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Integrated optimization of line planning and timetabling on high-speed railway network considering cross-line operation

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A B S T R A C T

With the implementation of cross-line operation in high-speed railway system, accessibility of cross-line passengers on the railway network has substantially improved. However, due to limitations of capacity, it is hard to schedule a conflict-free timetable based on a train line plan with many cross-line trains. In order to generate a feasible and optimal transportation plan, a novel methodology is introduced. The approach can simultaneously optimize both the line plans of cross-line trains and train timetable, aiming at having a trade-off between operating profit and direct service. Based on an event-activity network framework, a mixed integer programming model is established. Considering service quality would decline after optimizing line plan and train schedules, the objective of the model is set to minimize deviations from ideal schedules for main-line trains while maximize direct service frequency for cross-line passengers. To solve large-scale scenarios efficiently, an enhanced heuristic genetic algorithm is developed. Two smaller-scale cases are devised to validate the efficiency of the model and approach. Finally, the model and the algorithm are applied to a real-world scenario involving the Beijing-Shanghai High-speed Railway and its connecting lines. Also, comparative experiments, including a scenario without cross-line optimization, are conducted to evaluate the advantage of the proposed approach. The result shows the approach can help to quickly find a feasible solution and have a good balance between operating profit and passenger demand.

A R T I C L E I N F O

Keywords:
Railway network;
Optimization;
Train line planning;
Timetable scheduling;
Cross-line operation;
Passenger demand;
Origin-destination direct service frequency;
Genetic algorithm

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1. Introduction

High-speed railways have garnered increasing attention because of their advantages of speed, safety, convenience, and punctuality. With the large-scale construction of railway infrastructure and the continuous growth of high-speed railway passenger flow, the number of high-speed trains operating on the railway network is increased year by year. To meet the long-distance direct demands of passengers, railway departments need to operate cross-line trains with origin-destination stations (OD) on two different railways. However, this makes railway operators in a dilemma. On the one hand, because of the long-distance causing the small adjustment time domain of departure time at origin station, when many cross-line trains are operated, the travel time domain of main-line trains will be occupied by cross-line trains, and thus it is hard to operate trains and would have an effect on travel speed of main-line trains, and decrease railway line capacity. On the other hand, significantly reducing the number of cross-line trains makes it difficult to meet the direct travel needs of cross-line passengers, causing a large number of cross-line passengers transfer at transfer stations. Therefore, it is particularly important to optimize both train line plan and train timetable of main-line and cross-line trains comprehensively to adequately meet direct cross-line passenger demand. The problem is difficult but must be solved for well railway operation.

In this paper, we proposed a methodology to solve the problem of integration optimization train line plan planning and timetabling. Fig. 1 shows the framework of the methodology in this paper. In the model, the initial train line plan and ideal timetable provide input parameters. A timetable with ideal traveling times and ideal dwell times is termed an ideal timetable. In an ideal timetable, trains may violate track capacity constraints [1]. Because there are conflicts existing in the ideal timetable, we want to optimize train line plan for cross-line trains and the timetable to eliminate conflicts. The objectives are set to both minimize adjustment time and maximize OD direct frequency. In the methodology, we adjust origin-destination stations of some cross-line trains to change their line plan, but this will reduce some OD direct service frequency of cross-line passenger flow. Therefore, we add some short-distance trains to maintain service levels , allowing passengers to transfer between trains at stations where the line plan is changed. Based on the new line plan, we then optimize train arrival and departure times at stations considering basic train operation constraints. After several iterations, a satisfying result can be obtained.

The contribution of our paper can be summarized in three points. Firstly, we adopt a mixed integer programming model based on event-activity network to formulate train line plan and timetable collaboratively considering cross-line operation. This model will change train line plan of cross-line trains at some stations and optimize train timetable. Secondly, we develop a genetic algorithm to solve a very-large-scale problem. We can obtain an optimized train line plan and train timetable during each iteration. Thirdly, case studies based on the high-speed railway network composed of Beijing-Shanghai high-speed railway and its connecting lines are designed by using the proposed model and algorithm.

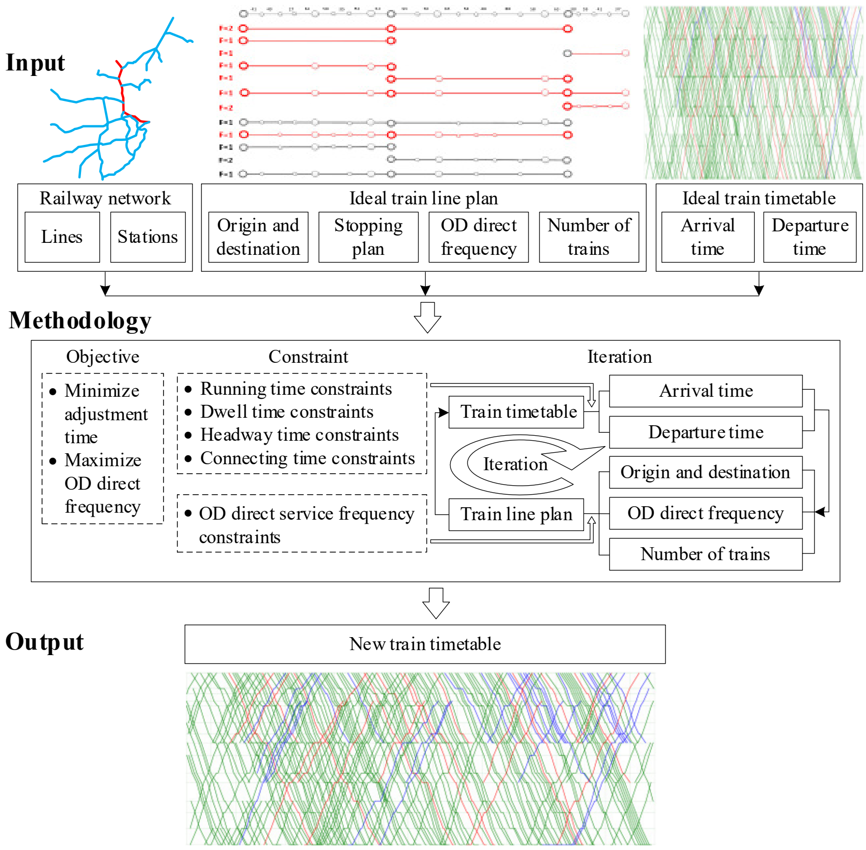


Fig. 1 Framework of integrated optimization methodology of train line plan planning and timetable scheduling

The structure of our paper is organized as follows. In the next section, we provide a summary of related work. Section 3 introduces a mixed integer programming model based on an event-activity network to describe the question. In Section 4, we develop a heuristic genetic algorithm to address the problem. Section 5 presents tests and case studies by taking Beijing-Shanghai high-speed railway and its connecting lines as an example to validate our formulas and algorithm. Finally, discussions and conclusions will be found int Section 6.

2. Literature review

2.1 Cross-line operation

With the construction of high-speed railway, the integrated operation of high-speed trains has become an inevitable trend. Many researchers studied the transportation organization modes for cross-line passengers. Wang *et al*. [2] analysed transportation modes of cross-line passenger flow, and put forward a combination mode with trains running on each line and passenger transfer. Because there were many off-line trains on Wuhan-Guangzhou high-speed corridor, which would have a great impact on capacity, Lei *et al*. [3] analysed the transportation organization mode of off-line trains on main line, and discovered that when off-line trains constitute 65 % of the traffic, capacity is most affected.

However, there were only a few researchers studying cross-line train organization problems, and most of which is metro research. Yang *et al*. [4] studied the cross-line train delays in urban rail transit, analysed cross-line train connection, and developed an optimization model aimed at curtailing delay times. Yang *et al*. [5] introduced a mixed integer non-linear programming model with the primary objective of maximizing passenger travel time while minimizing operation costs to optimize cross-line and main-line train planning, such as frequency, operation zone and stopping pattern. Still other researchers did some research on train transportation organization of main line with taking cross-line trains into consideration. Peng *et al*. [6] considered cross-line train timetable when scheduling trains, and put forward a train scheduling model aiming at minimizing train departure and arrival deviation time. Zhan *et al*. [7] focused on high-speed train stop planning when occurring a major disruption, and proposed a mixed integer programming model aimed at mitigating the frequency of train cancellations and deviations. In case study, the authors also considered the situation of cross-line trains.

2.2 Train line planning and timetable scheduling

There are two fundamental challenges in railway operation, that are train line planning operating at a strategic level, and timetable scheduling operating at a tactical level. Train line plan is the input of the train timetable, and it has a direct impact on which trains passengers choose to take. Chang *et al*. [8] devised a model with multi-objectives aimed at minimizing both the operation cost and the running time loss to solve the optimal allocation problem of high-speed train line plan. Goossens *et al*. [9] introduced a stop optimization model minimizing passenger travel time for solving line planning problems, allowing for the selection of the optimized stop combination from a predetermined stop pattern set. To meet the predicted passenger transport demand, Fu *et al*. [10] proposed a bi-level optimization model for determining line plans with the purpose of minimizing passengers' travel time at upper level while maximizing served passengers at lower level. They develop two consecutive stages heuristic algorithms corresponding to each classification. Parbo *et al*. [11] devised a bi-level programming model to present the skip-stop dilemma, and presented a heuristic approach suitable for large-scale networks. Shang *et al*. [12] introduced a skip-stop strategy, devised a space-time-state framework predicated upon multiple commodity problems, and used Lagrangian relaxation algorithm to dissect the primary problem into several subproblems.

Train timetable not only gives railway operators scheduled paths and times, but also provides different travel schemes for passengers. Therefore, a high-quality timetable can provide better service to passengers and generate more profits for the operators. Zhang *et al*. [13] introduced a minimum cycle time calculation model rooted in the PESP framework for macro-level capacity assessment and train scheduling, Additionally, they devised an iterative approximation technique aimed at enhancing solution efficiency. Gong *et al*. [14] examined the train timetabling problem accounting for the dynamics and randomness of passenger demand, and formulated a model with the aim of minimizing the expected service cost. Zhang *et al*. [15] devised a model minimizing total train travel time to solve integrated optimization of both maintenance planning and train timetabling. Aken *et al*. [16] considered the challenge of train timetable adjustments by developing a mathematical model. They applied three network aggregation techniques and introduced flexible short-turning possibilities to solve large-scale problems. Cacchiani *et al*. [17] presented a model and designed a heuristic iterative algorithm to obtain an optimal train timetable.

Usually, these two problems are solved successively; however, in recent years, many researchers shifted their focus towards the integrated optimization of train line planning problem and timetabling problem. In order to optimize both train departure schedules and stopping patterns , Yue *et al*. [18] established a mixed integer linear model rooted in a time-space network, aiming to maximize train profits, and proposed a column generation algorithm to effectively resolve this model. Yan *et al*. [19] proposed two models including a line planning model and a timetabling model respectively, and designed a combined method with taking travel time and timetable robustness into consideration. These two models could work iteratively under the designed feedback constraints. Meng *et al*. [20] introduced an integrated team-based model to determine train stopping pattern and timetable, which was aimed at maximize the transport profit and focused on passenger responses to service intervals, stop plans, train arrival/departure times, and the infrastructure capacity. Dong *et al*. [21] established a mixed-integer nonlinear model aimed at optimizing both stop plan and timetable comprehensively. The model's objective was to minimize train running time, waiting time and delay time. To solve the model efficiently, they devised an extended adaptive large-scale neighbourhood search algorithm. To accurately compute passenger waiting times and optimize train timetable, Niu *et al*. [22] developed a timetable optimization model under predetermined skip-stop pattern and OD passenger demand. Jiang *et al*. [23] proposed a method by adjusting train dwell time and train stop plan to increase more scheduled trains on a line. To address this problem, they introduced a Lagrangian-based heuristic algorithm. Cacchiani *et al*. [24] explored the integrated issue of train timetabling and stopping pattern under uncertain demand, and established mixed integer linear models to obtain resilient solutions. Yang *et al*. [25] introduced a collaborative optimization approach for addressing timetabling and stop planning, and establish a mathematical model with multi-objectives to minimize both dwell time and delays. Burggraeve *et al*. [26] designed an iterative method to integrate timetabling and line planning from scratch with considering passenger robustness, and the goal of this method is to minimize operator cost and travel time, and from which they could generate a robust timetable.

2.3 Summary

Table 1 compares previous studies on integration optimization of timetabling and line planning with our research within some key dimensions, such as objective function, cross-line operation or not, considering OD direct service frequency or not, solution algorithm, case size. The comparison reveals that numerous scholars have delved into the amalgamation of train line planning and timetabling. However, there is none of previous researches studying on cross-line operation, and only one of them considering OD direct service frequency but not cross-line passenger flow's neither. Therefore, our research is innovative, very important and necessary.

Table 1 Overview of contemporary research on the integration of line planning and timetable scheduling

Study	Objective	CLO	ODDF	Solution algorithm	Case size
Yue *et al*. (2016)	Maximize train profit	No	Yes	Column generation heuristic algorithm	1 line and 220 trains
Yan *et al*. (2019)	Minimize travel time, empty-seat-hour and the number of lines and overtakings, maximize timetable robustness	No	No	Gurobi	1 line and 25 trains within 3 hours
Meng *et al*. (2019)	Maximize total transporting profit	No	No	Lagrangian relaxation solution	1 line and 100 trains

Table 1 (Continuation)

Dong *et al.* (2020)	Minimize passenger waiting time, delays and running time	No	No	Adaptive large neighborhood search metaheuristics	1 line and 36 trains
Niu *et al.* (2015)	Minimize passenger waiting time	No	No	GAMS	1 line and 73 trains
Jiang *et al.* (2017)	Maximize the total profit	No	No	Heuristic algorithm	1line and 346 trains
Cacchiani *et al.* (2020)	Minimize train travel times	No	No	CPLEX	1 line and 36 trains
Yang *et al.* (2016)	Minimizing dwell time and delay time	No	No	CPLEX	line and 96 trains
Burggraeve *et al.* (2017)	Minimize operator cost and passenger travel time	No	No	Heuristic algorithm	Railway network with 7 line and 30 trains
This paper	Minimize adjustment time of main-line trains, maximize OD direct service frequency of cross-line passenger flow	Yes	Yes	Heuristic algorithm based on genetic algorithm	Railway network with 9 high-speed rail lines and 184 trains

Note: CLO = Cross-line operation; ODDF = OD direct service frequency;

3. Mathematical formulas

3.1 Problem statement

Our investigation focuses on a double-track high-speed rail network, and trains traversing in one direction entirely independent from trains operating in opposite direction. Therefore, we only consider train operation in one direction. In addition, we assume that there are adequate number of tracks in stations which can be used by trains to have stops.

Fig. 2 shows a simple high-speed railway network, which consists of 2 rail lines and 5 stations. Line 1 comprises four stations, while Line 2 consists of two stations. Station C is the connecting station of these two lines. 6 trains run on this railway network, including 2 main-line trains (T1 and T2) running from station A to station D and 4 cross-line trains (T3, T4, T5 and T6) running from A to E. The initial line plan and ideal train timetable of these simple network are as shown in Fig. 3. In ideal timetable, each train is operated at the time where the maximum benefit can be obtained. Because of this, usually there may exist many conflicts in an ideal timetable. In Fig. 3, it is obvious that there are two headway conflicts. Because of small adjustment time domain of long-distance cross-line trains and tough constraints between trains, it is hard to only adjust less arrival and departure times to make timetable feasible. Fig. 4 shows an example. In Fig. 4, all train paths are rescheduled except train T3's.

Therefore, we proposed an innovative methodology to decrease the repercussions of cross-line operation on main-line trains. Initially, our methodology, the origin or destination stations of some cross-line trains are changed. Then, some short-distance trains are added in the slot of timetable to make up the lost of train service. Finally, the new timetable is rescheduled. And Fig. 5 shows the changed train lien plan and rescheduled train timetable of the small network. We change the destination station of train T3 from station E to station B and add a new train T7 to execute the lost train line plan. Now the railway network accommodates seven operational trains, including 3 main-line trains (T1, T2 and T3) and 4 cross-line trains (T4, T5, T6 and T7). In the changed train line plan, cross-line passenger flow from station A to station D can take train T3 and transfer to train T5 or train T7 to accomplish their travel. In the rescheduled train timetable, only train path of T1 is rescheduled, and train T7 is added on the slot of the timetable.

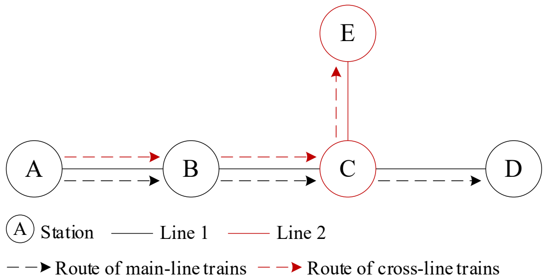


Fig. 2 A simple high-speed railway network

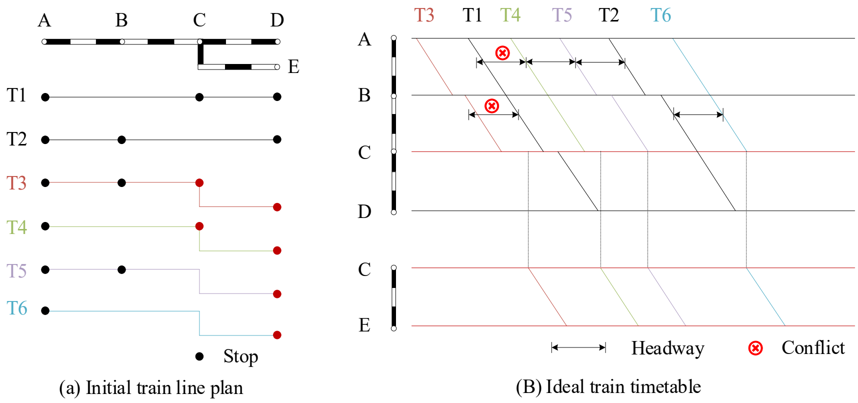


Fig. 3 Initial train line plan and ideal train timetable of the simple network

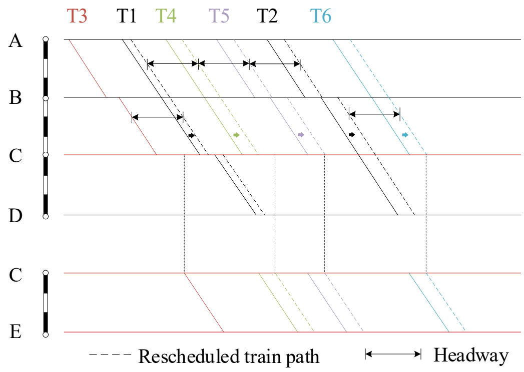


Fig. 4 An example for only adjusting arrival and departure times

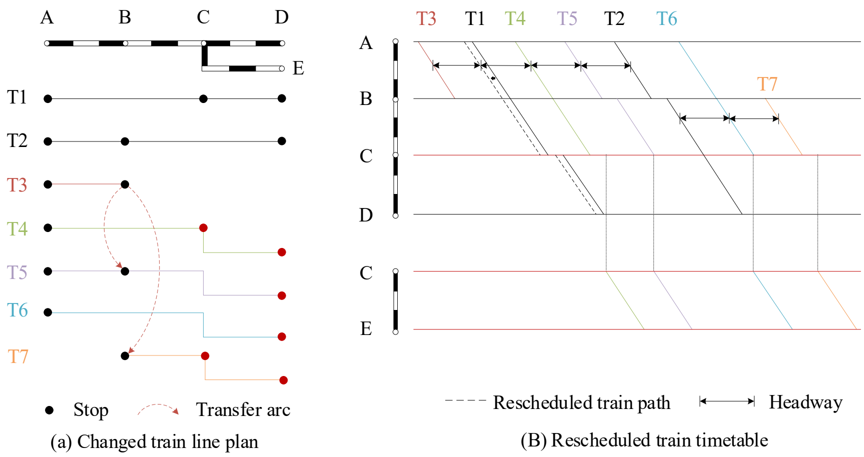


Fig. 5 Changed train line plan and rescheduled train timetable of the simple network

After the change of train line plan of cross-line trains, OD direct service frequency of cross-line passenger flow is decreased, causing passenger direct service demand cannot be well satisfied. For example, in the simple network, the initial direct service frequency from A to E and from B to E in Fig. 4 are 4 and 2, respectively. After applying the proposed methodology and changing the train line plan, we can find that in Fig. 5, these two direct service frequencies become 3 and 2, and passengers who travel from station A to station E can choose to take train T3 and transfer to train T5 or T7 at station B to accomplish their travel. Therefore, the proposed methodology should not only eliminate conflicts of timetable, but also decrease OD direct service frequency of passenger flow as less as possible.

3.2 Event-activity network

The proposed methodology is modelled on event-activity network. In the event-activity model $G = (E, A)$, events are regarded as nodes and activities are directed arcs from one event to another. In our problem of train operation organization, an event i is either a train departure event E_{dep} or a train arrival event E_{arr}, which is related to train index t_i, station index s_i, and scheduled time x_i; while an activity connecting two events is modelled as a constraint, and there are many types of activities, including train trip activities A_{trip}, train dwell activities A_{dwell}, train headway activities A_{head} and train connection activities A_{con}, which are constructed as follows.

a. $A_{trip} = \{(i,j) | i \in E_{dep}, j \in E_{arr}, t_i = t_j, s_i = s_j - 1\}$. A train trip activity is characterized a trip of one train leaving from an upstream station to next station.

b. $A_{dwell} = \{(i,j) | i \in E_{arr}, j \in E_{dep}, t_i = t_j, s_i = s_j\}$. A train dwell activity is described a train stop at a station.

c. $A_{head} = \{(i,j) | i \in E_{dep}(E_{arr}), j \in E_{dep}(E_{arr}), t_i \neq t_j, s_i = s_j\}$. A train headway activity is described a departure or arrival interval between two different trains departing or arriving at stations.

d. $A_{con} = \{(i,j) | i \in E_{arr}, j \in E_{dep}, t_i \neq t_j, s_i = s_j\}$. A train connection activity is described a connection of two different trains connecting at the connection station where passengers can transfer from one train to the other train.

Fig. 6 illustrates the event-activity graph of some trains in Fig. 5.

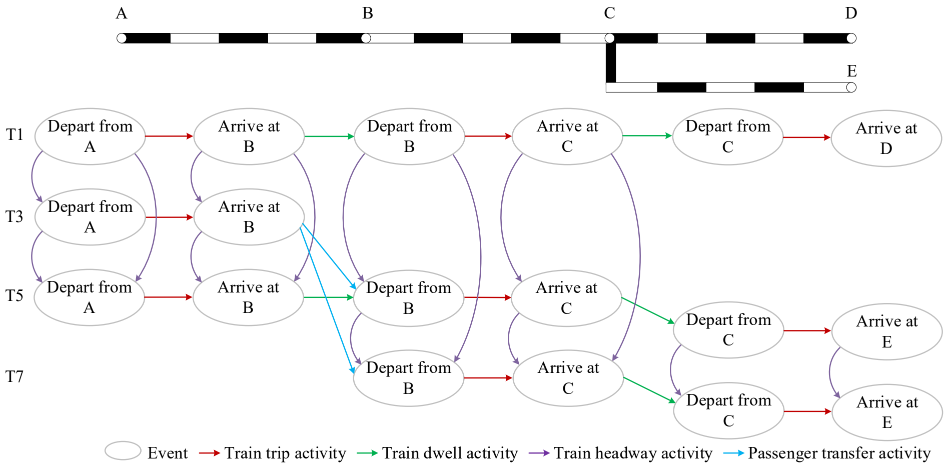


Fig. 6 Event-activity graph of some trains in Fig. 5

3.3 Notations

Table 2 presents indices, sets, parameters and variables integral to the proposed model.

Table 2 Definition of indices, sets, parameters and variables

Notation	Definition
Index	
i, j, i', j', k	Event index, $i, j, i', j', k \in E$
e	Activities index, $e = (i, j) \in A$
t_i	Train index, which denotes the train associated with event i, $t_i \in T$
s_i	Station index, which denotes the station associated with event i
Set	
E	Set of events, $E = E_{arr} \cup E_{dep}$
A	Set of activities, $A = A_{trip} \cup A_{dwell} \cup A_{head} \cup A_{con}$
T	Set of trains, $T = T_m \cup T_c$
T_m	Set of main-line trains
T_c	Set of cross-line trains
S_m	Set of stations on main line
S_i	Set of stations on cross lines where cross-line train t_i passes through
Parameter	
ori_i	Origin station on main-line of train t_i
des_i	Destination station on main-line of train t_i
ε	The maximum adjustable time of cross line trains at origin station
π_i	The scheduled departure and arrival time instant of main-line train t_i at station s_i in the ideal train timetable
c_i	The ideal time instant when cross-line train t_i departs at origin station s_i
λ_i	1 or 0, indicating whether train t_i stops or passes at station s_i
M	A sufficiently large positive integer
$trip_e^{min}$	The lower limit of running time for trip activity e, $e \in A_{trip}$
$trip_e^{max}$	The upper limit of running time for trip activity e, $e \in A_{trip}$
$dwell_e^{min}$	The lower limit of dwell time for dwell activity e, $e \in A_{dwell}$
$dwell_e^{max}$	The upper limit of dwell time for dwell activity e, $e \in A_{dwell}$
con_e^{min}	The lower limit of connecting time for connection activity e, $e \in A_{con}$
con_e^{max}	The upper limit of connecting time for connection activity e, $e \in A_{con}$
h_e	The minimum headway time of headway activity e, $e \in A_{head}$
td_i	The additional starting time for train t_i departing from station s_i
ta_i	The additional stopping time for train t_i arriving at station s_i
tws	The starting time instant of maintenance time
twe	The ending time instant of maintenance time
ξ	The discount of OD direct frequency of cross-line passenger flow
α, β	Weights of two sub-objectives
Variable	
x_i	The time instant when event i occurs in the revised train timetable, namely, when train t_i departs or arrives at station s_i in the new train timetable
ad_i	The adjustment of ideal event i, $i \in E$
θ_{ij}	1 or 0, indicating whether event i precedes or succeeds event j in the revised train timetable
ρ_i	1 or 0, indicating whether origin or destination station of cross-line train t_i was changed to station s_i

3.4 Model formulation

The mixed integer programming model has 2 part of objectives. In order to decrease the effect of cross-line operation on main-line trains, the first objective is to minimize adjustments to departure and arrival times of main-line trains, which ensures the obtained new timetable of main-line trains deviates from the ideal train timetable as little as possible, as expressed in Eq. 1. Moreover, altering the line plan of cross-line trains may lead to a reduction in the direct service frequency for cross-line passenger flow. To satisfy travel demand of cross-line passenger flow, the second objective is to maximize the OD direct service frequency after the adjustment of line plan of cross-line trains, as expressed in Eq. 2.

$$min\ z_1 = \sum_{i \in E, t_i \in T_m} ad_i \tag{1}$$

$$max\ z_2 = \sum_{t_i=t_j\in T_c, s_i\in S_m, s_j\in S_j} \lambda_i \cdot \lambda_j \cdot \left(1 - \sum_{t_i=t_k, s_k\in S_m} \rho_k\right) \tag{2}$$

We give different weights to these two sub-objectives and transform them into one objective, as expressed in Eq. 3.

$$min\ z = \alpha \cdot z_1 - \beta \cdot z_2 \tag{3}$$

There are some constraints must be considered, which are listed in Eqs. 4-15, including train timetable scheduling constraints and train line planning constrains.

(1) Train traveling time constraints between two neighboring stations

Traveling time for a train journey between two neighboring stations is not a certain interval but a flexible time ranging from $trip_e^{min}$ to $trip_e^{max}$, which is expressed in Eq. 4.

$$\begin{gathered} trip_e^{min} + \lambda_i \cdot td_i + \lambda_j \cdot ta_j \leq x_j - x_i \leq trip_e^{max} + \lambda_i \cdot td_i + \lambda_j \cdot ta_j \\ \forall e = (i,j) \in A_{trip}, i,j \in E \end{gathered} \tag{4}$$

(2) Train dwell time constraints at stations

Trains are mandated to pause at stations as designated by the line plan, that is $\lambda_i = 1$. Besides, train dwell time is also flexible. The constraints are expressed in Eq. 5.

$$dwell_e^{min} \leq x_j - x_i \leq dwell_e^{max}, \forall e = (i,j) \in A_{dwell}, i,j \in E \tag{5}$$

(3) Train headway time constraints between two arbitrary trains

The minimum headway can not only ensure safety operation of all trains, but also save occupation time of timetable and improve railway line capacity to some degree. The constraints is expressed in Eq. 6 and Eq. 7. In addition, trains cannot be overtaken by other trains on sections, but only at station, which is expressed in Eq. 8.

$$x_j - x_i + M \cdot (1 - \theta_{ij}) \geq h_e, \forall e = (i,j) \in A_{head}, i,j \in E \tag{6}$$

$$x_i - x_j + M \cdot \theta_{ij} \geq h_e, \forall e = (i,j) \in A_{head}, i,j \in E \tag{7}$$

$$\theta_{ij} = \theta_{i'j'}, \forall (i,j), (i',j') \in A_{head}, (i,i'), (j,j') \in A_{trip} \tag{8}$$

(4) Train connecting time constraints

If origin or destination station of one cross-line train is changed, some cross-line passenger flow necessitates transferring from one train to another train. Train connecting time constraints describe passenger transfer situations, as expressed in Eq. 9. At the same time, the new origin or destination station of the cross-line train should coincide with a stop on the original train's route, and the count of times a cross-line train changing its origin or destination station should less than 1, which is expressed in Eq. 10 and Eq. 11.

$$con_e^{min} \leq x_j - x_i \leq con_e^{max}, \forall e = (i,j) \in A_{con}, i,j \in E \tag{9}$$

$$\rho_i \leq \lambda_i, \forall t_i \in T_c, s_i \in S_m \tag{10}$$

$$\sum_{s_i} \rho_i \leq 1, \forall t_i \in T_c, s_i \in S_m \tag{11}$$

(5) Adjustment constraints of ideal schedules

The Adjustment constraints record the magnitude of left or right shifts ad_i of every main-line train event i. Meanwhile, we should guarantee the time instants of cross-line train departure event i at origin station within a certain range. These constraints are expressed in Eqs. 12 and 13.

$$ad_i \geq |x_i - \pi_i|, \forall t_i \in T_m, i \in E \tag{12}$$

$$\pi_i - \varepsilon \leq x_i \leq \pi_i + \varepsilon, \forall t_i \in T_c, s_i = ori_i \tag{13}$$

(6) OD direct service frequency constraints of cross-line passenger flow

The OD direct service frequency of cross-line passenger flow would be decreased after changing train line plan of cross-line trains, therefore, to satisfy travel demand, the model should ensure the OD direct service frequency of cross-line passenger flow above a certain level, so these constraints can be expressed in Eq. 14.

$$\sum_{t_i,s_i,s_j} \lambda_i \cdot \lambda_j \cdot \left(1 - \sum_{s_k} \rho_k\right) \geq \xi \cdot \sum_{t_i,s_i,s_j} \lambda_i \cdot \lambda_j \,, \forall t_i = t_j = t_k \in T_c, s_i \in S_m, s_j \in S_j, s_k \in S_m \tag{14}$$

(7) Maintenance time constraints

In China, there is a fixed maintenance time on high-speed railway every night. During this time, trains are forbidden to operate on rail lines. Therefore, any event i should not appear during maintenance time, as expressed in Eq. 15.

$$twe \leq x_i \leq tws, \forall i \in E \tag{15}$$

4. Solution approach

In the proposed model, the decision variables include time instants of events, adjustment of events, sequence of events and train line plan of cross-line trains. The integration optimization of timetabling and line planning is very complex, because it is a significant challenge to acquire a feasible initial solution which could meet the proposed constraints. Hence, devising an efficient optimization algorithm holds paramount importance [27-29]. In order to solve large-scale and complex rail network problem, we introduce a heuristic genetic algorithm (GA). The GA has a flexible chromosome encoding mechanism, which has a significant advantage in solving such problems and can achieve better results within a very short time [30-33].

The algorithm proceeds through the following steps.

Step 1: Input data. Read the information of railway network, initial line plan and ideal train timetable.

Step 2: Encoding and population initialization. Each gene in the chromosome represents the time instant of departure event i for train t_i at origin station ori_i, and the chromosome consists of n genes, where n denotes the total number of trains including main-line trains and cross-line trains. All genes are decimal coded. The population size is very important, which is usually depends on the size of problem. After generating chromosomes, feasibility of each chromosome should be checked.

Step 3: Fitness function and reproduction selection. The fitness function is defined as the reciprocal of objective function. Roulette wheel algorithm is employed for selecting new chromosomes. The chromosome with higher fitness is more likely to be chosen.

Step 4: Crossover and mutation. The single-point crossover is used to swap gene pairs within two linked chromosomes with a probability. The mutation is applied to each gene with a probability, and a novel value is generated within predetermined minimum and maximum thresholds. After crossover and mutation operation, feasibility of each chromosome should be checked.

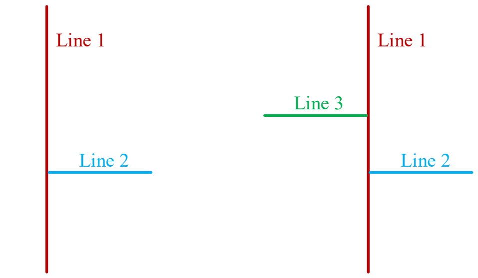
Step 5: Termination. If the number of generations reaches at maximum number of generations, output the saved best solution.

5. Case study

In this section, firstly, We evaluate the proximity of the solution derived from the proposed heuristic methodology to the optimal solution, and verify it is a repaid and effective approach. Then, we solve a large-scale case based on actual situation and analyze the solution.

5.1 Test

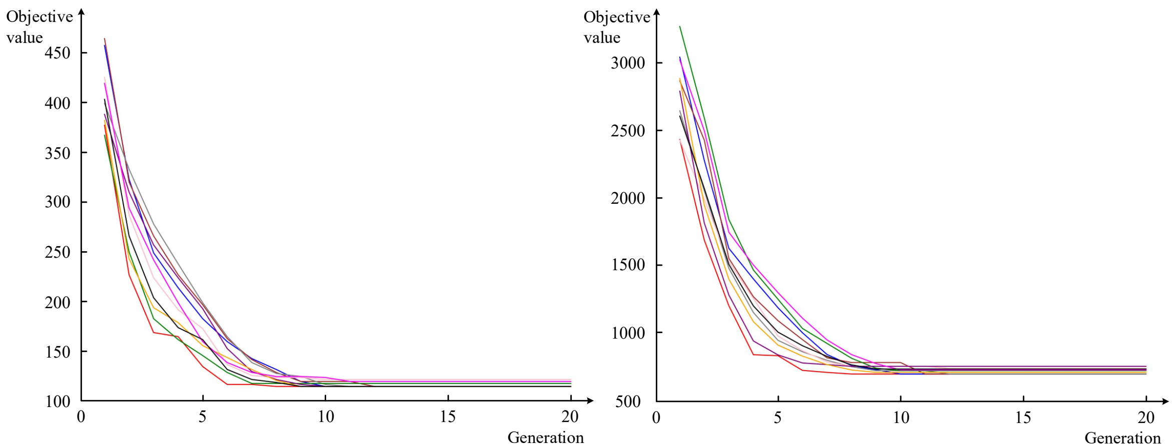
We establish two small railway networks to verify the proposed model and approach. Railway network 1 contains 2 rail lines, Line 1 and Line 2, where Line 1 is a mine line and Line 2 is a cross line. Line 1 encompasses 23 stations, while Line 2 encompasses 5 stations. 24 trains run on the network, including 8 main-line while 16 cross-line. Railway network 2 contains 3 rail lines, Line 3, Line 4 and Line 5, where Line 3 is a mine railway, Line 4 and Line 5 are two cross lines. There are 23 stations on Line 3, Line 4 features 5 stations, and Line 5 incorporates 10 stations. 60 trains run on the network, including 20 main-line while 40 cross-line. These two networks are shown in Fig. 7.



(a) Railway network 1 (b) Railway network 2

Fig. 7 Two simple railway networks

The model inputs comprise railway network, the initial train line plan and the ideal train timetable. To enhance the realism of the scenarios, we make an assumption that the new origin or destination stations of cross-line trains can be only selected in large stations. The heuristic algorithm is applied. The algorithmic parameters are configured as listed: a population size of 10, both crossover and mutation probability of 0.3, a maximum number of generations is 20. The heuristic genetic algorithm is programmed by using Python 3.7. To ascertain the robustness of the heuristic approach, we initially execute the algorithm 10 times, monitoring the fluctuation of objective values throughout each trial's iterations. Figure 8 illustrates the optimization processes across all 10 trials for both scenarios. All of them can converge at final solutions within 20 generations. We can find that 7 of the 10 trials finally converge at the solution 115 in Fig. 8 (a), and 4 of the 10 trials finally converge at the solution 691 in Fig. 8 (b). Both deviations of final solutions in these two figures are within 6 %. We choose solutions with the lowest objective values as the final results of these two cases.



(a) Optimization processes of case 1 (b) Optimization processes of case 2

Fig. 8 Optimization processes for all 10 trials of two small cases

Alternatively, we address the mixed integer programming model by optimization software CPLEX 12.10. The Python and CPLEX programs are conducted on a computing system operating on the 64-bit Windows 10 Pro platform, equipped with an Intel(R) Core(TM) i3-10100 CPU @ 3.60 GHz and 16.0 GB RAM.

We compare the outcomes derived from distinct computational methodologies, as shown in Table 3. It is obvious that Scenario 2 and 4 are able to obtain solutions closely to Scenario 1 and 3 with their gaps lower than 1 %. Compared with Scenario 1, there are the same adjustment time of main-line train and the number of changed train line plan of cross-line trains in Scenario 2, but they have difference in OD direct service frequency of cross-line passenger flow. Compared with Scenario 3, there are the same OD direct service frequency for cross-line passengers and alterations in line plan of cross-line trains in Scenario 4, but they have different adjustment time. In general, the application of the suggested heuristic method yields satisfactory outcomes.

However, when the case size (number of rail lines, stations and trains) is very small, there is no advantage for GA approach in calculation time. As the case size escalates, the amount of decision variables and constraints grows multiply, therefore, it takes more and more calculation time by using CPLEX solver, and GA approach has significant advantages when solving large-scale problems. Because of this, we find that calculation time in Scenario 2 is higher than that in Scenario 1, while calculation time in Scenario 4 is lower than that in Scenario 3 on the contrary.

Therefore, we can believe that the proposed heuristic genetic methodology is effective and rapid compared with CPELX solver.

Table 3 Comparison of solutions from CPLEX and GA

Scenario	1	2	3	4
	Case 1 with 24 trains		Case 2 with 60 trains	
	CPLEX	GA	CPLEX	GA
Objective	114	115	685	691
Adjustment time	126	126	716	722
OD direct service frequency (initial/new)	14/12	14/11	38/31	38/31
Number of changed train line plan	4	4	9	9
Calculation time	77 s	474 s	3102 s	2176 s
Gap	-	0.8 %	-	0.9 %

5.2 Real-world case

In this section, we deploy the proposed model and solution approach to the Beijing-Shanghai High-speed Railway and its HSR lines across China. Beijing-Shanghai High-Speed Railway (HSR) connects Beijing and Shanghai, connecting 23 stations. Up to now, there are 8 connecting lines of it, which are Tianjin-Shenyang HSR, Shijiazhuang-Jinan HSR, Jinan-Qingdao HSR, Xuzhou-Lanzhou HSR, Hefei-Bengbu HSR, Nanjing-Chongqing HSR, Nanjing-Hangzhou HSR, and Shanghai-Kunming HSR, as shown in Fig. 9.

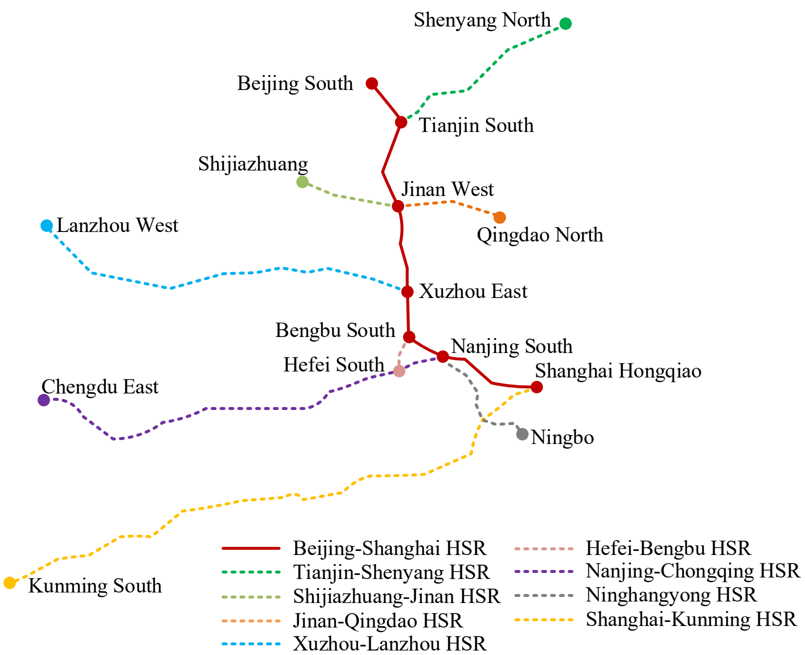


Fig. 9 Beijing-Shanghai High-speed Railway and its connecting lines

Table 4 lists some basic information of lines in this railway network, such as number of stations on each line, line length, and number of operated trains (only trains running in down direction are considered) on each line. It should be noted that trains which we count operated on each connecting line are only the cross-line trains of Beijing-Shanghai High-speed Railway.

Table 4 Basic information of lines in the railway network

Name	Number of stations	Length(km)	Number of operated trains
Beijing-Shanghai HSR	23	1318	Total 184 trains, including 61 main-line trains and 123 cross-line trains
Tianjin-Shenyang HSR	17	671	18
Shijiazhuang-Jinan HSR	11	298	1
Jinan-Qingdao HSR	10	230	26
Xuzhou-Lanzhou HSR	29	1434	24
Hefei-Bengbu HSR	5	132	13
Nanjing-Chongqing HSR	31	1630	13
Nanjing-Hangzhou HSR	11	256	21
Shanghai-Kunming HSR	52	2252	7

We apply the model and approach on the framework of railway network. Also, in order to make cases closer to the actual situation, we make an assumption that the new origin or destination stations of cross-line trains can be only selected in large stations, such as Beijing South, Tianjin South, Jinan West, Xuzhou East, Bengbu South, Nanjing South, and Shanghai Hongqiao. Fig. 10 illustrates 10 iterations of optimization process, all of them can converge at the final solutions within 20 generations, and the objective values vary from 2480 to 2601 within 5 % deviation. We can obtain the final solutions in acceptable calculation time. Additionally, we endeavour to address the problem using CPLEX. However, the model generates an immense number of variables and constraints, causing the solver to grapple with the complexity of such a colossal problem. Consequently, when the RAM reaches full capacity, the software halts without providing a solution.

We designate the solution yielding the minimum objective value as the ultimate outcome for the real-world case. The result shows that the total adjustment time is 2561 minutes, the number of changed train line plan of cross-line trains is 11, and the OD direct service frequency of cross-line passenger flow is 108 in the new train line plan while 117 in the initial train line plan. We analyse the distribution of the adjustments of main-line trains timetables in this case, as depicted in Fig. 11. The visualization demonstrates that 63.4 % of rescheduled timetable's arrival and departure times are equal to those in the ideal timetable, and only 0.4 % of them are adjusted more than 10 minutes. Besides, the line plans of cross-line trains after optimization are listed in Fig. 12.

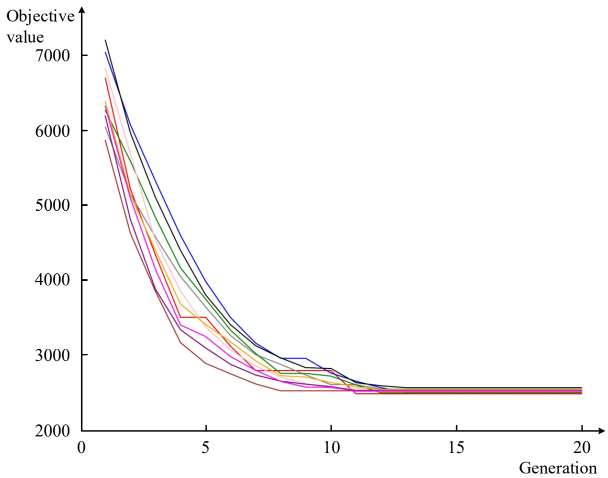


Fig. 10 Optimization process for all 10 trials of the real-world case

We also try to solve the real-world case when the line plans of cross-line trains are not modified, unfortunately, it is greatly difficult to acquire a feasible solution because the conflicts between each train paths cannot be completely resolved.

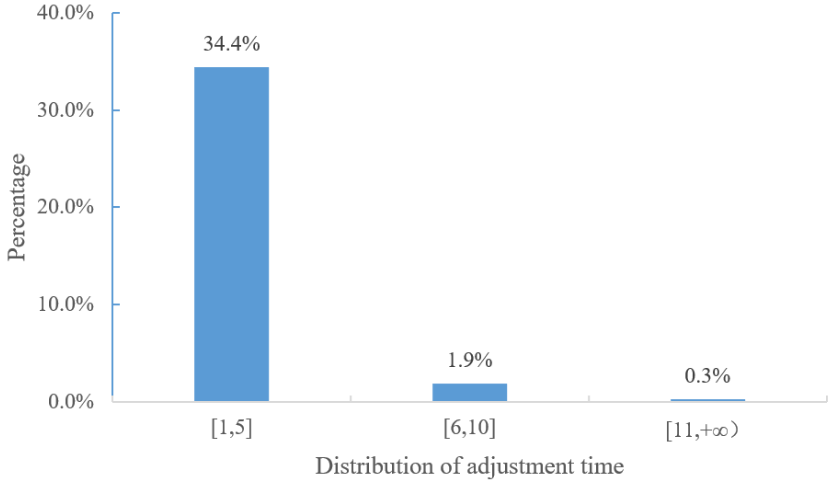


Fig. 11 Distribution of adjustment of main-line trains' departure and arrival times at stations

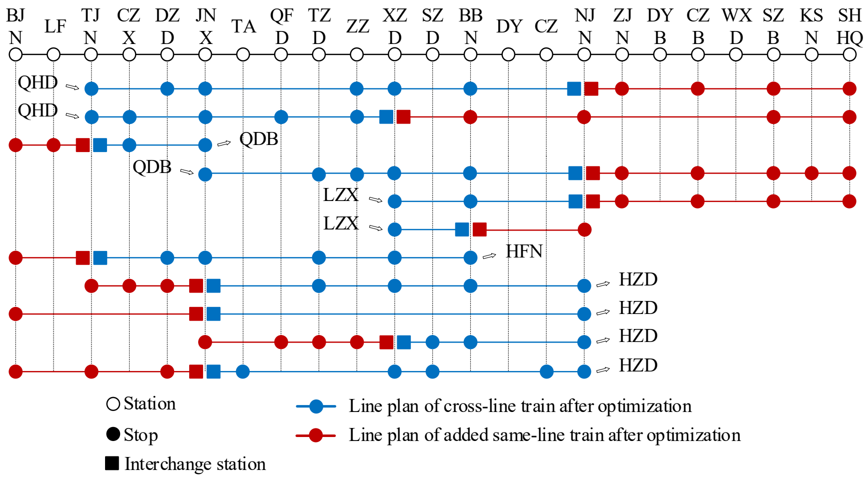


Fig. 12 Line plans of cross-line trains after optimization

6. Conclusion

This paper aims to address the comprehensive optimization challenge of train line planning and timetabling when taking cross-line operation into account. We develop an innovative method to change train line plans of cross-line trains, aiming to decrease the influence of cross-line operation on main-line trains. Leveraging an event-activity network framework, we formulate a sophisticated mixed integer programming model. The model tries to obtain a new timetable with less adjustment time of ideal main-line train timetable. At meantime, to satisfy travel demand of cross-line passengers, the model also takes into account the maximum OD direct service frequency for cross-line passengers. The constraints are formulated based on actual railway operation requirements and passenger demand. An improved heuristic algorithm is proposed to overcome difficulties in calculation efficiency when solving large-scale problems. The approach is based on the genetic algorithm, and firstly is tested on 2 small railway networks with 24 trains running on 2 lines and 60 trains running on 3 lines, respectively. The solutions are compared with those obtained by using CPLEX solver. The findings illustrate that the devised heuristic methodology effectively attains satisfactory solutions which are close to optimal solutions with much less calculation times when solving large-scale problems. Finally, the model and the methodology are deployed in an actual network with 184 trains operating across 9 high-speed rail corridors, and we obtain a satisfactory solution. In the future, we will do more researches on cross-line operations, especially in considering passenger demand, a regional railway network and rolling stock scheduling.

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