APEM journal

Advances in Production Engineering & Management
Volume 21 | Number 2 | June 2026 | pp 178–200
https://doi.org/10.14743/apem2026.2.568

ISSN 1854-6250
Journal home: apem-journal.org
Original scientific paper

Dynamic raw material ordering under demand uncertainty: A Markov chain–Newsvendor approach

Zhao, Y.[a], **Ma, Z.**[a], **Wang, Y.**[a,*], **Su, X.**[a]

[a]Guilin University of Electronic Technology, Guilin, Guangxi, P.R. China

ABSTRACT

Raw material ordering under demand uncertainty is often hindered by inaccurate forecasts, resulting in excess inventory, stockouts, and increased operating costs. Traditional Newsvendor models typically assume static demand distributions and therefore have limited ability to capture dynamic changes in demand. To address this limitation, this study proposes a Markov chain–based Newsvendor decision framework incorporating an [L, U] inventory boundary policy. The main contribution of the proposed framework is the integration of state-transition-based probabilistic demand forecasting with inventory optimization. Specifically, raw material demand is classified into a finite set of states, and the transition probability matrix is estimated from historical demand-state observations. The resulting predicted demand distribution is then incorporated into the Newsvendor model to determine the optimal order-up-to level and the corresponding replenishment quantity. A simulation-based case study of an electronics manufacturing firm is conducted using a generated 52-week demand dataset. The results show that the proposed MC-NM model reduces the expected total ordering cost from 4887.68 yuan to 4269.92 yuan, corresponding to a cost reduction of 12.6 %. These findings indicate that the proposed framework can reduce inventory-related costs while maintaining responsiveness to demand fluctuations, thereby providing practical decision support for raw material procurement under uncertainty.

ARTICLE INFO

Keywords:
Supply chain management;
Inventory management;
Ordering of raw materials;
Demand forecast;
Markov chain;
Newsvendor model;
Decision support system

**Corresponding author:*
wangyue@guet.edu.cn
(Wang, Y.)

Article history:
Received 19 October 2025
Revised 30 June 2026
Accepted 3 July 2026

1. Introduction

In modern enterprise operations, raw material ordering decisions are an important component of supply chain management [1]. Their effectiveness directly affects operational efficiency and cost control. As an upstream stage of production, raw material procurement directly affects production continuity, cost control, and quality assurance. First, the stability of raw material supply determines whether the production line can operate according to plan. Any delay or disruption in the procurement process may interrupt production and propagate down the supply chain, ultimately reducing overall performance and service levels [2] and causing substantial losses. Second, the purchase cost of raw materials usually accounts for a large proportion of the total cost of enterprises, and the appropriateness of the purchase quantity and timing has a decisive impact on capital turnover efficiency and inventory level. Third, the quality level and reliability of raw materials are often key determinants of product performance and market competitiveness. Improper procurement management may lead to rework, scrap and other hidden losses. Thus, raw material procurement is not only a fundamental component of production engineering but also a strategic lever for improving the overall operational efficiency and competitiveness of enterprises.

However, in the complex and changing market environment, the demand for raw materials is often subject to substantial uncertainty and dynamic fluctuations, which affect the optimal purchase and inventory decisions of enterprises [3]. This uncertainty is not only due to periodic price changes and fluctuations in market demand but may also be influenced by external factors such as supply chain disruptions, geopolitical events, and macroeconomic changes. For example, sharp fluctuations in international raw material prices can quickly propagate to enterprise procurement, leading to a significant increase in the difficulty of demand forecasting. A sudden supply chain disruption may cause a sharp drop in demand or an abnormal concentration of demand in the short term. Especially for seasonal, perishable or high-value raw materials, forecast errors can directly lead to serious consequences such as overstocking, excessive capital tied up in inventory or production stagnation. Traditional inventory decision-making methods often rely on empirical judgment [4] or assume a static demand distribution [5]. As a result, they may fail to capture demand evolution in time. Such methods can neither effectively prevent stockout risk nor avoid the high holding costs associated with excessive inventory. Therefore, they are unable to meet enterprises' needs for flexibility, agility, and accuracy in dynamic environments. Accordingly, it is important to develop a raw material ordering model that can support inventory management under dynamic and uncertain demand conditions.

As a mathematical tool for modeling random state-transition processes, the Markov chain is often used to describe the dynamic evolution of demand, prices, behavior, and related phenomena. In demand forecasting, Li *et al.* [6] used the Markov decision process to develop an optimal ambulance destination strategy that addresses ambulance unloading delays caused by emergency-room congestion, reduces patient waiting time, and rapidly restores ambulance service capacity; Chen *et al.* [7] proposed a market demand forecasting model based on a Markov chain, which analyzed customer behavior and market trends through the state transition probability matrix, providing a quantitative basis for marketing decisions of enterprises. For inventory decision-making under uncertain demand, Markov-chain forecasting also shows good applicability. Więcek and Kubek [8] analyzed the effects of time-series characteristics on the forecasting performance of ARIMA, SARIMA, and Markov-chain models for fuel demand, and found that the Markov chain performed best in dynamic demand scenarios, especially in long-term pattern recognition; Lin and Ho [9] proposed a new single-supplier, multi-buyer integrated inventory model, which can predict buyers' demand in a particular season through a Markov chain and historical demand data to minimize the cost; Chen and Song [10] studied multi-echelon inventory systems with Markov-modulated demand, where the demand distribution in each period is determined by the current state of an exogenous Markov chain, and showed that state-dependent order-up-to policies can be optimal under such fluctuating demand environments; Yuna *et al.* [11] used a Markov process to model intermittent demand and determine lower and upper stock limits, showing that Markov-based stock limits can help balance inventory holding costs and lost-sales costs. In addition, in the fields of energy structure [12] and energy consumption [13], power demand [14], equipment cycle inspection and maintenance [15], Markov-chain models show good adaptability and forecasting ability. In the field of supply chain management, researchers have also used Markov-chain models to study inventory optimization. Hashemi-Petroodi *et al.* [16] modeled the task allocation and worker movement process in the production system as a Markov decision process, characterizing transitions between states and showing that the Markov-chain method can effectively capture the dynamic characteristics of complex manufacturing environments, thereby providing new approaches for inventory demand forecasting and ordering decisions; Su and Wu [17] used Markov chain Monte Carlo to model the wind speed in different seasons, and then simulated the waiting time distribution of maintenance ships through a Markovian Monte Carlo method to evaluate the impact of random wind speed on the maintenance availability of offshore wind farms. These studies suggest that Markov-based prediction is suitable for scenarios with frequent demand fluctuations and limited historical data. It can provide timely decision support for raw material demand forecasting and procurement planning.

As a core tool for inventory optimization and supply chain decision-making, the Newsvendor model has been continually extended to address increasingly complex real-world decision settings. For purchasing and inventory optimization of seasonal or perishable goods, Ni [18] considered market characteristics and sales patterns and proposed an improved Newsvendor model integrating information recognition mechanism and discount promotion strategy, which provides analytical support for operational decisions concerning seasonal goods; Lei *et al*. [19] extended the Newsvendor model to analyze the situation of partial demand substitution of two products. The study shows that demand substitution reduces the demand variability through the formation of effective demand, thereby improving expected profits and order-quantity decisions. To address the management of perishable products, Puranam *et al*. [20] introduced uncontrollable transfer inventory into the traditional Newsvendor model, highlighted the complexity of inventory management introduced by random transfers, and expanded the theoretical scope of the Newsvendor model. This suggests that the single-period Newsvendor model is an important tool for supply chain management, and its classical formulation focuses on perishable goods in a single sales cycle. The model assumes that demand follows a known probability distribution [21], but in practical applications, the real demand distribution is often difficult to obtain, and the relevant parameter-estimation methods can be complex and prone to estimation errors [22]. To address demand uncertainty in complex Newsvendor settings, Gonzalez *et al*. [23] proposed a new method combining fog computing, an attention mechanism, and a gated recurrent unit; Hua *et al*. [24] extended the Newsvendor model to incorporate barter exchange, showing that retailers can handle unsold products through the barter platform, which can help manage demand uncertainty and improve profits, and analyzed the effects of relevant parameters on the optimal order quantity; Zhu [25] studied a new model for the Newsvendor problem based on the theory of focus selection; Sousa *et al*. [26] proposed a multi-project Newsvendor model based on fuzzy logic and an improved genetic algorithm to improve model performance under uncertain demand and optimize order-quantity decisions in complex scenarios.

Existing research on demand forecasting and inventory management provides substantial insights but also faces important limitations. The classical Newsvendor model, foundational for single-cycle optimization, relies on static, predefined demand distributions—a simplification that neglects real-world dynamics and restricts its utility in unstable markets. Meanwhile, contemporary approaches utilizing neural networks [27], machine learning [28], or deep learning focus on dynamic prediction but frequently decouple forecasting from optimization, leaving integration gaps unaddressed—particularly acute in raw material contexts where volatile demand challenges static assumptions. Limited progress has been made in combining Markov chains with Newsvendor frameworks; examples include Sprenkels' application of Markov decision processes to multi-product pricing [29] and Keskin *et al*.'s extension of the Newsvendor concept into multi-stage settings via declining inventory constraints [30], though these remain fragmented efforts. Systematic mechanisms integrating Markov-driven forecasts with Newsvendor decisions are still underdeveloped, especially for single periods where static models fail to capture temporal shifts [31], causing supply-demand misalignment during procurement. In practice, enterprises depend heavily on historical data mining for adaptive control [32]. Markov chains effectively map demand evolution and generate projections, while the Newsvendor model endures in short-lifecycle, non-replenishable scenarios [33-34]. A key priority is to combine these strengths, namely the predictive adaptability of Markov chains and the decision efficiency of the Newsvendor model, to develop raw material ordering systems balancing accuracy and flexibility.

In summary, although existing research has made progress in demand forecasting and inventory management, static Newsvendor models still struggle to reflect dynamic demand evolution, and many forecasting methods remain disconnected from inventory decision-making. Particularly in raw material inventory management, where real-time responsiveness and flexibility are important, there is a lack of a unified framework that can integrate dynamic demand forecasting with inventory optimization decisions. This research gap not only hinders the further development of inventory management methods, but also affects the practical effectiveness of enterprise applications in complex and changing environments. To address these limitations, the main contribution of this paper is to propose a Markov chain–Newsvendor decision model for raw material ordering (MC-NM model) with $[L, U]$ inventory constraints. Specifically, the framework uses Markov chains to characterize state transitions in raw material demand and implement dynamic demand forecasting, thereby overcoming the static-demand assumption of traditional Newsvendor models; introduces $[L, U]$ inventory constraints to control the upper and lower bounds of the order quantity and better reflect practical procurement and inventory management; and validates the proposed model through numerical analysis. The results show that the method can significantly reduce total inventory costs under demand fluctuations and improve resource-allocation efficiency and operational resilience in uncertain environments.

2. Problem statement and related assumptions

2.1 Problem statement and parameter explanation

In actual production and operations, companies often face the challenge of uncertainty in raw material demand. Let this uncertainty be represented by the random variable R for demand per cycle. Its fluctuations are influenced by various factors, such as periodic adjustments to production plans, fluctuations in the raw-material supply and demand market, potential disruptions in the supply chain, dynamic changes in supplier capacity, and the company's cash flow characteristics. These factors may cause R to exhibit significant stage-specific and temporal dynamics. Therefore, in raw material ordering, companies need to dynamically balance total cost C_T, shortage loss C_2, and inventory cost C_1 by capturing the changing patterns of demand distribution in real time. This enables them to effectively avoid ordering decision errors caused by demand changes and ensure that the company can achieve the optimal balance between costs and risks in a complex and ever-changing market environment.

To address the potential errors in ordering decisions caused by changes in the distribution patterns of raw material demand, this paper proposes a raw material ordering decision-making framework that uses state prediction as the input and Newsvendor optimization as the output while incorporating $[L, U]$-type inventory strategy constraints. In this framework, Markov chains are used to describe how demand R transitions over time within a finite state space $S = \{s_1, s_2, \dots, s_n\}$. Based on the transition probability matrix $P = [p_{ij}]$, historical states can be used to estimate future distributions, construct demand transition patterns across multiple states, provide time-dependent and traceable demand forecasting, and establish raw material ordering criteria under an $[L, U]$ inventory strategy. The Newsvendor model, on the other hand, addresses a given demand distribution $p(R)$ by minimizing expected costs to determine the optimal order quantity Q^*.

Compared with traditional Newsvendor models, which typically assume that demand follows a static or known distribution (such as a normal distribution), this paper proposes a dynamic modeling mechanism based on state transitions, using Markov chains as the computational basis for predicting the demand distribution. This transforms the Newsvendor decision problem from a static to a dynamic decision and introduces an $[L, U]$ inventory strategy to reduce bias in the theoretical optimal solution of the Newsvendor model, ensuring that the actual order quantity remains within a safe range. By predicting the demand distribution $\pi_{t+1} = e_i P$ for the next cycle based on the transition probabilities of the current cycle's state s_i, and using this as input for the Newsvendor model, the optimal raw material order quantity is determined by minimizing expected costs under the $[L, U]$ inventory constraints (determined from the demand forecast results). This ultimately constructs a raw material ordering Newsvendor decision model based on Markov chain demand forecasting (i.e., the MC-NM model). This model aligns with the flexibility required for production scheduling and dynamic procurement environments, thereby strengthening the integration of dynamic forecasting and decision-making. A summary of the model parameters and their meanings is provided in Table 1.

Table 1 Summary of model-related parameters and meanings

Notation	Meaning
L	Minimum inventory level
U	Maximum inventory level, $L \leq U$
K	Unit cost of raw materials
C_1	Holding cost per unit of raw material per cycle
C_2	Stockout cost per unit of raw material per cycle
C_3	The cost of each order of raw materials (independent of the order quantity)
R	Random variables of demand (in ascending order), $R_0, R_1, \cdots, R_m (0 \leq R_0, R_k < R_{k+1})$
$p(R)$	Demand probability, $\sum_{k=0}^{m} p\,(R_k) = 1$
I	Opening inventory (constant in this stage)
Q	Actual replenishment quantity; if $I < L$, $Q = U - I$; if $I \geq L$, $Q = 0$
Q^*	Theoretical optimal order-up-to level derived from the Newsvendor model
C_T	Total cost
X_t	System state at time t (random variable indicating the current state)
s_i	The i-th specific state in state space S
e_i	Unit state vector, indicating that the current state is s_i, with the i-th element being 1 and the rest being 0.
S	State space, a set containing all possible states of the system (e.g., $\{s_1, s_2, \ldots, s_n\}$)
P	Probability function used to represent conditional probabilities
p_{ij}	Single-step transition probability from state s_i to state s_j
P *(matrix)*	Transition matrix, containing all transition probabilities p_{ij}, with rows and columns corresponding to the current and next states, respectively.
$N = (n_{ij})$	n_{ij} denotes the number of transitions from state S_i to state S_j.
π_0	Initial state distribution, a vector representing the probabilities of each state of the system at $t = 0$, $(\sum \pi_0\,(s_i) = 1)$
π_t	The state distribution at time t, represented by a vector indicating the probabilities of each state.
π	Stationary distribution, a row probability vector representing the long-run probabilities of the states, satisfying $\pi P = \pi$
γ	Critical ratio, used to determine the optimal order quantity threshold
$\widehat{D}_h$	Smoothed demand value obtained using the three-point moving-average method in week t.

2.2 Model assumptions

To facilitate the description and construction of the model, this study makes the following assumptions:

- Timeliness and ordering constraints: Raw materials are time-sensitive and must be ordered before the start of each production cycle, with no additional purchases allowed during the cycle. The Newsvendor model, as a single-cycle ordering decision model, is suitable for raw materials subject to random demand and short cycles (such as perishable chemical raw materials, seasonal building materials, etc.), as its assumption of a single order aligns with practical scenarios involving continuously fluctuating raw material demand.
- Cost-risk trade-off: Excess raw material inventory generates holding costs (such as warehousing, discounting, or scrapping costs), while shortages lead to opportunity costs associated with production interruptions or emergency procurement (such as production line downtime and additional logistics costs). The Newsvendor model minimizes expected costs or maximizes profits by balancing these two types of costs.
- Dynamic demand modeling: The raw material demand for each production cycle is a finite-state random variable, whose distribution depends on the state of the previous cycle and evolves according to a defined transition probability matrix (such as the correlation between raw material consumption and production plans in historical data). By estimating the transition probability matrix, firms can generate the conditional probability distribution of next-period demand given the current state. This provides the Newsvendor model with dynamic, state-dependent inputs that replace traditional static demand assumptions.
- Data-driven forecasting: Companies can partition the demand state space and construct state transition matrices based on complete and stable historical raw material consumption data to ensure the feasibility and effectiveness of Markov chain forecasting.

- Idealized operating conditions: The lead time for raw material procurement is assumed to be negligible, and no natural deterioration or quality change in raw materials is assumed to occur during the cycle. These assumptions facilitate model development under idealized operating conditions.
- [L, U] Inventory strategy constraints: The lower inventory limit (L) and upper limit (U) of inventory are introduced into the ordering decision. When the inventory is lower than L, the purchase order is triggered to restore the inventory to the upper limit U; when the inventory is greater than or equal to L, the order is not triggered. The dynamic threshold is used to limit the quantity ordered in a single cycle, which both constrains deviations from the theoretical optimum of the Newsvendor model and reduces inventory risk.

3. Model construction

Fig. 1 shows the theoretical framework constructed in this paper. First, the historical demand data are used to divide the demand state and construct the transition matrix of the Markov chain. At the beginning of each cycle, the Markov chain is used to predict the demand distribution in the next period. Second, the predicted distribution is provided as input to the Newsvendor model, and the expected cost function is constructed. In practice, the [L, U]-type inventory strategy is introduced to modify the upper and lower limits of the optimal solution, so as to ensure that the order quantity can meet demand without causing excessive inventory. Finally, the actual demand observed by the enterprise will be fed back to the system, and the state distribution will be updated for the prediction of the next cycle. In this way, a closed-loop mechanism of prediction, optimization, adjustment, and feedback is formed to ensure that the model can continuously adapt to the dynamic demand fluctuations.

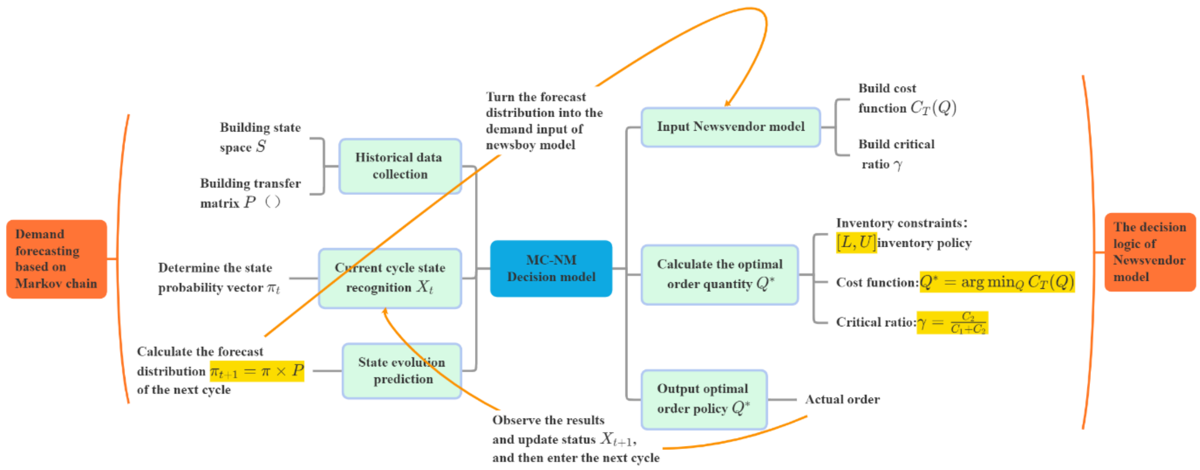

Fig. 1 Theoretical framework

3.1 Markov chain demand forecasting model

In the demand forecasting phase, this paper uses a Markov chain to describe the dynamic evolution of raw material demand. The basic idea is to divide the demand into several discrete states, such as "low, medium and high", and obtain the transition probability between different demand states from historical data. These probabilities constitute a transition matrix, which can reflect the dynamic evolution of demand in different periods.

A Markov chain is a set of discrete random variables that exhibit the Markov property. If a stochastic process, given the current state and all past states, has a conditional probability distribution for its future state that depends solely on the current state—that is, given the current state, it is conditionally independent of the past states (i.e., the historical path of the process)—then this stochastic process is said to have the Markov property. A process with the Markov property is typically referred to as a Markov process.

A Markov chain can be represented as:

$$P(X_{t+1} = s|X_t = s_t, X_{t-1} = s_{t-1}, \dots, X_0 = s_0) = P(X_{t+1} = s|X_t = s_t) \tag{1}$$

where $P(\,)$ denotes the probability function, representing the probability of a specific event; X_t represents the state of the system at time t; s_t represents the current state; s represents the possible next state.

From historical data, the transition probability p_{ij} between the demand states can be estimated, and the state transition matrix can be constructed P. When the current demand distribution is given by π_t, the demand distribution in the next cycle can be obtained as $\pi_{t+1} = \pi_t P$. This result provides a dynamic forecast input for the subsequent inventory optimization model.

Specifically, when the demand state of the current cycle is known, the Markov chain can infer the probability of each future state according to the transition probability, and thereby obtain the demand distribution of the next cycle. The advantage of this prediction method is that it can use historical observations to capture the randomness and dynamics of demand. Compared with the traditional static distribution assumption, it better reflects the characteristics of demand fluctuations over time in manufacturing. In this study, the Markov chain is selected because the weekly raw material demand is discretized into a finite number of demand states, and the subsequent Newsvendor decision requires a probability distribution rather than only a point forecast. The transition probability matrix can be estimated from the historical state sequence and used to generate a state-dependent demand distribution for the next ordering cycle. Therefore, the Markov chain provides a transparent and decision-oriented forecasting input that can be directly connected with the inventory optimization model.

The final demand forecast distribution will be used as input to the inventory optimization model, and combined with the subsequent Newsvendor model to determine the optimal ordering strategy. In order to keep the text concise, the relevant mathematical derivations and calculation details are provided in Appendix A.

3.2 Raw material ordering criteria under the [*L*, *U*] inventory strategy

The $[L, U]$-type inventory strategy determines the inventory lower bound L and upper bound U through modeling. When the inventory at the end of the previous period is less than the inventory lower bound L, an order is placed, and the order quantity raises the existing inventory to the upper bound U; when the inventory at the end of the previous period is greater than or equal to the inventory lower bound L, no order is placed.

The basic idea of the modeling approach is to achieve minimum total cost, minimum expected loss, or maximum expected profit. Because demand, inventory, and shortage quantities are random, the inventory cost and shortage loss cost within a cycle (usually measured in weeks or months) are also random. Therefore, the objective function should be based on the expected value (or average value) for a cycle.

The total cost includes ordering costs (ordering fees and raw material costs), inventory carrying costs, and stockout costs. Among these, the ordering cost can be denoted as $KQ + C_3$. The expected inventory holding cost can be denoted as $\sum_{R \le U = I+Q} C_1\,(I + Q - R)p(R)$, which represents the expense of maintaining surplus raw materials when supply exceeds demand. The shortage cost can be denoted as $\sum_{R > U = I+Q} C_2\,(R - I - Q)p(R)$, which reflects the opportunity loss when demand exceeds supply. Thus, the total cost in this cycle can be expressed as:

$$\begin{aligned} C_T = C_T(U) &= KQ + C_3 + \sum_{R \le U = I+Q} C_1(I + Q - R)p(R) + \sum_{R > U = I+Q} C_2\,(R - I - Q)p(R) \\ &= C_3 + K(U - I) + \sum_{R \le U} C_1\,(U - R)p(R) + \sum_{R > U} C_2\,(R - U)p(R) \end{aligned} \tag{2}$$

The upper-bound value U is the value that minimizes C_T. Solving the model yields the inequality used to determine the U value (i.e., the U_k value) as follows:

$$\sum_{R\leq U_{k-1}} p\,(R) < \frac{C_2 - K}{C_1 + C_2} \leq \sum_{R\leq U_k} p\,(R) \tag{3}$$

See Appendix B for the detailed derivation.

The value of U_k is calculated according to Eq. 3, i.e., let $U = U_k$, then the order quantity Q for this stage is $Q = U - I$. Thus, the order quantity is independent of the ordering fee.

If no raw materials are ordered at this stage (i.e., $C_3 = 0$, $I \geq L$), we then consider whether there is a value L ($L \leq U$) that satisfies the following relationship:

$$\begin{aligned} &KL + \sum_{R\leq L} C_1\,(L - R)p(R) + \sum_{R>L} C_2\,(R - L)p(R) \\ &\leq C_3 + KU + \sum_{R\leq U} C_1\,(U - R)p(R) + \sum_{R>U} C_2\,(R - U)p(R) \end{aligned} \tag{4}$$

L is selected from the discrete candidate values below U. If more than one candidate satisfies Eq. 4, the final value of L is determined by comparing their expected no-order costs and selecting the feasible candidate with the minimum expected cost.

When $L = U$, Eq. 4 is satisfied. When $L < U$, the expected storage cost $\sum_{R\leq U} C_1(U - R)p(R)$ on the right side of Eq. 4 is greater than the expected storage cost $\sum_{R\leq L} C_1\,(L - R)p(R)$ on the left side, and the expected stockout loss cost $\sum_{R>U} C_2\,(R - U)p(R)$ on the right side is less than the expected stockout loss cost $\sum_{R>L} C_2\,(R - L)p(R)$ on the left side.

After accounting for the corresponding increase and decrease, the relationship may still hold, and there may be more than one L (in which case the smallest is selected). Solving for the value of L is more complicated than solving for the value of U, but the value of L can still be obtained readily for specific problems because of $L \in [R_0, U]$.

To reduce the number of calculations, first set $L = R_0$ and substitute it into Eq. 4 for evaluation. If the relationship holds, the procedure is complete; if it does not hold true, then set $L = R_1$ and substitute it into the equation until a solution is found.

3.3 Optimal ordering Newsvendor decision model

After obtaining the probability distribution of raw material demand in the next cycle predicted by the Markov chain, this paper further uses the Newsvendor model to determine the optimal order quantity. The basic idea of the Newsvendor model is that in the single-cycle ordering decision, enterprises need to balance two types of risks: on the one hand, over-ordering increases inventory holding costs and ties up capital. On the other hand, under-ordering can lead to shortages, lost sales opportunities, and associated penalty costs.

To balance these two types of costs, the Newsvendor model introduces a critical ratio. By comparing the relative magnitudes of the shortage cost and the holding cost, the optimal order quantity Q^* is determined to minimize the expected total cost. On this basis, this paper further considers the practical management constraints, introduces [L, U] inventory boundary conditions, and constrains the optimal order quantity. This means that even if the optimal solution calculated by the model is Q^*, its implemented value must also lie within the [L, U] interval to better reflect practical constraints on cash flow, storage capacity, and supply contracts. To keep the text concise, the relevant mathematical derivations and calculation details are provided in Appendix C.

In summary, the optimal Newsvendor ordering model uses the future demand probability distribution generated by the Markov chain as the primary input. By constructing an expected total cost function that includes procurement costs, holding costs, stockout costs, and fixed ordering costs, and applying marginal analysis to identify the cost-minimizing solution, the optimal ordering strategy is derived for different demand scenarios. On this basis, the theoretical optimal solution is further checked for feasibility and adjusted to satisfy the [L, U] inventory constraints, ultimately forming an actual ordering plan that balances cost efficiency and inventory security.

4. Case study analysis

4.1 Case study background and simulated data generation

To verify the feasibility and effectiveness of the proposed MC-NM model, this study develops a simulation-based numerical case representing the raw material procurement scenario of a representative medium-sized electronics manufacturing firm. The hypothetical firm is assumed to be engaged in the assembly of electronic controllers, and its production process is highly dependent on a specific type of precision component, referred to as raw material M. In electronics manufacturing, product updates are frequent, supply chain links are complex, and raw material demand is often affected by production plans, market fluctuations, promotional cycles, and seasonal changes. Therefore, this scenario is suitable for evaluating the applicability of the proposed Markov chain–Newsvendor decision model under demand uncertainty.

It should be noted that the dataset used in this case is simulated rather than being directly extracted from actual enterprise ERP records. The simulated data are constructed to imitate the weekly material-outbound records that are commonly available in enterprise ERP systems. This design allows the proposed model to be validated under a transparent and reproducible numerical setting while still reflecting typical demand fluctuation characteristics in electronics manufacturing.

The inventory management cycle is set to one week, meaning that raw material ordering decisions are made once per week. A 52-week demand sequence is generated to represent one year of raw material consumption. To make the simulated data consistent with the state-dependent characteristics required by the Markov chain model, five discrete demand states, S_1 to S_5, are first defined. A state transition sequence with 52 observations is then generated through a controlled random-walk mechanism, so that adjacent weeks exhibit reasonable state transitions while avoiding unrealistic repeated cycles. For each demand state, a corresponding numerical demand interval is specified, and weekly demand values are generated by using the interval median as the benchmark and adding moderate random fluctuations. As a result, the simulated demand sequence reflects both the switching characteristics among different demand levels and the randomness commonly observed in raw material consumption.

The generated 52-week demand data are then used for demand smoothing, state classification, transition frequency calculation, transition probability estimation, next-period demand prediction, and ordering decision analysis. Therefore, the case study serves as a numerical validation of the proposed decision framework rather than a statistical inference based on real enterprise data. Table 2 presents the simulated weekly demand data for raw material M over the 52-week numerical case.

Table 2 Simulated weekly demand data for raw material M

Weeks number	Weeks 1 to 9	Weeks 10 to 18	Weeks 19 to 27	Weeks 28 to 36	Weeks 37 to 45	Weeks 46 to 52
Specific demand volume	52	90	110	150	170	138
	65	61	85	120	155	175
	80	55	65	95	125	160
	95	62	50	66	100	130
	110	85	68	53	78	105
	130	100	92	60	57	82
	160	125	115	88	66	60
	140	155	135	105	90	
	115	145	165	130	112	

To reduce the influence of extreme values, the following preprocessing steps are performed as needed before the data are used:

(1) Exclude weeks with zero demand due to equipment shutdowns or holidays. When forecasting demand, weekly observations with zero demand attributable to specific exogenous factors should be excluded. These factors include equipment shutdowns caused by maintenance, malfunctions, or upgrades that suspend production, as well as statutory holidays such as the Spring Festival and National Day, when companies or customers may suspend production or procurement activities. These zero-demand observations do not reflect underlying market demand but are outliers

caused by external factors. Retaining them may distort demand-pattern analysis or forecasting results and reduce model accuracy; therefore, they should be excluded. In this study, however, no zero-demand weeks are assumed, so no observations were removed.

(2) To improve the predictive accuracy and robustness of the model, the historical weekly demand data were smoothed before model estimation. Specifically, the three-point moving average method was used to smooth the demand values for each week, thereby reducing the interference of occasional fluctuations or outliers on the classification of demand states. This commonly used trend-identification method in time-series analysis can effectively filter out high-frequency fluctuations and extract a relatively stable demand trend.

The formula for calculating the three-point moving average is as follows:

$$\widehat{D}_h = \frac{D_{h-1} + D_h + D_{h+1}}{3}$$

In this formula, $\widehat{D}_h$ represents the smoothed value of demand for week t; D_{t-1} , D_t, and D_{t+1} represent the original demand data for weeks $t-1$, t, and $t+1$, respectively. To avoid missing data at the beginning and end, this paper only uses this method to calculate the smoothed values for demand data from week 2 to week 51. For week 1 and week 52, the two-point average method is used for boundary treatment, that is:

$$\widehat{D}_1 = \frac{D_1 + D_2}{2}, \quad \widehat{D}_{52} = \frac{D_{51} + D_{52}}{2}$$

The smoothed demand sequence is more stable in terms of volatility than the original data and retains the characteristics of overall demand trend changes, enabling more accurate and reasonable state classification and Markov chain state modeling. Table 3 shows the smoothed historical data.

Table 3 Smoothed simulated demand data for raw material *M*

Weeks number	Weeks 1 to 9	Weeks 10 to 18	Weeks 19 to 27	Weeks 28 to 36	Weeks 37 to 45	Weeks 46 to 52
Demand after smoothing	58.5	88.7	113.3	145	151.7	141.7
	65.7	68.7	86.7	121.7	150	157.7
	80	59.3	66.7	94	126.7	155
	95	67.3	61	71.3	101	131.7
	111.7	82.3	70	59.7	78.3	105.7
	133	103.3	91.7	67	67	82.3
	143.3	126.7	114	84.3	71	71
	138.3	141.7	138.3	107.7	89.3	
	115	136.7	150	135	113.3	

Since the smoothed data are used only to determine which demand range each week's demand falls into, the data items are not strictly rounded to integers.

(3) Standardize the demand data for state classification. This removes differences in measurement scale and magnitude, thereby facilitating subsequent classification into demand states (such as high, medium, and low demand levels) for analysis or modeling.

According to the design of the simulated demand scenario, the demand for raw material M is assumed to have pronounced seasonal characteristics. This paper divides demand into five states based on quantity ranges, with the following specific criteria:

State S_1: Weekly demand ≤ 60 units
State S_2: 60 units $<$ weekly demand ≤ 90 units
State S_3: 90 units $<$ weekly demand ≤ 120 units
State S_4: 120 units $<$ weekly demand ≤ 150 units
State S_5: 150 units $<$ weekly demand

For ease of calculation, in the subsequent solution, the median of each interval is taken as the specific demand for that interval, i.e., the specific demand for intervals S_2, S_3, and S_4 is taken as 75, 105, and 135 units, respectively, and the specific demand for intervals S_1 and S_5 is taken as 45 and 165 units, respectively.

After state classification, the 52-week data are used to assign a state label to each week and construct a state sequence (see Table 4), which provides the basic input for the subsequent transition-probability estimation and demand-distribution prediction of the Markov chain model.

In addition, to make the model more realistic, the company's procurement of raw material M is subject to minimum order quantity and storage capacity restrictions. Therefore, in the subsequent modeling, an [L, U] inventory strategy will be used to set minimum and maximum inventory thresholds to reflect the company's actual management constraints.

Table 4 Status tag after smoothing

Weeks number	Weeks 1 to 9	Weeks 10 to 18	Weeks 19 to 27	Weeks 28 to 36	Weeks 37 to 45	Weeks 46 to 52
Status tag	S_1	S_2	S_3	S_4	S_5	S_4
	S_2	S_2	S_2	S_4	S_4	S_5
	S_2	S_1	S_2	S_3	S_4	S_5
	S_3	S_2	S_2	S_2	S_3	S_4
	S_3	S_2	S_2	S_1	S_2	S_3
	S_4	S_3	S_3	S_2	S_2	S_2
	S_4	S_4	S_3	S_2	S_2	S_2
	S_4	S_4	S_4	S_3	S_2	
	S_3	S_4	S_4	S_4	S_3	

4.2 Markov chain parameter estimation based on simulated data

To dynamically predict the distribution of future raw material demand, this paper constructs a Markov chain model based on the simulated 52-week demand data to obtain the transition probabilities between different states.

State space construction

According to the classification of demand states for raw material M mentioned earlier, this paper divides the weekly demand for raw materials into five finite states: S_1, S_2, S_3, S_4, S_5. Based on this, the state space can be defined as:

$$S = \{S_1, S_2, S_3, S_4, S_5\}$$

The historical 52-week demand data are smoothed and classified into states to form a state sequence, such as:

$$\{S_1, S_2, S_2, S_3, S_3, S_4, S_4, S_4, S_3, S_2, \dots, S_2\}$$

A total of 51 consecutive state-transition pairs are obtained for subsequent frequency statistics.

State transition frequency table

From the state sequence, the number of transitions from the current state S_i to the next period state S_j is counted to construct the transition-frequency matrix $N = (n_{ij})$. Here, n_{ij} denotes the number of transitions from state S_i to S_j. The statistical results are shown in Table 5.

Table 5 Frequency table of state transitions

Current status S	Convert to S_1	Convert to S_2	Convert to S_3	Convert to S_4	Convert to S_5	Total
S_1	0	3	0	0	0	3
S_2	2	11	5	0	0	18
S_3	0	5	2	5	0	12
S_4	0	0	5	8	2	15
S_5	0	0	0	2	1	3
Total	2	19	12	15	3	51

Calculation of the transition probability matrix

According to the definition of a Markov chain, the state transition probability matrix $P = (p_{ij})$ can be obtained by normalizing the frequency matrix:

$$p_{ij} = \frac{n_{ij}}{\sum_{j=1}^{3} n_{ij}}$$

Based on this, the transition probability matrix (rounded to three decimal places) is:

$$P = \begin{bmatrix} 0.000 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.111 & 0.611 & 0.278 & 0.000 & 0.000 \\ 0.000 & 0.417 & 0.166 & 0.417 & 0.000 \\ 0.000 & 0.000 & 0.333 & 0.534 & 0.133 \\ 0.000 & 0.000 & 0.000 & 0.667 & 0.333 \end{bmatrix}$$

The sum of each row is 1, which is consistent with the properties of a Markov transition matrix.

4.3 [*L, U*] Inventory strategy setting and optimal Newsvendor ordering decision analysis

Based on the simulated demand distribution obtained using the Markov chain forecasting process, this section further applies the Newsvendor model to determine the optimal ordering decision under the $[L, U]$ inventory boundary framework. The purpose of this section is to conduct a numerical validation of the proposed MC-NM model and compare its decision results with those of the traditional static Newsvendor model.

The cost parameters and inventory parameters used in this section are assumed values selected for the simulation scenario. They are not drawn directly from the accounting records of a real enterprise. However, the relative cost relationships are set according to the operational characteristics of electronics manufacturing. In particular, the unit stockout cost is assumed to be much higher than the unit holding cost, because production interruption and emergency procurement usually cause greater losses than ordinary inventory storage. The initial inventory, fixed ordering cost, unit procurement cost, unit holding cost, and unit stockout cost are therefore used as scenario parameters for numerical analysis.

Under this simulation setting, the demand state of the 52nd week is assumed to be S_2. Based on the estimated transition probability matrix, the next-period demand distribution is predicted and then used as the input of the Newsvendor model. The $[L, U]$ inventory strategy is introduced to constrain the final ordering decision, ensuring that the calculated order quantity is not only cost-minimizing in theory but also feasible within the assumed inventory boundary. The comparison with the traditional Newsvendor model is conducted under the same simulated cost and inventory parameter settings, so that the performance difference can be attributed to whether dynamic Markov-based demand forecasting is incorporated.

Parameter settings

To enhance the responsiveness of the ordering strategy, the target control range for inventory levels is set to $[L, U]$, meaning that the total inventory level at any given time must be controlled within this range to avoid the extreme risks of stockout losses due to insufficient inventory or a sharp increase in storage costs due to inventory exceeding the storage-capacity limit. Based on the characteristics of raw material management in the electronics manufacturing industry, the following cost parameters and inventory constraints are set, as shown in Table 6.

Table 6 Assumed cost parameters for the simulation case

Parameters	Notation	Numerical value	Economic meaning
Unit procurement cost	K	50 yuan/unit	Purchase price per unit of raw material M
Unit holding cost	C_1	8 yuan/unit/week	Storage, insurance, and capital occupation costs
Unit stockout cost	C_2	120 yuan/unit	Production downtime losses and emergency procurement premiums
Fixed order fee	C_3	200 yuan/time	Logistics and management costs (irrelevant to order volume)
Initial inventory	I	45 units	Assumed initial inventory level at the end of week 52

Parameter rationality explanation: The assumed unit stockout cost is $C_2 = 120$ yuan/unit, which is much higher than the unit holding cost C_1= 8 yuan/unit/week. This reflects the importance of production continuity in electronics manufacturing. In this type of production environment, losses caused by production interruption and emergency procurement are usually much greater than ordinary inventory storage costs. The ratio $C_2/K = 2.4$ indicates that the potential loss caused by one unit of stockout is 2.4 times the unit procurement cost. Therefore, the assumed cost structure is consistent with the characteristics of production-critical raw materials in electronics manufacturing.

Demand-distribution forecast

In the simulation scenario, the demand state in week 52 is S_2. Therefore, the unit state vector can be constructed as follows:

$$e_{S_2} = [0,1,0,0,0]$$

Based on this, we can predict the demand distribution for the next cycle:

$$\pi_{t+1} = \pi_t \cdot P = e_{S_2} \cdot P$$

$$\pi_{53} = e_{S_2} \times P = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \begin{bmatrix} 0.000 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.111 & 0.611 & 0.278 & 0.000 & 0.000 \\ 0.000 & 0.417 & 0.166 & 0.417 & 0.000 \\ 0.000 & 0.000 & 0.333 & 0.534 & 0.133 \\ 0.000 & 0.000 & 0.000 & 0.667 & 0.333 \end{bmatrix}$$
$$= [0.111 \quad 0.611 \quad 0.278 \quad 0.000 \quad 0.000]$$

Thus, the probability of demand in the next period being in state S_1 is 11.1 %, in state S_2 is 61.1 %, in state S_3 is 27.8 %, and is 0 % for states S_4 and S_5.

The forecast results are used as dynamic distribution inputs to the Newsvendor model to calculate the expected inventory cost function and determine the optimal order quantity.

Critical-ratio calculation and solution for the inventory upper limit U and lower limit L

The critical ratio is calculated from the cost parameters of the Newsvendor model:

$$\gamma = \frac{C_2}{C_1 + C_2} = \frac{120}{8 + 120} = 0.9375$$

The demand distribution is converted into a cumulative probability distribution, as shown in Table 7.

Table 7 Cumulative probability distribution

Demand ceiling	60 units	90 units	120 units	150 units	> 150 units
Status	S_1	S_2	S_3	S_4	S_5
Probability density	0.111	0.611	0.278	0	0
Cumulative probability $\sum P(R)$	0.111	0.722	1.000	1.000	1.000

Calculation of inventory upper limit U:

U denotes the target inventory upper limit. When the initial inventory $I < L$, the order quantity $Q = U - I$. In this study, U is calculated using the critical ratio method of the Newsvendor model together with the demand distribution predicted by the Markov chain. The specific steps for calculating U are as follows:

Based on the demand distribution forecast results above, we can conclude that:

$$P(R \leq 60) = 0.111, P(R \leq 90) = 0.722, P(R \leq 120) = 1.000$$

The inventory upper limit U is the minimum U satisfying $P(R \leq U) \geq \gamma$. Based on the cumulative probability distribution:

$$P(R \leq 90) = 0.722 < 0.9375$$
$$P(R \leq 120) = 1.000 \geq 0.9375$$

Therefore, $U = 120$ units is the theoretical optimal value for the upper limit of inventory U (i.e., the theoretical optimal order quantity $Q*$ in the Newsvendor model, as detailed below).

Calculation of the lower bound of inventory L:

L denotes the minimum inventory level that triggers an order. When $I \geq L$, no order is placed. To determine L, it is necessary to compare the expected costs at different inventory levels to ensure that:

$$E[C_T(I = L)] \leq E[C_T(I = L + 1)] \tag{5}$$

where $E[C_T(I = L)]$ represents the expected total cost under the no-order condition when the initial inventory is L, and $E[C_T(I = L + 1)]$ represents the expected total cost under the no-order condition when the initial inventory is $(L + 1)$. Requiring the lower inventory bound L to satisfy Eq. 5 means that, when L is chosen as the lower bound, its expected cost should not exceed the cost of adding one unit of inventory $(L + 1)$. This ensures that L is the cost-optimal trigger point. In other words, L should be the optimal value immediately before additional inventory no longer reduces the expected cost.

As noted above, Eq. 4 applies only when no purchase order is placed in the current stage (i.e., C_3= 0, $I \geq L$). Therefore, the left side of the equation represents the total expected cost when the initial inventory $I = L$ (no purchase order, $Q = 0$).

The value of L can be determined using Eq. 4. Since the upper bound $U = 120$ units has already been calculated, the value of L is restricted to candidate values of 45 units, 75 units, or 105 units. The specific determination process is as follows:

The calculation gives the lower inventory limit L as 75 units. See Appendix D for the specific determination process.

In summary, based on the $[L, U]$ inventory strategy logic described in this paper and the above calculations, the final lower inventory limit L is determined to be 75 units, indicating that when the initial inventory $I \geq 75$ units, no purchase order is required; when $I < 75$ units, an order is required to reach the upper inventory limit $U = 120$ units.

Optimal order quantity and expected cost analysis

Using Eq. (C-3) and Eq. (C-4), together with Table 7, the optimal order quantity $Q*$ is obtained as the minimum Q satisfying the condition: $\sum P(Q^*) \geq \gamma$:

$$P(90) = 0.722 < 0.9375, \quad P(120) = 1.0 \geq 0.9375$$

It follows that the theoretically optimal order quantity $Q*$ = 120 units.

As shown in Table 6, the initial inventory $I = 45$ units < $L = 75$ units, which satisfies the ordering condition and triggers a raw-material order. The target inventory level is the upper inventory bound U, that is:

$$Target\ level = U = 120 \text{ units}$$

Therefore, the actual order quantity is:

$$Q = max(Target\ level - I, 0) = 120 - 45 = 75 \text{ units}$$

The theoretical optimal solution obtained using the Newsvendor model is $Q^* = 120$ units. After applying the inventory upper-limit constraint, the actual order quantity is $Q = 75$ units.

It should be noted that Q represents the replenishment quantity, while $(I + Q)$ represents the total available inventory after ordering. Therefore, the expected holding cost and expected stockout cost should be calculated using the post-order available inventory level $(I + Q)$, rather than the replenishment quantity Q alone. Under the current decision, the available inventory after ordering is 120 units.

According to the Markov chain prediction results, the next-period demand takes the values 45, 75, and 105 units with probabilities 0.111, 0.611, and 0.278, respectively, while the probabilities of demand states above 120 units are zero. Therefore, no expected stockout cost occurs under the MC-NM decision. The expected total cost is calculated as follows:

$$\begin{aligned} C_T(Q) &= KQ + C_3 + \sum_{s_j < I+Q} C_1\left(I + Q - s_j\right)p\left(s_j\right) + \sum_{s_j > I+Q} C_2\left[s_j - (I + Q)\right]p\left(s_j\right) \\ &= 50 \times 75 + 200 + 8[(120 - 45) \times 0.111 + (120 - 75) \times 0.611 + (120 - 105 \times 0.278)] + 120 \times 0 \\ &= 3750 + 200 + 319.92 = 4269.92 \text{ yuan} \end{aligned}$$

In the total cost $C_T(Q^*)$, KQ represents the procurement cost, C_3 represents the fixed ordering cost, $\sum_{s_j<I+Q} C_1(I + Q - s_j)\, p(s_j)$ represents the expected holding cost, and $\sum_{s_j>I+Q} C_2[s_j - (I + Q)]p(s_j)$ represents the expected stockout cost. Therefore, the expected total cost under the MC-NM joint decision-making model is 4269.92 yuan.

4.4 Comparative analysis

The traditional Newsvendor model assumes that the demand probability distribution is known, fixed, and does not change over time or with decisions. It typically bases ordering decisions on the overall distribution of historical data. The frequency of each demand state can be obtained from the state labels shown in Table 4. From these frequencies, the demand probability and cumulative distribution probability of each state can be obtained. Table 8 shows the demand state distribution and probability details of raw material M in the simulated numerical case under the traditional Newsvendor model.

Table 8 Distribution and probability of raw material M

Demand status S	Frequency	Probability $P(R)$	Cumulative probability $\sum P(R)$	Demand quantity R value (units)
S_1	3	0.059	0.059	45
S_2	19	0.373	0.432	75
S_3	12	0.235	0.667	105
S_4	15	0.294	0.961	135
S_5	2	0.039	1.000	165

From the above, we know that the critical ratio $\gamma = 0.9375$, and the optimal order quantity Q^* should satisfy:

$$\sum P(R) \geq \gamma = 0.9375$$

From the data in Table 8, when $R = 135$ units, $\sum P(R) = 0.961 \geq \gamma = 0.9375$, so the optimal order quantity Q^* under the traditional Newsvendor model is 135 units.

However, due to the constraints of the $[L, U]$ inventory strategy, orders can only be placed up to the inventory upper limit of U = 120 units. Since the initial inventory $I = 45$ units, the order quantity $Q = U - I = 120 - 45 = 75$ units. The available stock after ordering is 120 units.

The total cost at this time is:

$$\begin{aligned} C_T(Q) &= KQ + C_3 + \sum_{s_j<I+Q} C_1\left(I + Q - s_j\right)p\left(s_j\right) + \sum_{s_j>I+Q} C_2\left[s_j - (I + Q)\right]p\left(s_j\right) \\ &= 50 \times 75 + 200 + 8[(120 - 45) \times 0.059 + (120 - 75) \times 0.373 \\ &\quad + (120 - 105) \times 0.235] + 120[(135 - 120) \times 0.294 + (165 - 120) \times 0.039] \\ &= 3750 + 200 + 197.88 + 739.80 = 4887.68 \text{ yuan} \end{aligned}$$

The comparison shows that the traditional Newsvendor model based on the static demand distribution assumption produces a less favorable inventory decision, with an expected inventory cost of 4887.68 yuan. The MC-NM model proposed in this paper combines Markov chain prediction and $[L, U]$ inventory constraints, and the expected inventory cost decreases to 4269.92 yuan, with a cost reduction of 12.6 %. This result shows that the MC-NM model can better capture demand changes under dynamic conditions. It can also support more effective ordering strategies and reduce inventory risk and capital pressure in uncertain environments. For the electronics manufacturing industry and other enterprises with pronounced demand fluctuations and strong dependence on raw materials, the model can not only improve the analytical basis and flexibility of inventory decision-making but also provide a practical methodological pathway for enterprises to use ERP-type demand records in dynamic inventory management.

4.5 Robustness and general applicability discussion

The robustness and general applicability of the proposed MC-NM framework are mainly related to demand-state classification, transition probability estimation, inventory cost parameters, and data availability. First, the framework is applicable to raw material demand with seasonal, cyclical, or fluctuating patterns, as long as the demand can be divided into a finite number of states. When the demand pattern changes, the state boundaries and transition matrix can be updated without changing the basic structure of the model. Second, the transition probability matrix directly affects the predicted demand distribution. If the transition probabilities indicate a higher likelihood of moving to high-demand states, the model tends to generate a higher order-up-to level. Conversely, if demand is more likely to remain in low or medium states, the ordering decision becomes more conservative.

Inventory cost parameters also affect the final ordering decision through the Newsvendor critical ratio and the $[L, U]$ inventory policy. To further examine this effect, a sensitivity analysis was conducted by changing the unit holding cost C_1 and unit stockout cost C_2 while keeping the demand transition matrix and other parameters unchanged. The results are shown in Table 9. To keep the presentation concise, only the sensitivity analysis results are reported in Table 9, while the detailed calculation process is omitted because it follows the same procedure described in Section 4.3 and Appendix D.

The results show that the proposed MC-NM model maintains a lower expected total cost than the traditional Newsvendor model under all tested cost settings. When the unit stockout cost decreases, the cost saving decreases; when the unit stockout cost increases, the cost saving increases. In the high holding-cost scenario, the critical ratio decreases, and the model correspondingly reduces the inventory upper bound and replenishment quantity. This indicates that the proposed framework can respond to changes in inventory cost structures and adjust ordering decisions accordingly.

Overall, the proposed framework is not limited to the specific 52-week simulated case used in this paper. Its core logic can be extended to other raw material procurement scenarios by redefining demand states, re-estimating the transition matrix, and adjusting inventory cost parameters according to practical operating conditions. However, the accuracy of the model still depends on the representativeness of historical demand data and the reasonableness of state classification. Future research may further validate the framework using larger real-world datasets and broader operational settings.

Table 9 Sensitivity analysis under different inventory cost parameters

Hypothetical scenario	C_1	C_2	γ	U	L	Q	MC-NM cost	Traditional Newsvendor cost	Cost saving (%)
Baseline	8	120	0.9375	120	75	75	4269.92	4887.68	12.6
Low stockout cost	8	90	0.9184	120	75	75	4269.92	4702.73	9.2
High stockout cost	8	150	0.9494	120	75	75	4269.92	5072.63	15.8
High holding-cost	50	120	0.7059	90	75	45	3658.40	5224.10	30.0

5. Conclusion

This paper proposes a Markov chain–based Newsvendor (MC-NM) decision framework for raw material ordering under demand uncertainty. The primary scientific contribution of the proposed framework is that it integrates state-transition-based probabilistic demand forecasting with Newsvendor inventory optimization under an $[L, U]$ inventory boundary policy. Unlike the traditional Newsvendor model, which relies on a static historical demand distribution, the proposed framework uses a Markov chain to generate a state-dependent demand probability distribution and then transforms this distribution into an ordering decision. In this way, the model provides a transparent and decision-oriented mechanism for linking dynamic demand forecasting with raw material procurement decisions.

The simulation-based numerical case in an electronics manufacturing scenario shows that the proposed MC-NM model reduces the expected total ordering cost from 4887.68 yuan to 4269.92 yuan, achieving a 12.6 % cost saving compared with the traditional Newsvendor model. The additional sensitivity analysis further indicates that the proposed framework maintains lower expected costs under different inventory cost settings. These results suggest that incorporating demand-state transitions into ordering decisions can improve the alignment between demand fluctuations and procurement planning, while also supporting cost control under changing inventory cost conditions.

However, this study still has several limitations. First, the validation is based on a 52-week simulated demand dataset rather than a large-scale real-world enterprise ERP dataset. Second, the model assumes that demand can be divided into finite states and that the transition probability matrix can be estimated from historical state sequences. Third, several operational assumptions are simplified, such as negligible procurement lead time, no deterioration of raw materials during the cycle, and fixed cost parameters. These assumptions help clarify the model mechanism but may limit the direct generalization of the numerical results.

The framework proposed in this paper can be extended to other industries where raw material demand is volatile, seasonal, or state-dependent, such as electronics manufacturing, machinery manufacturing, chemical materials, and seasonal building materials. In future research, the model should be further validated using larger real-world datasets from ERP or supply chain management systems. Future extensions may also consider multi-period replenishment, multi-item coordination, procurement lead time, and integration with machine-learning-based forecasting methods.

Acknowledgments

This paper is supported by the National Natural Science Foundation of China under contract no. 72364008, 72464008, Guangxi Project of Philosophy and Social Science for Planning no. 24JYF027.

References

[1] Pahinggis, M., Sucita, I.K. (2022). Analysis of implementation supply chain management material procurement effect on construction project performance, *Logic: Jurnal Rancang Bangun dan Teknologi*, Vol. 22, No. 2, 89-96, doi: 10.31940/logic.v22i2.89-96.

[2] Hosseini, S., Ivanov, D., Dolgui, A. (2020). Ripple effect modelling of supplier disruption: Integrated Markov chain and dynamic Bayesian network approach, *International Journal of Production Research*, Vol. 58, No. 11, 3284-3303, doi: 10.1080/00207543.2019.1661538.

[3] Yang, C., Jian, M., Fang, X. (2026). Coordination of material supply chains under multidimensional stochastic risks, *Advances in Production Engineering & Management*, Vol. 21, No. 1, 5-15, doi: 10.14743/apem2026.1.557.

[4] Fan, P., Wang, J., Zhang, L., Zhang, S. (2025). A multi-objective approach for optimizing emergency material locations in natural disasters, *Advances in Production Engineering & Management*, Vol. 20, No. 2, 205-223, doi: 10.14743/apem2025.2.536.

[5] Shinde, A., Mehta, P., Amit, R.K. (2021). An experimental investigation of newsvendor decisions under ambiguity, *International Journal of Production Research*, Vol. 59, No. 19, 5960-5971, doi: 10.1080/00207543.2020.1797206.

[6] Li, M., Carter, A., Goldstein, J., Hawco, T., Jensen, J., Vanberkel, P. (2021). Determining ambulance destinations when facing offload delays using a Markov decision process, *Omega*, Vol. 101, Article No. 102251, doi: 10.1016/j.omega.2020.102251.

[7] Chen, H., Chen, H., Zhang, W., Yang, C., Cui, H. (2021). Research on marketing prediction model based on Markov prediction, *Wireless Communications and Mobile Computing*, Vol. 2021, No. 1, Article No. 4535181, doi: 10.1155/2021/4535181.

[8] Więcek, P., Kubek, D. (2024). The impact time series selected characteristics on the fuel demand forecasting effectiveness based on autoregressive models and Markov chains, *Energies*, Vol. 17, No. 16, Article No. 4163, doi: 10.3390/en17164163.

[9] Lin, Z.-P., Ho, S.-P. (2021). Supply chain inventory model with Markov chain demand, *Journal of Marine Science and Technology–Taiwan,* Vol. 29, No. 4, 498-513, doi: 10.51400/2709-6998.1585.

[10] Chen, F., Song, J.-S. (2001). Optimal policies for multiechelon inventory problems with Markov-modulated demand, *Operations Research*, Vol. 49, No. 2, 226-234, doi: 10.1287/opre.49.2.226.13528.

[11] Yuna, F., Erkayman, B., Yılmaz, M. (2025). Markov approach for inventory control with meta-heuristics in intermittent demand environment, *PLOS One*, Vol. 20, No. 6, Article No. e0325658, doi: 10.1371/journal.pone.0325658.

[12] D'Amico, G., Karagrigoriou, A., Vigna, V. (2024). Forecasting the power generation mix in Italy based on grey Markov models, *Energies*, Vol. 17, No. 9, Article No. 2184, doi: 10.3390/en17092184.

[13] Karakoyun, E.C., Avci, H., Kocaman, A.S., Nadar, E. (2023). Deviations from commitments: Markov decision process formulations for the role of energy storage, *International Journal of Production Economics*, Vol. 255, Article No. 108711, doi: 10.1016/j.ijpe.2022.108711.

[14] Owusu, F.K., Amoako-Yirenkyi, P., Frempong, N.K., Omari-Sasu, A.Y., Mensah, I.A., Martin, H., Sakyi, A. (2023). Seemingly unrelated time series model for forecasting the peak and short-term electricity demand: Evidence from the Kalman filtered Monte Carlo method, *Heliyon*, Vol. 9, No. 8, Article No. e18821, doi: 10.1016/j.heliyon.2023.e18821.

[15] Havinga, M.J.A., de Jonge, B. (2020). Condition-based maintenance in the cyclic patrolling repairman problem, *International Journal of Production Economics*, Vol. 222, Article No. 107497, doi: 10.1016/j.ijpe.2019.09.018.

[16] Hashemi-Petroodi, S.E., Thevenin, S., Kovalev, S., Dolgui, A. (2023). Markov decision process for multi-manned mixed-model assembly lines with walking workers, *International Journal of Production Economics*, Vol. 255, Article No. 108661, doi: 10.1016/j.ijpe.2022.108661.

[17] Su, C., Wu, L. (2024). Opportunistic maintenance optimisation for offshore wind farm with considering random wind speed, *International Journal of Production Research*, Vol. 62, No. 5, 1862-1878, doi: 10.1080/00207543.2023.2202280.
[18] Ni, G. (2023). Optimal decisions on price and inventory for a newsboy-type retailer with identifiable information and discount promotion, *PLOS One,* Vol. 18, No. 7, Article No. e0288874, doi: 10.1371/journal.pone.0288874.
[19] Lei, L., Ru, J., Shi, R., Zhang, J. (2022). A two-product newsvendor problem with partial demand substitution, *Production and Operations Management*, Vol. 31, No. 3, 1157-1173, doi: 10.1111/poms.13599.
[20] Puranam, K., Novak, D.C., Lucas, M. (2022). Extending the newsvendor model to account for uncontrolled inventory transfers, *Annals of Operations Research*, Vol. 317, No. 1, 213-226, doi: 10.1007/s10479-015-2080-5.
[21] Oroojlooyjadid, A., Snyder, L.V., Takáč, M. (2020). Applying deep learning to the newsvendor problem, *IISE Transactions*, Vol. 52, No. 4, 444-463, doi: 10.1080/24725854.2019.1632502.
[22] Zhang, Y., Gao, J. (2017). Assessing the performance of deep learning algorithms for newsvendor problem, In: *Proceedings of 24th International Conference, ICONIP 2017,* Guangzhou, China, 912-921, doi: 10.1007/978-3-319-70087-8_93.
[23] Gonzalez, J., Avelar Sosa, L., Bravo, G., Cruz-Mejia, O., Mejia-Muñoz, J.-M. (2024). Fog computing and industry 4.0 for newsvendor inventory model using attention mechanism and gated recurrent unit, *Logistics*, Vol. 8, No. 2, Article No. 56, doi: 10.3390/logistics8020056.
[24] Hua, G., Zhang, Y., Cheng, T.C.E., Wang, S., Zhang, J. (2020). The newsvendor problem with barter exchange, *Omega*, Vol. 92, Article No. 102149, doi: 10.1016/j.omega.2019.102149.
[25] Zhu, X., Li, K.W., Guo, P. (2023). A bilevel optimization model for the newsvendor problem with the focus theory of choice, *4OR*, Vol. 21, No. 3, 471-489, doi: 10.1007/s10288-022-00520-6.
[26] Sousa, J.M.C., Luís, R., Santos, R.M., Mendonça, L., Vieira, S.M. (2024). Fuzzy multi-item newsvendor problem: An application to inventory management, *Mathematics*, Vol. 12, No. 11, Article No. 1652, doi: 10.3390/math12111652.
[27] Pirayesh Neghab, D., Khayyati, S., Karaesmen, F. (2022). An integrated data-driven method using deep learning for a newsvendor problem with unobservable features, *European Journal of Operational Research*, Vol. 302, No. 2, 482-496, doi: 10.1016/j.ejor.2021.12.047.
[28] Sun, Y.Y., Yang, J.H., Zhai, L.Y., Liu, N. (2025). Enhanced product defect forecasting using partitioned attributes and ensemble machine learning, *Advances in Production Engineering & Management*, Vol. 20, No. 2, 157-172, doi: 10.14743/apem2025.2.533.
[29] Sprenkels, L., Atan, Z., Adan, I. (2025). Multi-product newsvendor problem with dynamic pricing: A decision interpolation approximation, *Optimization Letters*, Vol. 19, No. 4, 699-716, doi: 10.1007/s11590-025-02185-6.
[30] Keskin, N.B., Li, C., Song, J.-S. (2024). The blockchain newsvendor: Value of freshness transparency and smart contracts, *Management Science*, Vol. 71, No. 8, 6666-6682, doi: 10.1287/mnsc.2021.02949.
[31] Ji, R., Kamrad, B. (2019). Newsvendor model as an exchange option on demand and supply uncertainty, *Production and Operations Management*, Vol. 28, No. 10, 2456-2470, doi: 10.1111/poms.13056.
[32] Cao, T., Yang, Y., Zhu, H., Yu, M. (2025). The big data newsvendor problem under demand and yield uncertainties, *International Journal of Production Economics*, Vol. 279, Article No. 109409, doi: 10.1016/j.ijpe.2024.109409.
[33] Käki, A., Liesiö, J., Salo, A., Talluri, S. (2015). Newsvendor decisions under supply uncertainty, *International Journal of Production Research*, Vol. 53, No. 5, 1544-1560, doi: 10.1080/00207543.2014.952798.
[34] Ghosh, S., Mukhoti, S. (2023). Non-parametric generalised newsvendor model, *Annals of Operations Research*, Vol. 321, No. 1-2, 241-266, doi: 10.1007/s10479-022-05112-5.

Appendix A

Mathematical derivation and calculation details for Markov chain demand forecasting

In the main text, Eq. 1 states that the probability of $X_{t+1} = s$ conditioned on $X_t = s_t, X_{t-1} = s_{t-1}, \dots, X_0 = s_0$, equals the probability of $X_{t+1} = s$ conditioned on $X_t = s_t$.

This formula represents the core property of Markov chains—the Markov property—which states that the future state $X_{t+1} = s$ of the system depends only on the current state X_t and is independent of previous states $X_{t-1}, X_{t-2}, \dots, X_0$.

Definition 1: State space. The state space is the set of all possible system states, denoted as $S = \{s_1, s_2, \dots, s_n\}$. The state space may be finite or infinite (e.g., in continuous-state systems).

Definition 2: Transition probability matrix. Transition probabilities describe the probability of a system transitioning from one state to another. For the transition probability from state s_i to state s_j, denoted as $P(s_i \to s_j)$ or p_{ij}, the following conditions are satisfied:

$$\sum_j p_{ij} = 1 \tag{A-1}$$

The sum of the transition probabilities for each state is 1. Transition probabilities are usually represented by a transition probability matrix P:

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix} \tag{A-2}$$

Definition 3: Initial state distribution. The initial state distribution represents the state distribution of the system at the initial time $t = 0$, denoted as $\pi_0 = [\pi_0(s_1), \pi_0(s_2), \ldots, \pi_0(s_n)]$. It specifies the probability associated with ~~a~~each possible initial state.

In actual business operations, raw material demand often exhibits temporal fluctuations, with different periods potentially corresponding to different demand "states." Therefore, Markov chains can serve as an effective modeling tool. Assume that all possible raw material demand states constitute a discrete state space $S = \{s_1, s_2, \ldots, s_n\}$, where each state si represents a specific demand level interval. Based on the transition frequency between states in historical data, companies can construct a transition probability matrix $P = [p_{ij}]$, where:

$$p_{ij} = P(X_{t+1} = s_j | X_t = s_i)$$

and $\sum_{j=1}^{n} p_{ij} = 1$. When the system is in state s_i at time t, the probability of transitioning to state s_j at the next time step is determined by the transition matrix. Given the current state distribution vector π_t, the next-cycle predicted distribution is:

$$\pi_{t+1} = \pi_t P \tag{A-3}$$

Using this recursive formula, companies can dynamically predict the possible demand levels and their associated probabilities for the next period based on the known current demand state distribution π_t and state transition rules P, thereby providing data input for inventory optimization models.

In addition, to facilitate calculation and decision-making, this paper further uses a "unit state vector" to represent the current exact state of the prediction system. Let $e_i \in R_n$ denote a unit vector with a value of 1 at the i-th position and 0 at all other positions, indicating that the current system is in state s_i. Then, the state distribution predicted for the next cycle can be simplified as follows:

$$\pi_{t+1} = e_i P \tag{A-4}$$

This method makes the state prediction process of Markov chains more tractable and structurally clear, and provides probabilistic inputs derived from state evolution for subsequent Newsvendor models, enabling the entire inventory optimization process to have a logical closed loop of prediction-driven decision-making.

Appendix B

In the ordering rules of raw materials under the [L, U] inventory strategy, the procedure for determining the upper inventory limit U is as follows.

From Eq. 2 in the text:

$$C_T(U_{k+1}) = C_3 + K(U_{k+1} - I) + \sum_{R \le U_{k+1}} C_1\,(U_{k+1} - R)p(R) + \sum_{R > U_{k+1}} C_2\,(R - U_{k+1})p(R)$$

$$C_T(U_k) = C_3 + K(U_k - I) + \sum_{R \le U_k} C_1\,(U_k - R)p(R) + \sum_{R > U_k} C_2\,(R - U_k)p(R)$$

$$C_T(U_{k-1}) = C_3 + K(U_{k-1} - I) + \sum_{R \le U_{k-1}} C_1\,(U_{k-1} - R)p(R) + \sum_{R > U_{k-1}} C_2\,(R - U_{k-1})p(R)$$

To obtain the minimum U-value for $C_T(U_k)$, U_{k+1} should satisfy the following inequality:

$$C_T(U_{k+1}) \ge C_T(U_k) \tag{B-1}$$

$$C_T(U_{k-1}) \ge C_T(U_k) \tag{B-2}$$

Eq. (B-1) gives:

$$\begin{aligned}
&C_T(U_{k+1}) - C_T(U_k) \\
&= \left(\left\{C_3 + K(U_{k+1} - I) + \sum_{R \le U_{k+1}} C_1\,(U_{k+1} - R)p(R) + \sum_{R > U_{k+1}} C_2\,(R - U_{k+1})p(R)\right\} - \right. \\
&\qquad \left.\left\{C_3 + K(U_k - I) + \sum_{R \le U_k} C_1\,(U_k - R)p(R) + \sum_{R > U_k} C_2\,(R - U_k)p(R)\right\}\right) \\
&= \left(\left\{C_3 + K(U_{k+1} - I) + \sum_{R \le U_k} C_1\,(U_{k+1} - R)p(R) + \sum_{R \ge U_{k+1}} C_2\,(R - U_{k+1})p(R)\right\} - \right. \\
&\qquad \left.\left\{C_3 + K(U_k - I) + \sum_{R \le U_k} C_1\,(U_k - R)p(R) + \sum_{R > U_k} C_2\,(R - U_k)p(R)\right\}\right) \\
&= K(U_{k+1} - U_k) + C_1(U_{k+1} - U_k) \sum_{R \le U_k} p\,(R) - C_2(U_{k+1} - U_k) \sum_{R > U_k} p\,(R) \\
&= (U_{k+1} - U_k)\left\{K + C_1 \sum_{R \le U_k} p\,(R) - C_2 \sum_{R > U_k} p\,(R)\right\} \ge 0
\end{aligned}$$

Because $(U_{k+1} - U_k) = R_{k+1} - R_k > 0$, we obtain $K + C_1 \sum_{R \le U_k} p\,(R) - C_2 \sum_{R > U_k} p\,(R) \ge 0$, i.e.:

$$\begin{aligned}
K + C_1 \sum_{R \le U_k} p\,(R) - C_2 \sum_{R > U_k} p\,(R) &= K + C_1 \sum_{R \le U_k} p\,(R) - C_2 \left\{1 - \sum_{R \le U_k} p\,(R)\right\} \\
&= K + (C_1 + C_2) \sum_{R \le U_k} p\,(R) - C_2 \ge 0
\end{aligned}$$

Therefore:

$$\sum_{R \le U_k} p\,(R) \ge \frac{C_2 - K}{C_1 + C_2} \tag{B-1}$$

From Eq. (B-2):

$$\begin{aligned}
&C_T(U_{k-1}) - C_T(U_k) \\
&= \left(\left\{C_3 + K(U_{k-1} - I) + \sum_{R \le U_{k-1}} C_1\,(U_{k-1} - R)p(R) + \sum_{R > U_{k-1}} C_2\,(R - U_{k-1})p(R)\right\} - \right. \\
&\qquad \left.\left\{C_3 + K(U_k - I) + \sum_{R \le U_k} C_1\,(U_k - R)p(R) + \sum_{R > U_k} C_2\,(R - U_k)p(R)\right\}\right) \\
&= \left(\left\{C_3 + K(U_{k-1} - I) + \sum_{R \le U_{k-1}} C_1\,(U_{k-1} - R)p(R) + \sum_{R > U_{k-1}} C_2\,(R - U_{k-1})p(R)\right\} - \right. \\
&\qquad \left.\left\{C_3 + K(U_k - I) + \sum_{R \le U_{k-1}} C_1\,(U_k - R)p(R) + \sum_{R \ge U_k} C_2\,(R - U_k)p(R)\right\}\right) \\
&= K(U_{k-1} - U_k) + C_1(U_{k-1} - U_k) \sum_{R \le U_{k-1}} p\,(R) - C_2(U_{k-1} - U_k) \sum_{R > U_{k-1}} p\,(R) \\
&= (U_{k-1} - U_k)\left\{K + C_1 \sum_{R \le U_{k-1}} p\,(R) - C_2 \sum_{R > U_{k-1}} p\,(R)\right\} \ge 0
\end{aligned}$$

Because$(U_{k-1} - U_k) = R_{k-1} - R_k < 0$, we obtain $K + C_1 \sum_{R \le U_{k-1}} p\,(R) - C_2 \sum_{R > U_{k-1}} p\,(R) \le 0$, i.e.:

$$K + C_1 \sum_{R \leq U_{k-1}} p\,(R) - C_2 \sum_{R > U_{k-1}} p\,(R) = K + C_1 \sum_{R \leq U_{k-1}} p\,(R) - C_2 \left\{ 1 - \sum_{R \leq U_{k-1}} p\,(R) \right\}$$
$$= K + (C_1 + C_2) \sum_{R \leq U_{k-1}} p\,(R) - C_2 \leq 0$$

Therefore:

$$\sum_{R \leq U_{k-1}} p\,(R) \leq \frac{C_2 - K}{C_1 + C_2} \tag{B-4}$$

Combining Eq. (B-3) and Eq. (B-4) gives the non-strict bounds for U_k. Since U_k is defined as the smallest discrete candidate for which the cumulative demand probability reaches or exceeds the critical ratio, the cumulative probability at the preceding candidate U_{k-1} must be strictly below the critical ratio. Therefore, the inequality for determining U_k can be written as:

$$\sum_{R \leq U_{k-1}} p\,(R) < \frac{C_2 - K}{C_1 + C_2} \leq \sum_{R \leq U_k} p\,(R) \tag{B-5}$$

Appendix C

Mathematical derivation and calculation details for the optimal Newsvendor ordering decision model

After obtaining the predicted demand distribution $\pi_{t+1} = \{p(s_1), p(s_2), \ldots, p(s_n)\}$, the company needs to determine the order quantity Q at the beginning of the cycle to meet demand while minimizing losses caused by stockouts and overstocking. Given the predicted distribution, the expected total inventory cost function can be written as:

$$C_T(Q) = KQ + C_3 + \sum_{s_j < Q} C_1(Q - s_j)\, p(s_j) + \sum_{s_j > Q} C_2(s_j - Q)\, p(s_j) \tag{C-1}$$

In this expected total inventory cost function, K is the unit procurement cost, C_1 is the unit holding cost (i.e., the holding cost associated with excess raw material inventory), C_2 is the unit stockout cost (i.e., the opportunity loss caused by failure to meet production demand), C_3 is the fixed cost per order, I is the initial inventory quantity, and Q is the raw material order quantity for the current period.

If an inventory limit U is imposed, the order quantity should satisfy $Q = U - I$. The first term represents the procurement cost for the current period; the second term represents the fixed costs of the ordering operation itself; the third term represents the holding cost incurred when the post-order available inventory level $(I + Q)$ exceeds actual demand R, while the fourth term represents the shortage cost incurred when actual demand R exceeds the post-order available inventory level $(I + Q)$.

To minimize costs, the optimal decision minimizes $C_T(Q)$ and determines the raw material order quantity $Q*$:

$$Q^* = arg \min_{Q} C_T(Q) \tag{C-2}$$

If demand is discrete and the probability of each state is known, the critical ratio method can be used for efficient decision-making. The core idea of this method is: if the stockout cost is much greater than the holding cost, a larger order quantity is preferable to avoid stockouts; otherwise, ~~orders~~ordering should be more conservative to reduce inventory risk.

Define the critical ratio:

$$\gamma = \frac{C_2}{C_1 + C_2} \tag{C-3}$$

The critical ratio γ is a trade-off factor between 0 and 1 that reflects the importance a company places on "stockout losses" relative to "excess inventory costs."

Based on this ratio, the optimal order quantity for raw materials can be obtained as follows:

$$\sum_{s_j \leq Q^*} p(s_j) \geq \gamma \tag{C-4}$$

That is, find the minimum order quantity Q^* for which the cumulative probability is not less than the critical ratio. The critical ratio method requires little additional derivation and is straightforward to implement, making it especially suitable for scenarios with discrete demand and clearly defined states.

Appendix D

Calculation of the lower inventory bound L

The calculation in this appendix is used to determine the lower inventory threshold L by comparing different candidate inventory levels. It should be noted that this comparison is different from the final operating cost calculation in Section 4.3. In Section 4.3, the actual procurement cost is calculated according to the actual replenishment quantity Q, namely KQ. In this appendix, however, the calculation is used only to compare candidate values of L under the no-order condition. Therefore, the term $(K \times L)$ represents the material value associated with the candidate inventory level L in the threshold comparison, rather than the actual procurement cost of the current ordering cycle. The actual replenishment quantity still follows the [L, U] policy, namely $Q = U - I$ when $I < L$ and $Q = 0$ when $I \geq L$.

Determination of L is conducted in two steps. First, Eq. 4 is used as a feasibility condition to examine whether a candidate inventory level is economically acceptable under the no-order condition. Second, since more than one candidate may satisfy Eq. 4, Eq. 5 is further used to identify the cost-optimal trigger point. In other words, satisfying Eq. 4 only indicates that a candidate L is feasible; the final lower threshold should be the candidate at which the expected no-order cost reaches the lowest value before it starts to increase again.

First, substitute $U = 120$ units into the right side of Eq. 4 to obtain:

$$C_3 + KU + \sum_{R \leq U} C_1\,(U - R)p(R) + \sum_{R > U} C_2\,(R - U)p(R)$$

$$= 200 + 20 \times 120 + 8 \times [(120 - 45) \times 0.111 + (120 - 75) \times 0.611 + (120 - 105) \times 0.278] + 120 \times 0$$
$$= 200 + 6000 + 8 \times (75 \times 0.111 + 45 \times 0.611 + 15 \times 0.278)$$
$$= 6200 + 8 \times 39.99 = 6200 + 319.92 = 6519.92 \text{ yuan}$$

Next, let $L = 45$ units and substitute it into the left side of Eq. 4 to obtain:

$$KL + \sum_{R \leq L} C_1\,(L - R)p(R) + \sum_{R > L} C_2\,(R - L)p(R)$$
$$= 50 \times 45 + 8 \times 0 + 120[(75 - 45) \times 0.611 + (105 - 45) \times 0.278]$$
$$= 2250 + 120(30 \times 0.611 + 60 \times 0.278) = 2250 + 4201.2 = 6451.2 \text{ yuan}$$

Since 6451.2 yuan $<$ 6519.92 yuan, $L = 45$ units satisfies the condition of Eq. 4.

Next, let $L = 75$ units and substitute it into the left side of Eq. 4 to obtain:

$$KL + \sum_{R \leq L} C_1\,(L - R)p(R) + \sum_{R > L} C_2\,(R - L)p(R) =$$
$$= 50 \times 75 + 8 \times (75 - 45) \times 0.111 + 120 \times (105 - 75) \times 0.278$$
$$= 3750 + 8 \times 30 \times 0.111 + 120 \times 30 \times 0.278 = 3750 + 1027.44 = 4777.44 \text{ yuan}$$

Since 4777.44 yuan $<$ 6519.92 yuan, $L = 75$ units satisfies the condition of Eq. 4.

Next, let $L = 105$ units and substitute it into the left side of Eq. 4 to obtain:

$$KL + \sum_{R \leq L} C_1\,(L - R)p(R) + \sum_{R > L} C_2\,(R - L)p(R)$$
$$= 50 \times 105 + 8 \times [(105 - 45) \times 0.111 + (105 - 75) \times 0.611]$$
$$= 5250 + 8 \times (60 \times 0.111 + 30 \times 0.611) = 5250 + 199.92 = 5449.92 \text{ yuan}$$

Since 5449.92 yuan $<$ 6519.92 yuan, $L = 105$ units satisfies the condition of Eq. 4.

Under the premise of Eq. 5, comparing the results for L values of 45, 75 and 105 units, we obtain:

$E[C_T(45)] = 6451.2 > E[C_T(75)] = 4777.44$, not satisfied.
$E[C_T(75)] = 4777.44 < E[C_T(105)] = 5449.92$, satisfied.

This means that $L = 75$ units is the cost-minimizing point among the feasible candidates. Therefore, $L = 75$ units is selected as the final lower inventory threshold. Under this threshold, when the opening inventory is lower than 75 units, replenishment is triggered and the inventory is restored to the upper level $U = 120$ units; when the opening inventory is greater than or equal to 75 units, no replenishment order is placed in the current cycle.