

Flexible Job-shop Scheduling Problem with parallel operations using Reinforcement Learning: An approach based on Heterogeneous Graph Attention Networks

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ABSTRACT

The Flexible Job-shop Scheduling Problem (FJSP) has received considerable scholarly attention as a classic problem. However, in practical industrial manufacturing scenarios, it is common for an operation to have multiple preceding parallel operations. This not only necessitates adhering to the sequential relationships inherent in FJSP but also requires ensuring that preceding operations are completed simultaneously whenever feasible. We term this scenario as the Flexible Job-shop Scheduling Problem with Parallel Operations (FJSP-PO), a pervasive challenge encountered across nearly every production line in real-world discrete manufacturing applications. Despite its prevalence, there is a noticeable scarcity of research on FJSP-PO in existing literature. Given the objective of synchronizing multiple preceding operations, FJSP-PO presents a broader solution space and more intricate optimization challenges compared to traditional FJSP. To address this, we propose an Attention Restart method based on Heterogeneous Graph Attention Networks (AR-HGAT). Leveraging a heterogeneous graph network structure and reinforcement learning, AR-HGAT learns the implicit features of operations and machines through node-level and semantic-level attention mechanisms. The AR mechanism is utilized to determine the optimal scheduling of operations at specific time slots. Compared to existing FJSP methods, our AR-HGAT approach demonstrates superior performance in terms of inference time and solution effectiveness. Furthermore, we conducted a comparative analysis using authentic operational data from companies and contrasted it with results obtained from an online tree search algorithm, thereby providing empirical validation of the effectiveness of the proposed AR-HGAT method.

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Problem prilagodljivega terminiranja proizvodnje z vzporednimi operacijami z uporabo učenja z ojačitvijo: Pristop, ki temelji na heterogenih grafnih omrežjih pozornosti

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POVZETEK

Problem prilagodljivega terminiranja proizvodnje (FJSP) je bil kot klasičen problem deležen velike znanstvene pozornosti. Vendar pa je v praktičnih scenarijih industrijske proizvodnje običajno, da ima operacija več predhodnih vzporednih operacij. Problem torej ne zahteva le upoštevanja zaporednih razmerij, ki so značilna za FJSP, temveč tudi zagotavljanje, da se predhodne operacije končajo hkrati, kadar koli je to izvedljivo. Ta scenarij označujemo kot problem prilagodljivega terminiranja proizvodnje z vzporednimi operacijami (FJSP-PO), ki je zelo razširjen problem, s katerim se srečujemo na skoraj vsaki proizvodni liniji v realnih diskretnih proizvodnih aplikacijah. Kljub njegovi razširjenosti je v obstoječi literaturi opazno pomanjkanje raziskav o FJSP-PO. Glede na cilj sinhronizacije več predhodnih operacij predstavlja FJSP-PO širši prostor rešitev in bolj zapletene optimizacijske izzive v primerjavi s tradicionalnim FJSP. Da bi rešili ta problem, predlagamo metodo ponovnega zagona pozornosti, ki temelji na heterogenih grafnih omrežjih pozornosti (AR-HGAT). AR-HGAT z uporabo strukture heterogenih grafnih omrežij in učenja z ojačitvijo uči skrite značilnosti operacij in strojev s pomočjo mehanizmov pozornosti na ravni vozlišč in na semantični ravni. Mehanizem AR se uporablja za določitev optimalnega razporejanja operacij v določenih časovnih intervalih. V primerjavi z obstoječimi metodami FJSP naš pristop AR-HGAT izkazuje boljšo učinkovitost v smislu časa sklepanja in učinkovitosti rešitve. Poleg tega smo izvedli primerjalno analizo z uporabo resničnih obratovalnih podatkov iz podjetij in jih primerjali z rezultati, pridobljenimi s spletnim algoritmom iskanja po drevesu, s čimer smo zagotovili empirično potrditev učinkovitosti predlagane metode AR-HGAT.

PODATKI O ČLANKU

Ključne besede:

Prilagodljivo terminiranje;
Prilagodljiv problem terminiranja proizvodnje (FJSP);
Enotni model terminiranja;
Vzporedne operacije;
Učenje z ojačitvijo;
Heterogena grafna omrežja;
Metoda ponovnega zagona pozornosti na podlagi heterogenih grafnih omrežij pozornosti (AR-HGAT)

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