined by Kim and Alden [12] by analytically approximating the density function as well as the variance of the period. The number of commodities produced in time t, denoted as $N(t)$, and its asymptotic normalcy, was used by Tan [10] to estimate the delivery schedule variance. Tan [13] addressed the output variance of series-parallel production systems with no inter-station buffers. This study is an extension of Tan [8]. El Abbadi *et al.* [14] and Marinas *et al.* [15] studied production variance in serial manufacturing lines with unreliable machines in a Bernoulli reliability case.

The current research on production variation might not be enough for real-world applications, according to these evaluations. In addition, with the exception of Tan's work on series-parallel systems [13], all of the systems discussed in the aforementioned literatures are serial systems. The unreliability of a module-based assembly (MOBA) system is the subject of this paper's discussion of production variation. MOBA systems have become widely adopted in the modern manufacturing sector to achieve efficient production and adaptability. They combine agile manufacturing principles to optimize both product-focused and process-focused production modes. Several concerns must be considered while designing and controlling such systems, including variations in production, along steady-state throughput, and average buffer level. In this study, we introduced a novel and efficient technique known as the MAP approximation-based compression method.

The MAP is a helpful framework of math for modeling point processes having irregular, non-Markovian dynamics or a unique pattern. Notable instances are Neuts [16, 17], Lucantoni [18, 19], Montoro-Cazorla and Pérez-Ocón [20], Visagan and Ganesh [21], and Fan [22].

Because of the intricate nature and interconnections of MOBA systems, it is quite challenging to assess the variability in production of these systems. The MAP estimation method in this study offers a foundation for a viable strategy to assess production variation in complicated and unreliable assembly systems. This work examines two important issues related to production variance: (i) the issue of output variance; and (ii) the issue of delivery schedule variance.

The remaining portion of this paper is organized in the following manner. Section 2 provides a description of the MOBA system. Section 3 discusses the estimation using the MAP method used to estimate the production variance. In Section 4, two deviations for the production are estimated. Numerical examples are examined in Section 5. Lastly, we provide some last comments in Section 6.

3. Model description

This section provides a description of a MOBA system, as seen in Fig. 1.

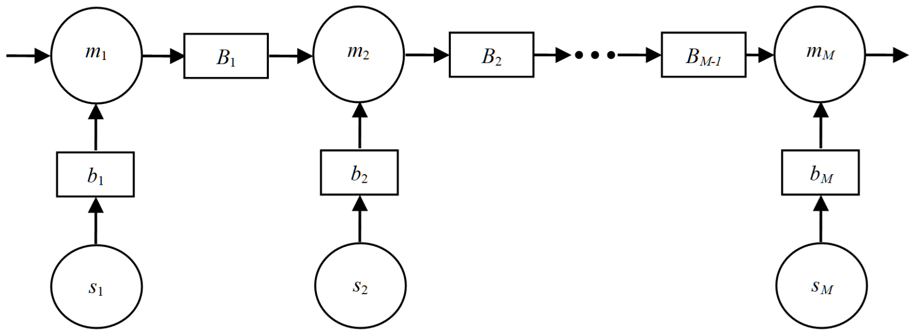

Fig. 1 A MOBA system

The MOBA system has a primary production lane equipped with M stations for assembly and M lines for sub-fabrication. Every assembly station carries out a single assembly operation by extracting a module from its sub-fabrication line and joining it with a subassembly obtained from the upstream station. The primary assembly line has *M1* inter-station buffers. The following assumptions define the assembly stations and the inter-station buffers.

- The assembly time it takes to assemble station m_i is distributed exponentially at a rate of μ_i, where i = 1, … , M. The assembly times of the assembly stations are not dependent on each other.
- When it comes to assembly activities, every single assembly station is prone to erratic failures. The duration till failure and the duration until repair for the station m_i follow exponential distributions with rates α_i and β_i, respectively, for i = 1, … , M.
- The inter-station buffer B_i has a capacity to store N_i components, where N_i is a positive integer more than or equal to 1 and less than infinity, for i = 1, … , *M1*.
- If assembly station mi completes work on a component while inter-station buffer B_{i-1} is empty, station m_i will not receive any new parts until m_i finishes a part, for i = 2, … , M. The first station is always well-supplied, as there is an endless amount of basic assembly available.
- If assembly station m_i completes work on a part while inter-station buffer B_i is at maximum capacity, station m_i is unable to proceed until an empty buffer space becomes vacant at B_i for i = 1, … , M–1. The final station remains unimpeded, as there is an endless capacity for transporting completed goods.

In an actual MOBA system, every single subsidiary manufacturing line may have several stations for either machining or assembly. To keep things simple, we suppose it is a subsidiary manufacturing line is made up of a solitary processing machine as well as a supply buffer designed to store the outputs generated by the processing machine. The processing machines and supply buffers are defined by the following assumptions.

- The sub-fabrication lines are managed separately to achieve rate of module filling θ_i at assembly station m_i, where $0 \le \theta_i \le 1$ for $i = 1, \dots, M$.
- When there is a shortage of stock in the buffer of supplies b_i, the time it takes for the next module to become available follows an exponential distribution with a rate of δ_i, where i ranges from 1 to M. In other words, there is an infinite supply of raw materials and components for the sub-fabrication line, as well as the machine's processing time s_i follows an exponential pattern.

4. The MAP approximation

Within this part, we initially present the effective assembly times. Based on that, the procedure by which every subsystem leaves the MOBA system can be modeled as a MAP. Then we describe Diamond and Alfa's [23] discussion on MAP approximation, which is used in our study.

Random failures and maintenance of assembly stations, and also fill rates and delays at sub-fabrication lines, have an impact on assembly operations in a MOBA system. The duration of station's effective assembly m_i is defined as the duration between the availability of an item of work or a station m_i and the commencement of an operation at station m_i, provided that a module may be accessed via the supply buffer. It includes the time needed to complete a procedure that involves the assembling of the workpiece.

The efficient assembly duration of the station m_i is proved by Liu *et al.* [24] to follow a PH-distribution with the representation(τ_i, T_i) ,which is given by

$$\tau_i = (1-\theta_i, \theta_i, 0,0), T_i = \begin{pmatrix} T_i' - \alpha_i I_2 & \alpha_i I_2 \\ \beta_i I_2 & -\beta_i I_2 \end{pmatrix} \tag{1}$$

where I_2 is an identity matrix of order 2,$T_i' = \begin{pmatrix} -\delta_i & \delta_i \\ 0 & -\mu_i \end{pmatrix}$, for $i = 1, \dots, M$.

4.1 The MAP structure of a MOBA system

A MAP, or Markovian Arrival Process, is a type of Markov process that may be represented by a map of size m. The transitions in this process are categorized based on whether they result in an arrival or not. The symbol m refers to the order of the MAP. The associated rates are divided into two matrices E and F of size m. Matrix F has only nonnegative components, whereas matrix E comprises negative diagonal members and nonnegative off-diagonal elements. The infinitesimal generator Q, which is the sum of matrices E and F, is indivisible. Therefore, the Markov chain Q exhibits positive recurrence. The matrix that determines the likelihood of the system's phase transitioning the information on the time between two consecutive arrival epochs is given as $P = -E^{-1}F$. Let η be the stationary probability vector of P. It is clear that $\eta P = \eta$.

The MOBA system may be studied by examining the MAP of each subsystem's departure procedure, which is based on the PH-distribution of the effective assembly time of assembly stations. The information is explained as follows.

Subsystem 1 shown in Fig. 2 is a PH/PH/1/N_1 queue. Its leaving procedure may be represented as a MAP_1 of order $(N_1 + 1)\times 16$ with the matrix identifier (E_1, F_1), determined by

$$E_1 = \begin{bmatrix} T_1 \otimes I_4 & T_1^0 \tau_1 \otimes I_4 & \square & \square & \square \\ \square & T_1 \oplus T_2 & T_1^0 \tau_1 \otimes I_4 & \square & \square \\ \square & \square & \ddots & \ddots & \square \\ \square & \square & \square & T_1 \oplus T_2 & T_1^0 \tau_1 \otimes I_4 \\ \square & \square & \square & \square & I_4 \otimes T_2 \end{bmatrix} \tag{2}$$

$$F_1 = \begin{bmatrix} 0 & \square & \square & \square \\ I_4 \otimes T_2^0 \tau_2 & 0 & \square & \square \\ \square & \ddots & \ddots & \square \\ \square & \square & I_4 \otimes T_2^0 \tau_2 & 0 \end{bmatrix} \tag{3}$$

where I_4 is identity matrix of order 4, $T_i^0 = -T_i e$ for i = 1 ,..., M and e is the column vector of ones, the operator ⊗ denotes the Kronecker product, see Diamond *et al.* [23], the operator ⊕ is defined as $A \oplus B = A \otimes I + I \otimes B$.

A MAP order reduction to 2 is required to make this paper's technique work for a big MOBA system. As a result, a MAP_1* of order 2 with the matrix identifier should be used to approximate the MAP_1 of higher order; Section 4.2 will provide further information on this.

As depicted in Fig. 2, subsystem 2 regards the MAP_1* as input. It may be referred to as a MAP/PH/1/N_2 queue, where the process of departure is a newly constructed MAP_2 that includes the matrix identifier (E_2^*, F_2^*), as follows:

$$E_2 = \begin{bmatrix} E_1^* \otimes I_4 & F_1^* \otimes I_4 & \square & \square & \square \\ \square & E_1^* \oplus T_3 & F_1^* \otimes I_4 & \square & \square \\ \square & \square & \ddots & \ddots & \square \\ \square & \square & \square & E_1^* \oplus T_3 & F_1^* \otimes I_4 \\ \square & \square & \square & \square & I_2 \otimes T_3 \end{bmatrix} \tag{4}$$

$$F_2 = \begin{bmatrix} 0 & \square & \square & \square \\ I_2 \otimes T_3^0 \tau_3 & 0 & \square & \square \\ \square & \ddots & \ddots & \square \\ \square & \square & I_2 \otimes T_3^0 \tau_3 & 0 \end{bmatrix}, \tag{5}$$

where I_2 is identity matrix of order 2.

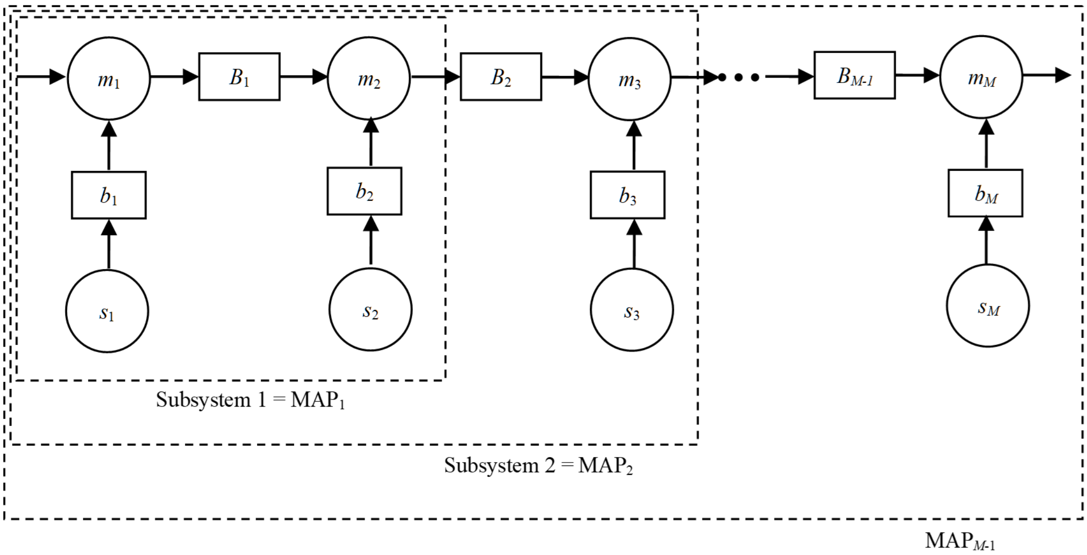

Fig. 2 The MAP approximation of a MOBA system

The MAP_2's order is 8 times the product of N_2 and 1. The estimation of MAP_2 involves the utilization of a second-order MAP_2* in conjunction with the matrix descriptor (E_2^*, F_2^*), following a analogous discourse to that over the MAP_1. Continuing in this manner, it is evident that the ultimate system may clearly be described by a MAP_{M-1} using the matrix identifier (E_{M-1}, F_{M-1}), as follows:

$$E_{M-1} = \begin{bmatrix} E_{M-2}^* \otimes I_4 & F_{M-2}^* \otimes I_4 & \square & \square & \square \\ \square & E_{M-2}^* \oplus T_M & F_{M-2}^* \otimes I_4 & \square & \square \\ \square & \square & \ddots & \ddots & \square \\ \square & \square & \square & E_{M-2}^* \oplus T_M & F_{M-2}^* \otimes I_4 \\ \square & \square & \square & \square & I_2 \otimes T_M \end{bmatrix}, \tag{6}$$

and

$$F_{M-1} = \begin{bmatrix} 0 & \square & \square & \square \\ I_2 \otimes T_M^0 \tau_M & 0 & \square & \square \\ \square & \ddots & \ddots & \square \\ \square & \square & I_2 \otimes T_M^0 \tau_M & 0 \end{bmatrix} \tag{7}$$

The order of MAP_{M-1} is $8 \times (N_{M-1} + 1)$.

4.2 Higher order MAP approximation using MAP of order 2

In a MOBA system with several stations and substantial buffer capacity between them, the resultant MAP grows exponentially in order. It is required to approximate all MAPs of higher order, such MAP_1, MAP_2, … , MAP_{M-2}, to those of order 2, as mentioned in Section 4.1. Keep in mind that Diamond and Alfa [23] offered this approximation model.

5. Variance of output and delivery schedule

Two major issues with production variance are discussed here. The output variance as well as the delivery schedule variance are these. They may be calculated using two efficient techniques that are also provided.

Define O_{t1} as the number of arrivals in the interval [0, t] for the Markovian Arrival Process (MAP) with the matrix descriptor (E, F). According to Andersen and Nielsen [25], we have

$$Var[O_{t1}] = (\lambda^* - 2\lambda^{*2} + 2\varphi F(e\varphi - Q)^{-1}Fe)t - 2\varphi F(I - exp\{Qt\})(e\varphi - Q)^{-2}Fe \tag{8}$$

the vector φ irepresents the stationary probabilities of the irreducible infinitesimal generator Q = $E + F$, $\lambda^* = \varphi Fe$ is the stationary arrival rate of the MAP.

The stationary sequence of the arrival times between events is denoted by X_n, and $D_{n1} = \sum_{i=1}^{n} X_i$. Utilizing Andersen and Nielsen [25], we get

$$\begin{aligned} Var[D_{n1}] &= (\frac{2}{\lambda^*}\varphi(I + E^{-1}F + e\varphi)^{-1}(-E)^{-1}e - \frac{1}{\lambda^{*2}})n \\ &-\frac{2}{\lambda^*}\varphi(I - (-E^{-1}F)^n)(I + E^{-1}F + e\varphi)^{-2}(-E^{-1}F)(-E)^{-1}e \end{aligned} \tag{9}$$

$Var[O_{t1}]$ and $Var[D_{n1}]$ represent the output variance as well as the delivery schedule variance. It is important to acknowledge both Eq. 10 and Eq. 11 can produce valuable outcomes when time t and part number n approach infinity.

Because $\lim_{t\to+\infty} exp\{Qt\} = e\varphi$, it follows from Eq. 8 that

$$\lim_{t\to+\infty}(Var[O_{t1}]/t) = \lambda^* - 2\lambda^{*2} + 2\varphi F(e\varphi - Q)^{-1}Fe \tag{10}$$

and as $\lim_{t\to+\infty}(E^{-1}F)^n = e\varphi$, it follows from Eq. 9 that

$$\lim_{t\to+\infty}(Var[D_{n1}]/n) = \frac{2}{\lambda^*}\varphi(I + E^{-1}F + e\varphi)^{-1}(-E)^{-1}e - \frac{1}{\lambda^{*2}} \tag{11}$$

The asymptotic variance rate of O_{t1}, as t approaches infinity, refers to the maximum rate at which the output variance may change over time. $Var[O_{t1}]/t$, as defined by Tan [9]. Fig. 3 demonstrates the computational methods for calculating the production variance, as outlined in Eqs. 8 and 9.

Fig. 3 may be utilized to examine the production variation through the calculation of the output variance as well as delivery schedule variance utilizing the following two algorithms (see Appendix A).

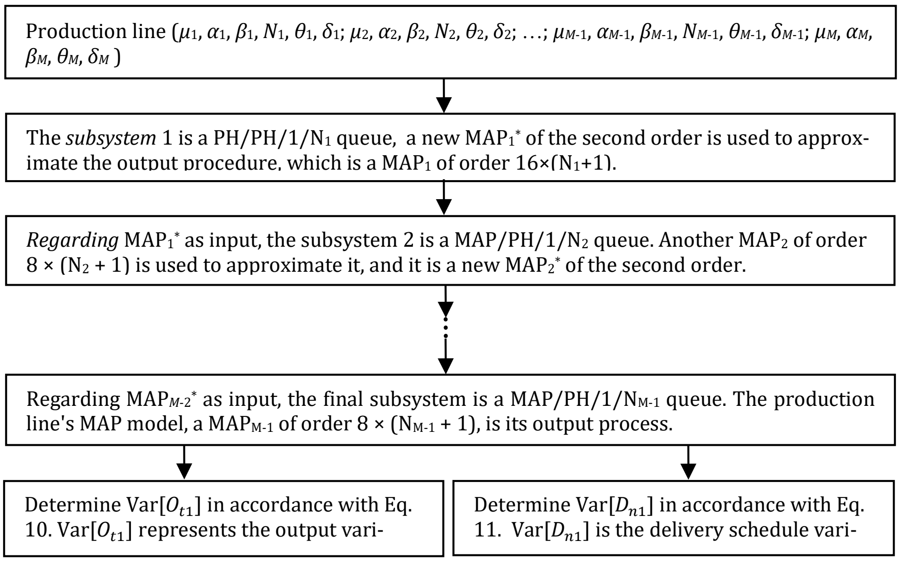

Fig. 3 Computing the output variance and delivery schedule variance

6. Numerical examples

The approach provided in the preceding section may be used to assess a broad spectrum of MOBA systems with finite inter-station buffers. In this section, an comparison between approximation and simulation results will be made to state the accuracy of this method. And then some numerical experiments show that this approach is computationally very efficient. And, some basic observations are also given to show how the production variance is influenced by system parameters. Finally, some discuss about the limit of this approach will be held.

6.1 Accuracy of the approximation

Diamond and Alfa [23] has proved the MAP approximation on higher order is acceptable and has a better performance than renewal approximation by using simulation method. In our case, the Figs. 4 and 5 depict the approximation versus simulation results. A five-station MOBA system with a certain set of system parameters is studied, where μ = 2.125, α = 0.135, β = 1.452, θ = 0.61, δ = 2.031, N = 5. All the data are obtained from the bumper factory. Curves are used to demonstrate the approximation result while the solid point as the simulation ones, as seen in the figure, the estimation yields satisfactory outcomes.

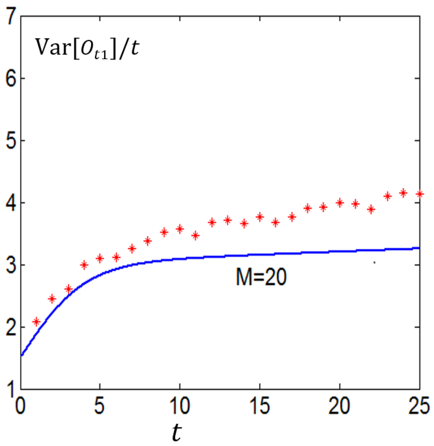

Fig. 4 Simulation and analytical approximation of output variance (Curve as approximation; point as simulation)

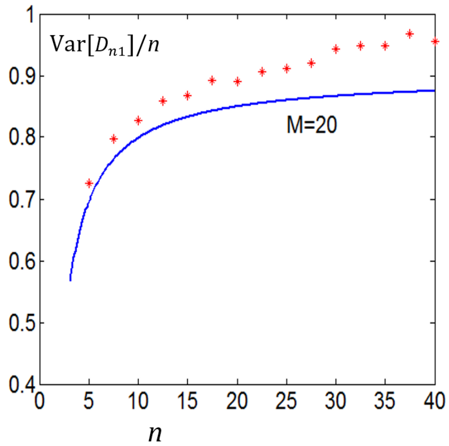

Fig. 5 Simulation and analytical approximation of delivery time (Curve as approximation; point as simulation)

6.2 Numerical examples for two/three-station MOBA system

First we investigate the relationship between the two production variances and the inter-station buffer capacity. A two-station MOBA system with two different sets of system parameters is studied, where μ_1 = 2.125, α_1 = 0.135, β_1 = 1.452, θ_1 = 0.61, δ_1 = 2.031, μ_2 = 3.725, α_2 = 0.231, β_2 = 2.524, θ_2 = 0.91, δ_2 = 1.215 in system 1 and μ_1 = 3.125, α_1 = 0.135, β_1 = 1.452, θ_1 = 0.8, δ_1 = 2.725, μ_2 = 6.725, α_2 = 0.081, β_2 = 2.024, θ_2 = 0.8, δ_2 = 2.326 in system 2. This two system are used for different bumpers. Figs. 6 and 7 show the output variance over time for various buffer capacity. It has been demonstrated that, in general, the output variance over time grows as the buffer capacity increases in both systems. Nevertheless, it is not feasible to establish a strong correlation between the delivery schedule variance as well as the capacity of the buffer. Figs. 8 and 9 show a clear correlation between the variability in delivery schedule and the size of the buffer for both systems.

Secondly, the effect of the assembly station's failure rate and repair rate on the production variance is studied. For a MOBA system with three identical assembly stations and sub-fabrication lines where μ = 2.726, β = 2.426, θ = 0.8, δ = 1.823 and N = 3 for both inter-station buffers, Figs. 10 and 11 illustrate the relationship between the output variation for each time period as well as the delivery schedule variance for each item, with respect to the failure rate. It's shown that in this system, as the failure rate increases, the delivery schedule variance for each individual item increases, while output variance per unit time increases first and then decreases. For this three-station MOBA system, when failure rate of each assembly station is fixed as α = 0.715, it can be seen from Figs. 12 and 13 that as the repair rate increases, the delivery schedule variance per unit part diminishes, because the rate of change in output variance increases with time, and this is dependent on the variable t, may fall somewhat in the middle.

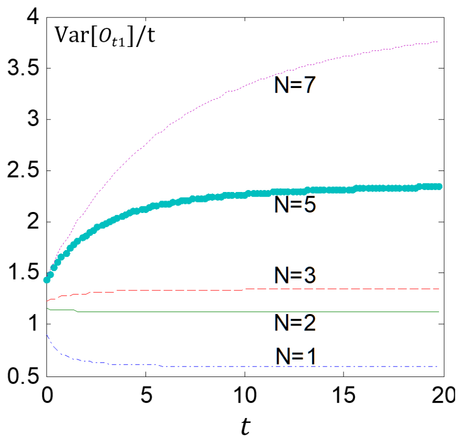

Fig. 6 Impact of buffer capacity on output variance in system 1

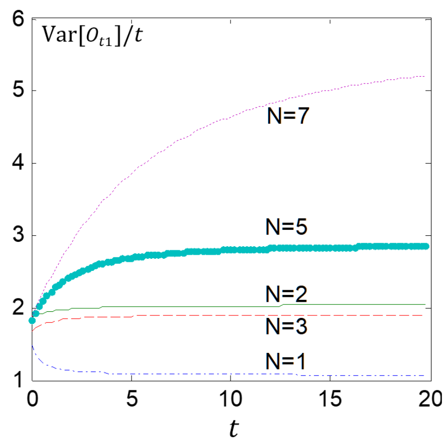

Fig. 7 Impact of buffer capacity on output variance in system 2

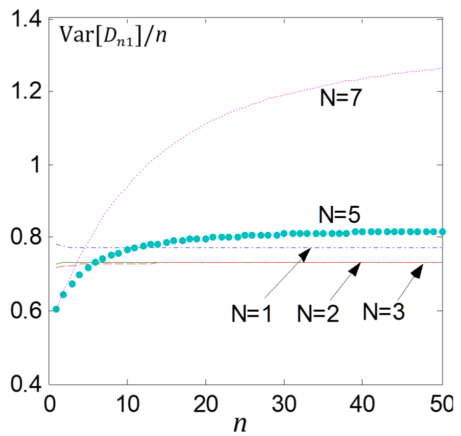

Fig. 8 Impact of buffer capacity on delivery time variance in system 1

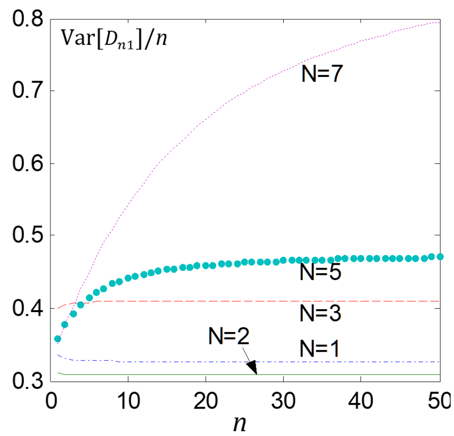

Fig. 9 Impact of buffer capacity on delivery time variance in system 2

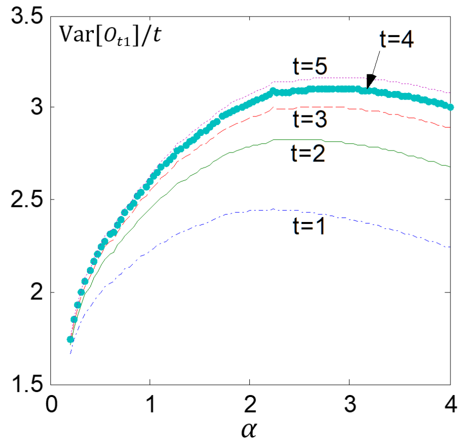

Fig. 10 Impact of failure rate on the output variance for a three-station MOBA system

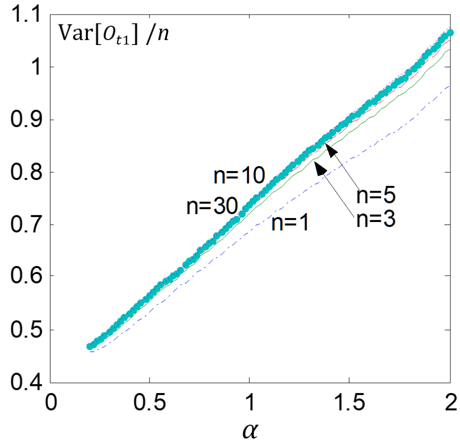

Fig. 11 Impact of failure rate on the delivery time variance for a three-station MOBA system

This observation may inspire the factory an efficient way to control the delivery schedule variance, that is, put in more effort on increasing the repair rate when it is in a lower level at the very beginning, and then focus on decreasing failure rate when the repair rate stays in a steady level. As for the output variance, failure rate may play a minor role after a certain level, but the increasing effect is quite obvious when it's in a lower level.

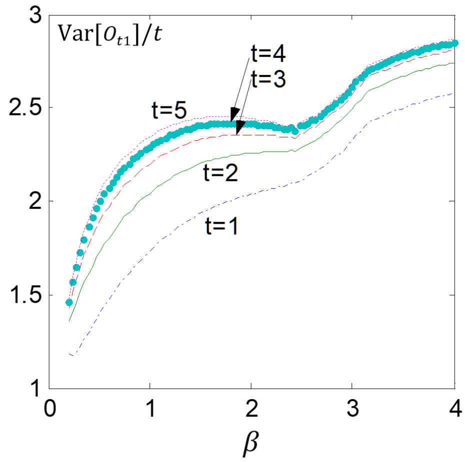

Fig. 12 Impact of repair rate on the output variance for a three-station MOBA system

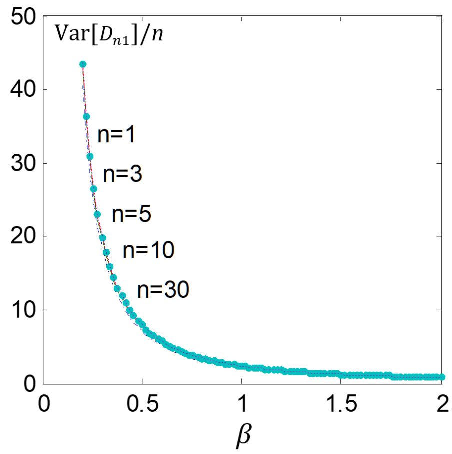

Fig. 13 Impact of repair rate on the delivery time variance for a three-station MOBA system

Thirdly, we investigate the effect of the module fill rate and the delay rate of sub-fabrication line on the production variance. A three-station MOBA system with μ = 2.726, α = 0.715, β = 2.426 for each assembly station and N = 3 for each inter-station buffer is studied. With each sub-fabrication line having a delay rate of δ = 1.823, Figs. 14 and 15 show the output variance for each time period as well as the delivery schedule variance for each item as a function of the module fill rate for this MOBA system. It can be seen that as the module fill rate increases from 0.6 to 1, while the delivery schedule variance per unit component grows first and subsequently drops, the output variance for each time period falls initially as well as then increases. Figs. 16 and 17 depict the effect of the sub-fabrication line delay rate on the output variance per unit time as well as the delivery schedule variance for each item for this MOBA system when the module fill rate of each sub-fabrication line is θ = 0.9. It's shown that as the delay rate of sub-fabrication line increases, the delivery schedule variance per unit component reduces, while the output variance for each time period decreases first, then increases and decreases again finally.

Some rules can be found from the Figs. 14-17. The trend of each variance is not monotonous as the module fill rate changes, and the effects are even opposite which means a balance point between them. By contrast, the effects of sub-fabrication line delay rate has a great influence on the delivery schedule variance at lower level which suggests the operators to increase it to a proper state as soon as possible.

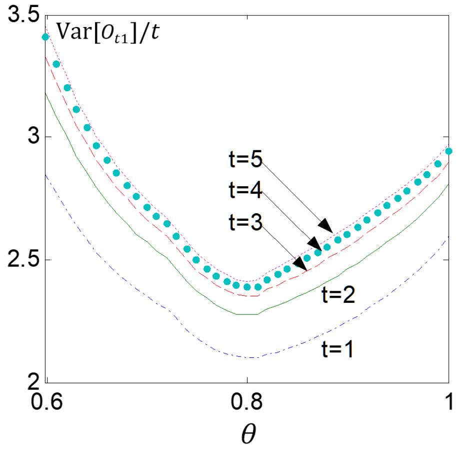

Fig. 14 Impact of module fill rate on the output variance for a three-station MOBA system

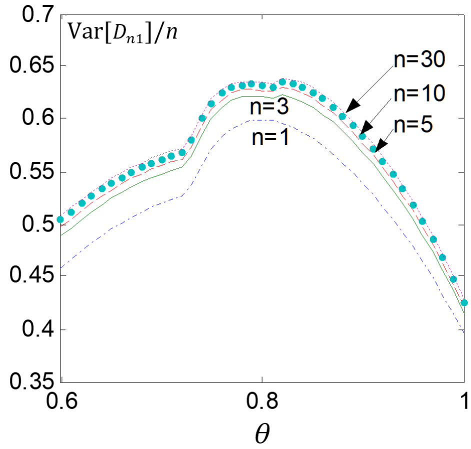

Fig. 15 Impact of module fill rate on the delivery-time variance for a three-station MOBA system

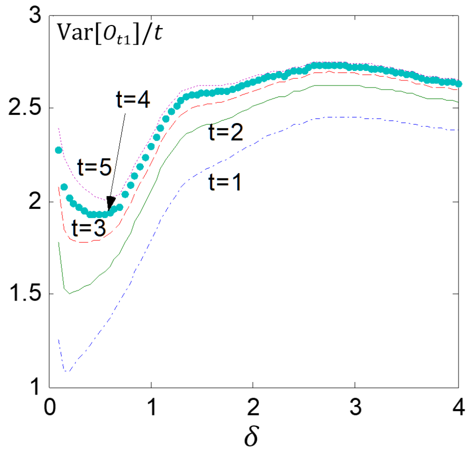

Fig. 16 Impact of sub-fabrication line delay rate on the output variance for a three-station MOBA system

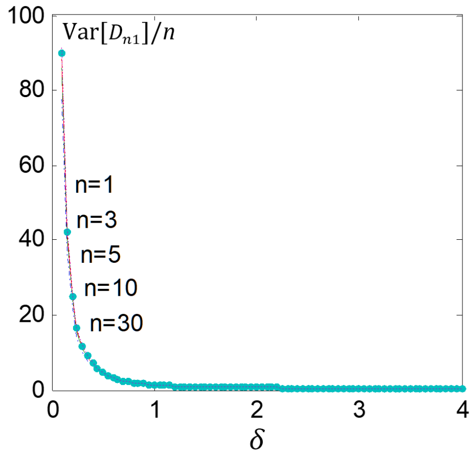

Fig. 17 Impact of sub-fabrication line delay rate on the delivery-time variance for a three-station MOBA system

6.3 Discuss about the limit of this method

The approach outlined in this research is executed via the use of software. Due to its reliance on recursion, the approach exhibits a high level of computing efficiency.

To explain the limitation of this strategy, we analyzed how the length of the line affected the production variance. We studied a MOBA system of 2, 4, 6, 10, 80 identical stations with μ = 2.726, α = 0.715, β = 2.426, θ = 0.8, δ = 1.823 and capacities for buffering between stations of N = 4. It's shown in Figs. 16 and 17, as the line length increases, output variance during each time period falls as delivery schedule variance per unit of component rises. In fact, very few assembly line can hold more than 80 stations which means this method can be used widely from a practical point of view.

Seen from the Figs. 8 and 9, the variance increases rapidly when the buffer capacity larger than 7 for both the output and the delivery schedule. Although our method can deal with a much larger buffer capacity, the result will be unacceptable for the factory.

This method for calculating production variance is quite effective. To compute the output variance as well as delivery schedule variance for big MOBA systems, it provides the findings instantly. By utilizing the MAP approximation on a per-subsystem basis, we can prevent a fast growth in the number of states as the number of stations and inter-station buffer capacity increase. Therefore, the overall computational effort for the production variance of the MOBA system is kept within a reasonable range.

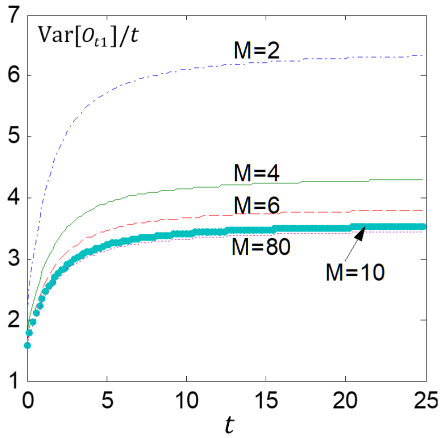

Fig. 18 Impact of line length on the output variance of MOBA system

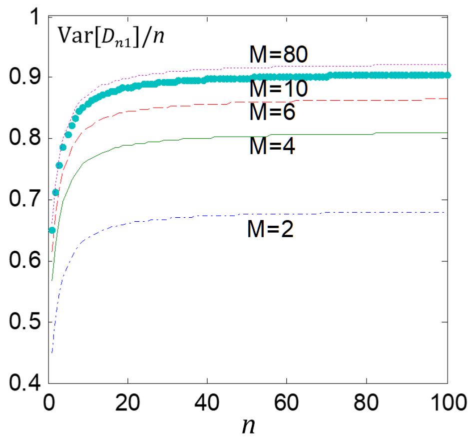

Fig. 19 Impact of line length on the delivery-time variance of MOBA system

7. Concluding remarks

The article covered the topic of production variance in MOBA systems that aren't trustworthy. For an unstable MOBA system with finite inter-station buffers, an efficient method is suggested to find the output variance as well as the delivery schedule variance using the MAP approximation. Using this method, the number of states in the manufacturing line's subsystems is drastically reduced. Therefore, the overall system's computational effort is kept within a tolerable range. When dealing with big MOBA systems, the numerical examples show that this method works. Additionally, numerical experiments are used to study the correlations between the system characteristics and the production variance. As indicated in section 1, it's known that study on production variance of assembly systems is imperfect. It is thought that this study would address a significant lack in the existing research, and the MAP approximation method discussed in this work offers a foundation for a potentially effective way to evaluating production variation in intricate and unreliable assembly systems.

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Appendix A

In this appendix we come up with two algorithms to calculate the output variance as well as due-time variance of production variance.

Algorithm 1. Calculation of the output variance.

INPUT: The parameters μ_1, α_1, β_1, N_1, θ_1, δ_1; μ_2, α_2, β_2, N_2, θ_2, δ_2; …; $\mu_{M\text{-}1}$, $\alpha_{M\text{-}1}$, $\beta_{M\text{-}1}$, $N_{M\text{-}1}$, $\theta_{M\text{-}1}$, $\delta_{M\text{-}1}$; μ_M, α_M, β_M, θ_M, δ_M.
OUTPUT: The output variance Var[O_{t1}].
Calculation:

Calculate the matrix identifier (E_1, F_1) of MAP$_1$ in the first step.
Calculate the matrix identifier (E_1^*, F_1^*) of the MAP$_1$* of the second order in the second step.
Calculate the matrix identifier (E_2, F_2) of MAP$_2$ in the third step.
Calculate the matrix identifier (E_2^*, F_2^*) of the MAP$_2$* of the second order in the fourth step.
Continuously calculate the matrix identifier ($E_{M\text{-}1}$, $F_{M\text{-}1}$) of the MAP $_{M\text{-}1}$ in the fifth step.
Calculate the variance of O_{t1} in the sixth step.

Algorithm 2. Calculation of the due-time variance.

INPUT: The parameters μ_1, α_1, β_1, N_1, θ_1, δ_1; μ_2, α_2, β_2, N_2, θ_2, δ_2; …; $\mu_{M\text{-}1}$, $\alpha_{M\text{-}1}$, $\beta_{M\text{-}1}$, $N_{M\text{-}1}$, $\theta_{M\text{-}1}$, $\delta_{M\text{-}1}$; μ_M, α_M, β_M, θ_M, δ_M.
OUTPUT: The due-time variance Var[D_{n1}].
Calculation:

Calculate the matrix identifier (E_1, F_1) of MAP$_1$ in the first step.
Calculate the matrix identifier (E_1^*, F_1^*) of the MAP$_1$* of the second order in the second step.
Calculate the matrix identifier (E_2, F_2) of MAP$_2$ in the third step.
Calculate the matrix identifier (E_2^*, F_2^*) of the MAP$_2$* of the second order in the fourth step.
Continuously calculate the matrix identifier ($E_{M\text{-}1}$, $F_{M\text{-}1}$) of the MAP $_{M\text{-}1}$ in the fifth step.
Calculate the variance of D_{n1} in the sixth step.